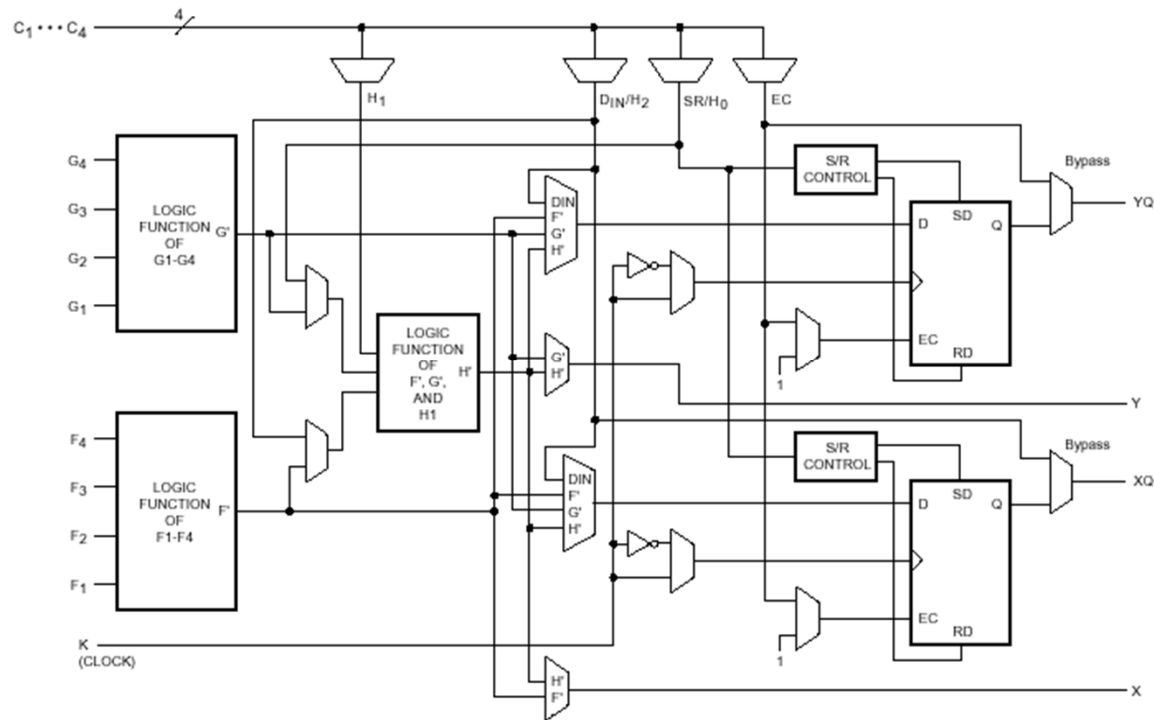

ECE 428 Programmable ASIC Design

FPGA Implementation of Combinational Logic

Haibo Wang
ECE Department
Southern Illinois University
Carbondale, IL 62901

Combinational Logic Implemented by Xilinx XC4000 CLB



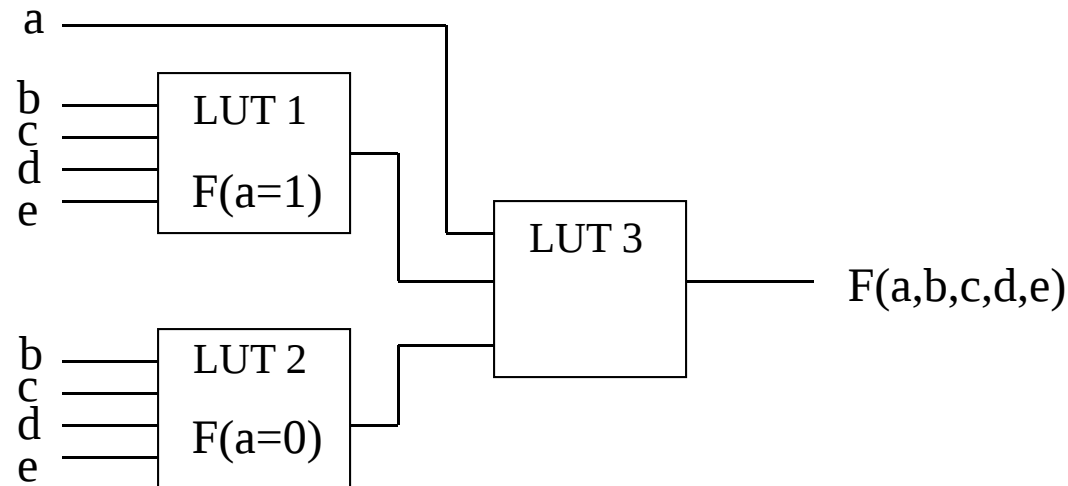
- ❑ Any function of up to four variables, plus any second function of up to four unrelated variables, plus any third function of up to three unrelated variables
- ❑ Any single function of five variables
- ❑ Any function of four variables together with some functions of six variables
- ❑ Some functions of up to nine variables.

Combinational Logic Implemented by Xilinx XC4000 CLB

❑ Why can any five-input function be implemented by a XC4000 CLB?

$$F(a, b, c, d, e) = a \bullet F(a=1) + a' \bullet F(a=0)$$

— Both $F(a=1)$ and $F(a=0)$ are four-input functions



Combinational Logic Implemented by Xilinx XC4000 CLB

- ❑ Any function of four variables together with some functions of six variables can be implemented by a single CLB

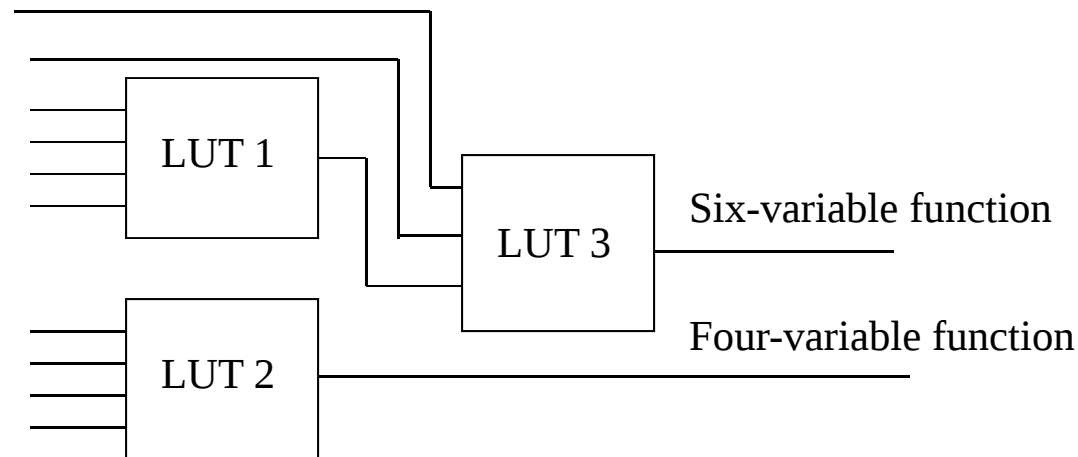
— What kind of six variable functions can be implemented with another four variable function within a single CLB?

$$F(a,b,c,d,e,f) = a \cdot b \cdot F1 + a \cdot b' \cdot F2 + a' \cdot b \cdot F3 + a' \cdot b' \cdot F4$$

$$F1 = F(a=1, b=1); F2 = F(a=1, b=0);$$

$$F3 = F(a=0, b=1); F4 = F(a=0, b=0)$$

Condition: Among F1-F4, three of them are constant (e.g. F1=1, F2=F3=0)



Example: A Six-Input Majority Function

- ❑ **Six-input majority function:** the function output is 1 if three or more than three inputs are 1.

$$F(a, b, c, d, e, f) = \sum_i m_i \quad (m_i \text{ Minterm of function } F)$$

Number of inputs with logic 1	Minterms
6	$a \cdot b \cdot c \cdot d \cdot e \cdot f$
5	$a \cdot b \cdot c \cdot d \cdot e \cdot f' \quad a \cdot b \cdot c \cdot d \cdot e' \cdot f \quad a \cdot b \cdot c \cdot d' \cdot e \cdot f \quad a \cdot b \cdot c' \cdot d \cdot e \cdot f$ $a \cdot b' \cdot c \cdot d \cdot e \cdot f \quad a' \cdot b \cdot c \cdot d \cdot e \cdot f$
4	$a \cdot b \cdot c \cdot d \cdot e' \cdot f' \quad a \cdot b \cdot c \cdot d' \cdot e \cdot f' \quad a \cdot b \cdot c' \cdot d \cdot e \cdot f' \quad a \cdot b' \cdot c \cdot d \cdot e \cdot f'$ $a' \cdot b \cdot c \cdot d \cdot e \cdot f' \quad a' \cdot b \cdot c \cdot d' \cdot e \cdot f' \quad a' \cdot b \cdot c' \cdot d \cdot e \cdot f' \quad a' \cdot b' \cdot c \cdot d \cdot e \cdot f'$ $a' \cdot b \cdot c \cdot d \cdot e' \cdot f' \quad a' \cdot b \cdot c' \cdot d \cdot e \cdot f' \quad a' \cdot b' \cdot c \cdot d \cdot e \cdot f' \quad a' \cdot b' \cdot c \cdot d' \cdot e \cdot f'$ $a \cdot b' \cdot c' \cdot d \cdot e \cdot f' \quad a' \cdot b \cdot c' \cdot d \cdot e \cdot f' \quad a' \cdot b' \cdot c \cdot d \cdot e \cdot f'$
3	$a \cdot b \cdot c \cdot d' \cdot e' \cdot f' \quad a \cdot b \cdot c' \cdot d \cdot e' \cdot f' \quad a \cdot b' \cdot c \cdot d \cdot e' \cdot f' \quad a' \cdot b \cdot c \cdot d \cdot e' \cdot f'$ $a \cdot b \cdot c' \cdot d' \cdot e \cdot f' \quad a \cdot b' \cdot c \cdot d' \cdot e \cdot f' \quad a' \cdot b \cdot c \cdot d' \cdot e \cdot f' \quad a \cdot b' \cdot c' \cdot d \cdot e \cdot f'$ $a' \cdot b \cdot c' \cdot d \cdot e \cdot f' \quad a' \cdot b' \cdot c \cdot d \cdot e \cdot f' \quad a \cdot b \cdot c' \cdot d' \cdot e' \cdot f' \quad a \cdot b' \cdot c \cdot d' \cdot e' \cdot f'$ $a' \cdot b \cdot c \cdot d' \cdot e' \cdot f' \quad a \cdot b' \cdot c' \cdot d \cdot e' \cdot f' \quad a' \cdot b \cdot c' \cdot d \cdot e' \cdot f' \quad a' \cdot b' \cdot c \cdot d \cdot e' \cdot f'$ $a \cdot b' \cdot c' \cdot d' \cdot e \cdot f' \quad a' \cdot b \cdot c' \cdot d' \cdot e \cdot f' \quad a \cdot b' \cdot c \cdot d' \cdot e \cdot f' \quad a' \cdot b' \cdot c' \cdot d \cdot e \cdot f'$

Example: A Six-Input Majority Function

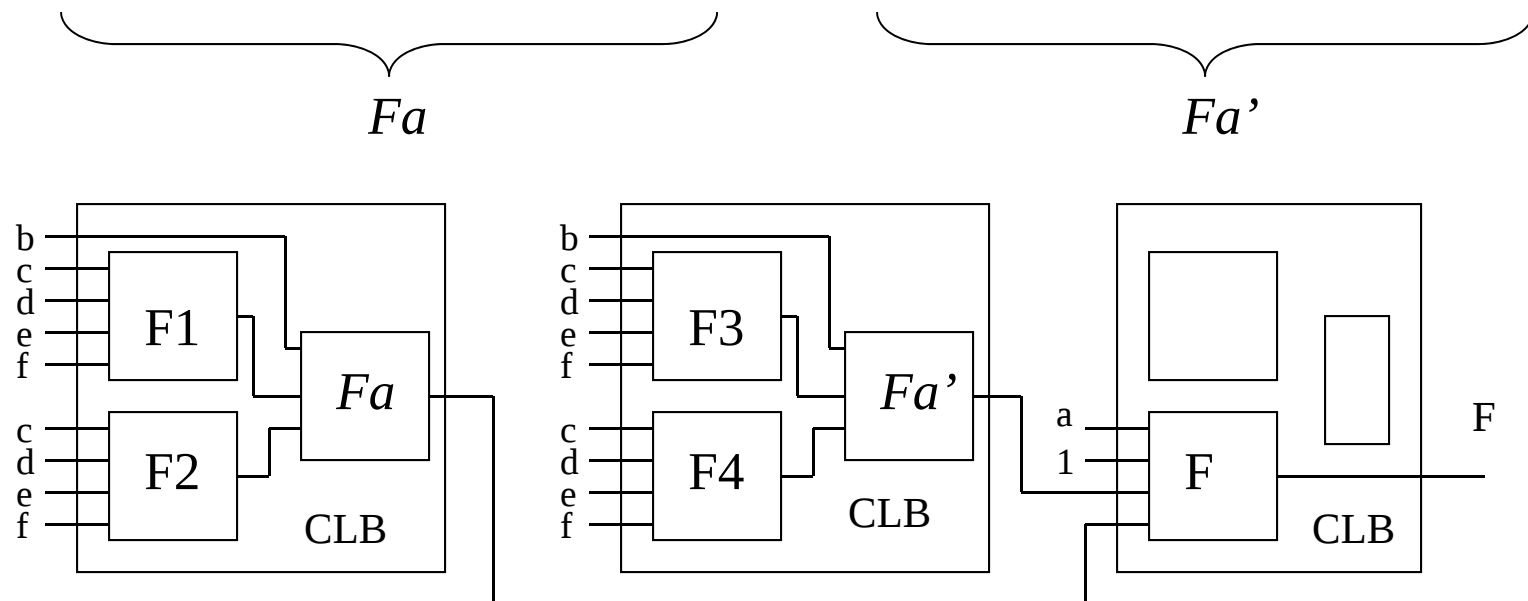
$$F(a=1,b=1) = e + c + f \cdot c' + e' \cdot d \quad (F1)$$

$$F(a=1,b=0) = F(a=0,b=1) = c \cdot d + e \cdot f + d \cdot f + e \cdot d + f \cdot c + e \cdot c \quad (F2, F3)$$

$$F(a=0,b=0) = c \cdot d \cdot f + c \cdot d \cdot e + e \cdot f \cdot c + e \cdot f \cdot d \quad (F4)$$

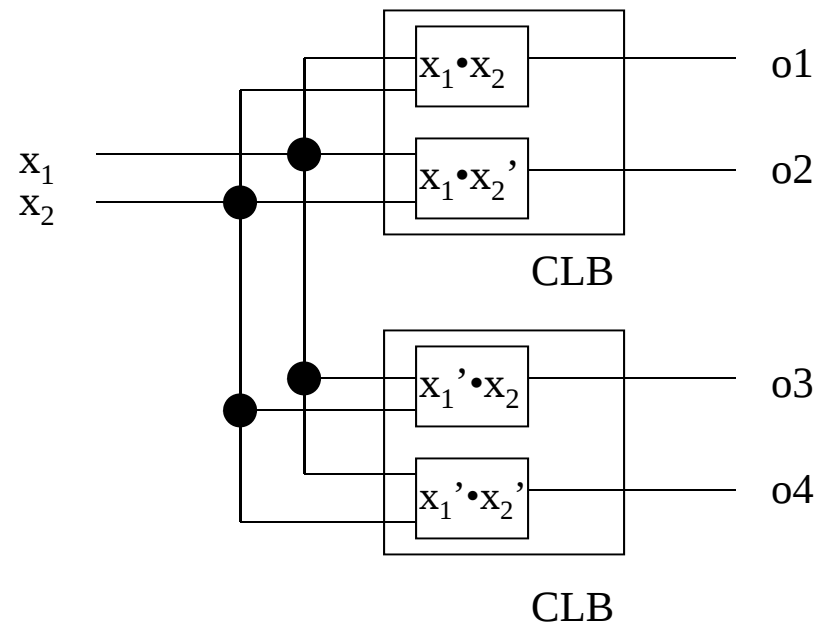
$$F = a \cdot b \cdot F(a=1,b=1) + a \cdot b' \cdot F(a=1,b=0) + a' \cdot b \cdot F(a=0,b=1) + a' \cdot b' \cdot F(a=0,b=0)$$

$$= a \cdot \{b \cdot F(a=1,b=1) + b' \cdot F(a=1,b=0)\} + a' \cdot \{b \cdot F(a=0,b=1) + b' \cdot F(a=0,b=0)\}$$



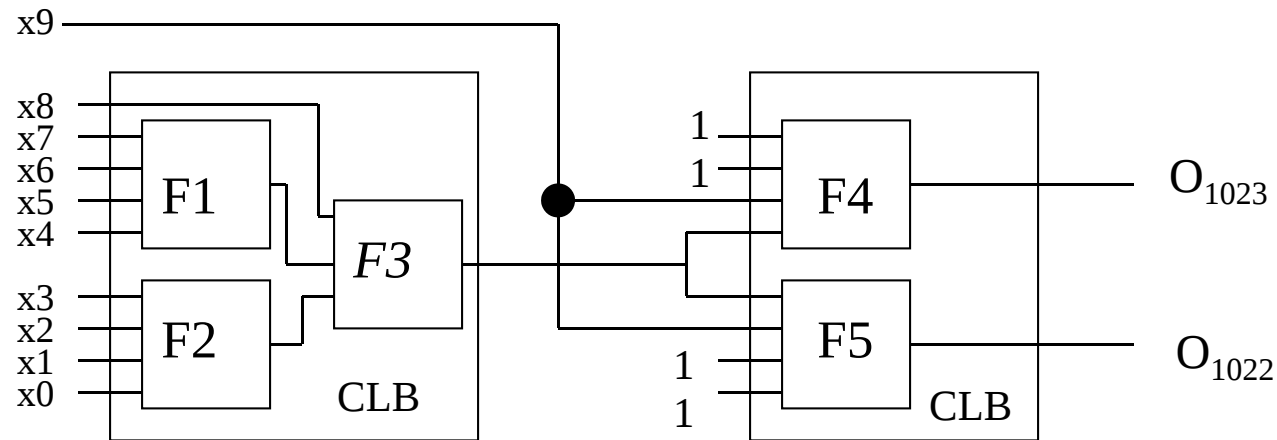
Decoding Circuits

❑ 2-to-4 Decoding circuit



Decoding Circuits

❑ 10-to-1024 Decoding circuit



$$F1 = x4 \cdot x5 \cdot x6 \cdot x7$$

$$F2 = x0 \cdot x1 \cdot x2 \cdot x3$$

$$F3 = x8 \cdot F1 \cdot F2$$

$$F4 = x9 \cdot F3$$

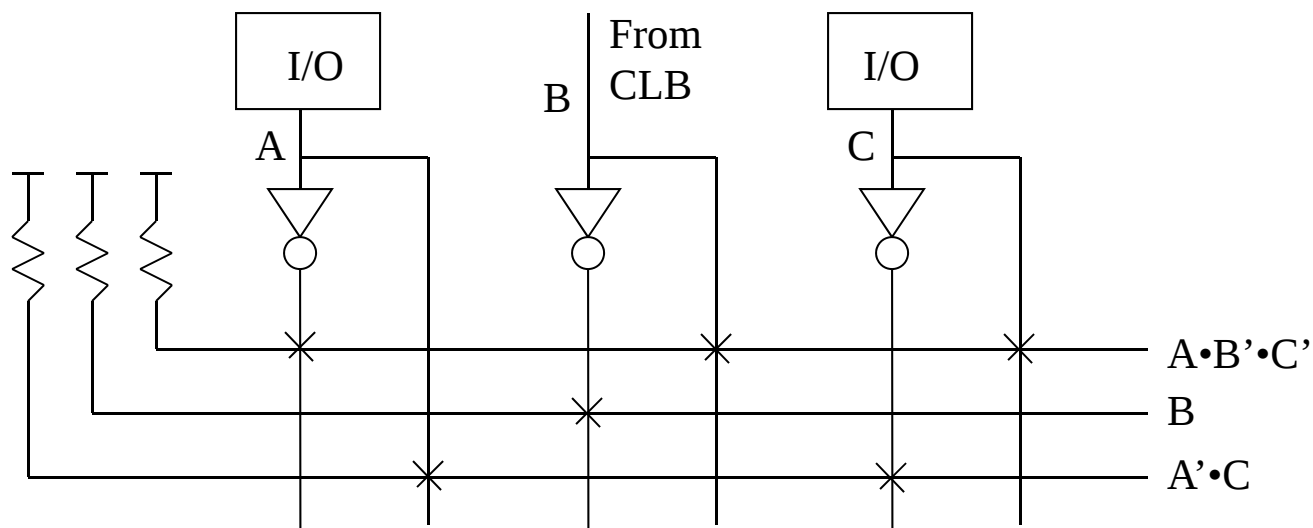
$$F5 = x9' \cdot F3$$

Disadvantages

- It needs 1024 CLBs; expensive to implement.
- It is a two level implementation, resulting large delay.

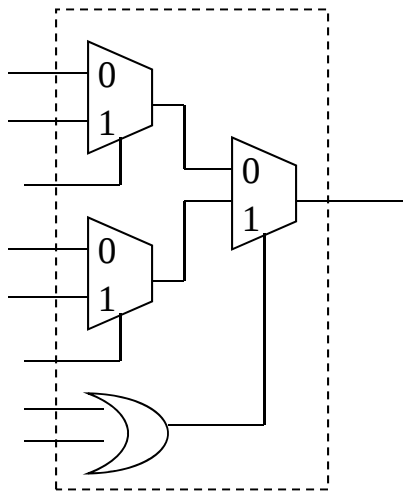
Dedicated Decoding Circuits in Xilinx FPGAs

- ❑ Four dedicated programmable decoding circuits are included in Xilinx FPGAs.
- ❑ The number of decoder inputs ranges from 42 to 132 for different devices.
- ❑ The decoding circuits use wired-AND gate structures (like the AND plane in PAL).

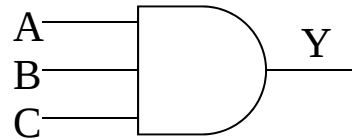


Combinational Logic implemented by Actel ACT1

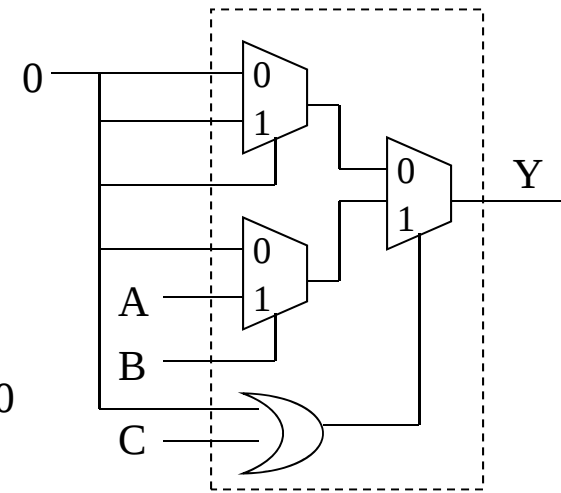
□ Example



Actel ACT1 Cell



$$Y = A \cdot B \cdot C = C \cdot (AB) + C' \cdot 0 \\ = C \cdot (B \cdot A + B' \cdot 0) + C' \cdot 0$$

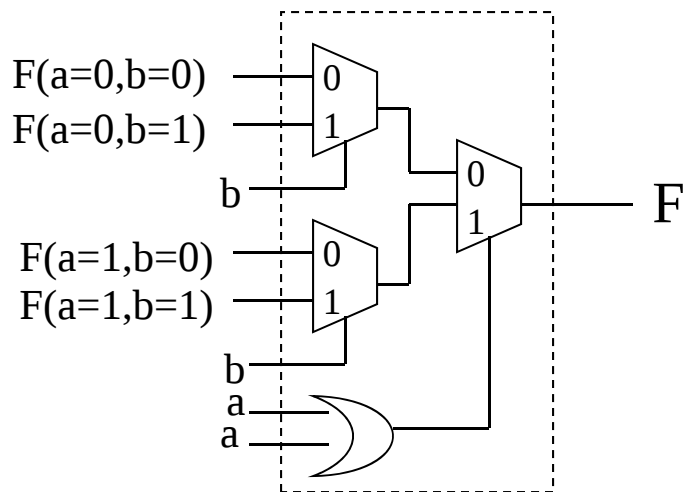


3-input AND gate implementation

Combinational Logic implemented by Actel ACT1

- ❑ For a general three-input function

$$\begin{aligned} F(a, b, c) &= a \cdot F(a=1) + a' \cdot F(a=0) \\ &= a \cdot (b \cdot F(a=1, b=1) + b' \cdot F(a=1, b=0)) + a' \cdot (b \cdot F(a=0, b=1) + b' \cdot F(a=0, b=0)) \end{aligned}$$



Note: $F(a=1, b=1)$, $F(a=1, b=0)$, $F(a=0, b=1)$ and $F(a=0, b=0)$ must be one of the following values: c , c' , 0 , 1

- ❑ If for every variable its complement is also available, any three-input combinational function can be implemented by a single ACT cell.
- ❑ Some functions with inputs up to eight can be implemented by a single ACT1 cell.

Example: A Six-Input Majority Function

- **Six-input majority function:** the function output is 1 if three or more than three inputs are 1.

$$F(a=1, b=1, c=1) = 1 \quad (F1)$$

$$F(a=1, b=1, c=0) = F(a=1, b=0, c=1) = F(a=0, b=1, c=1) = e+f+d \quad (F2, F3, F4)$$

$$F(a=1, b=0, c=0) = F(a=0, b=1, c=0) = F(a=0, b=0, c=1) = e \cdot f + d \cdot f + e \cdot d \quad (F5, F6, F7)$$

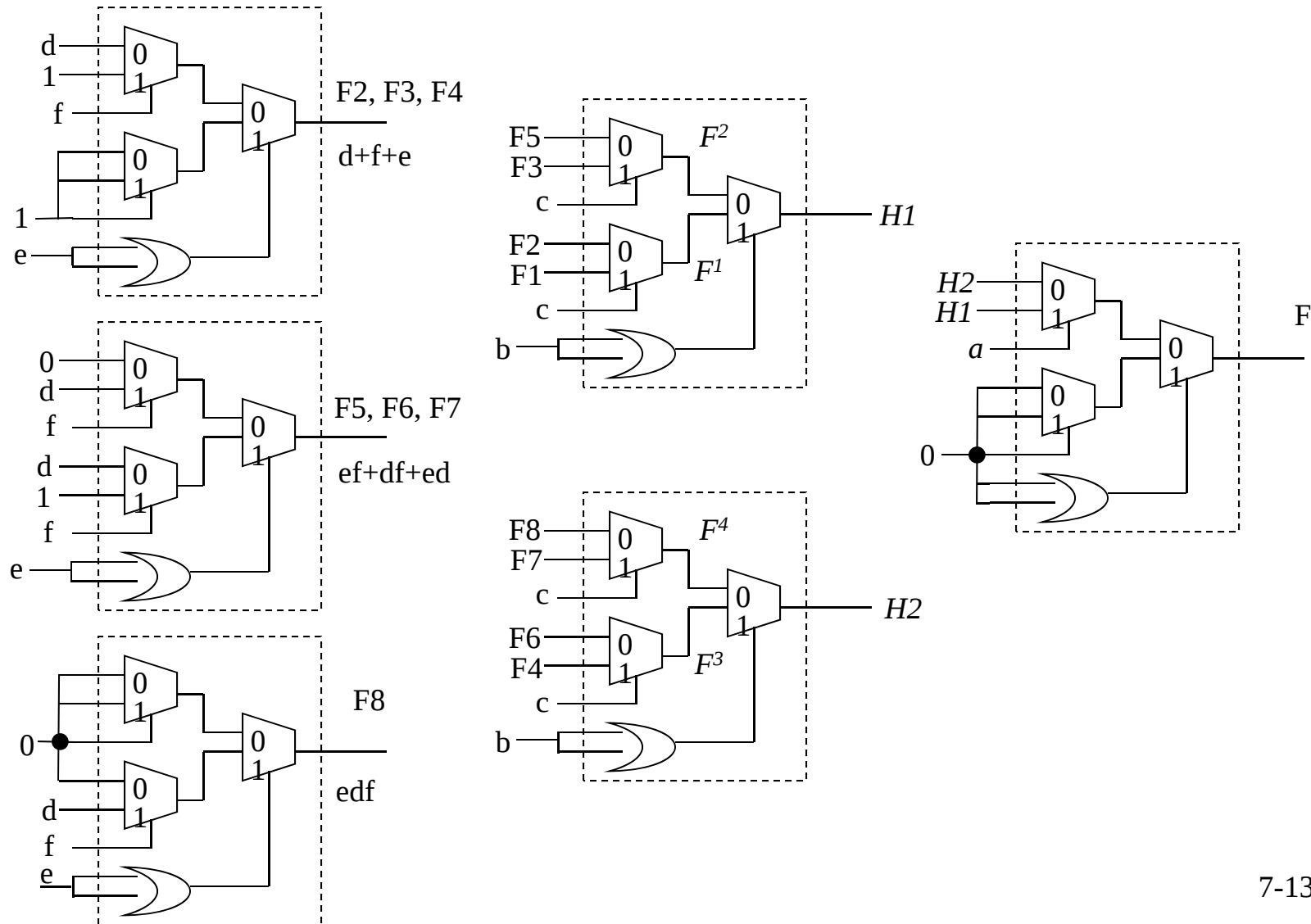
$$F(a=0, b=0, c=0) = e \cdot f \cdot d \quad (F8)$$

$$F = abcF1 + abc'F2 + ab'cF3 + a'bcF4 + ab'c'F5 + a'bc'F6 + a'b'cF7 + a'b'c'F8$$

$$= ab(\underbrace{cF1 + c'F2}_{F^1}) + ab'(\underbrace{cF3 + c'F5}_{F^2}) + a'b(\underbrace{cF4 + c'F6}_{F^3}) + a'b'(\underbrace{cF7 + c'F8}_{F^4})$$

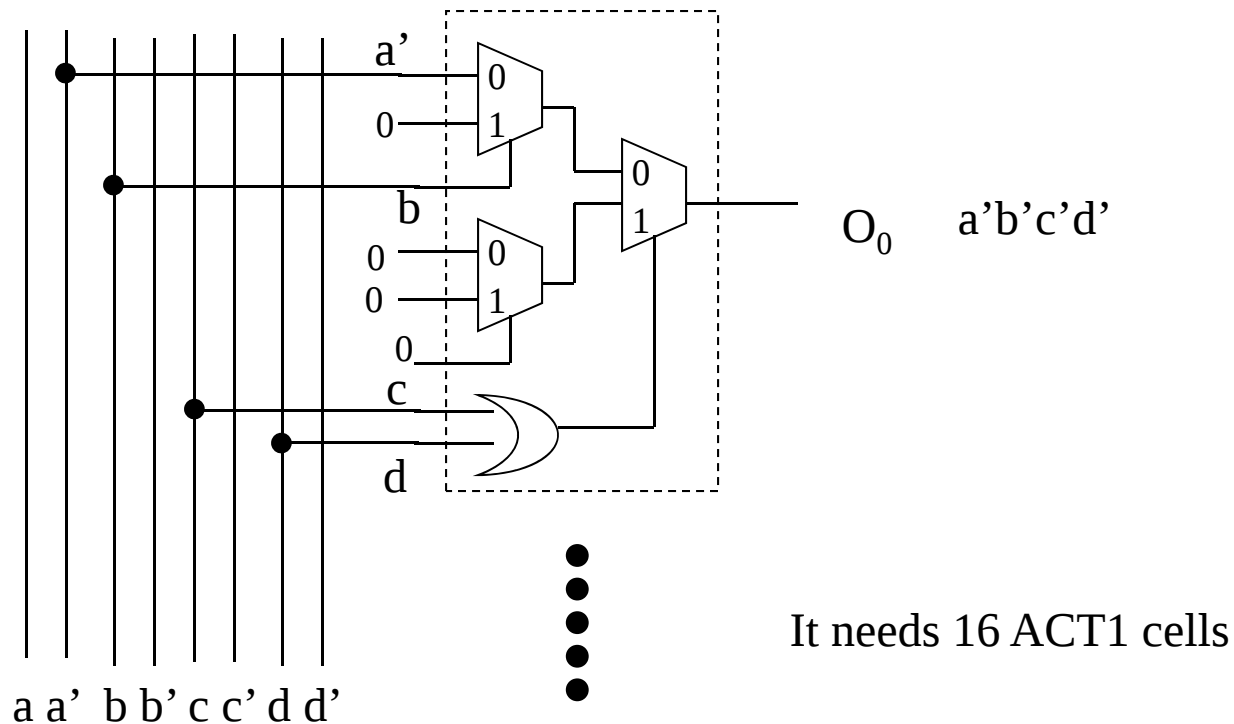
$$= a(\underbrace{bF^1 + b'F^2}_{H1}) + a'(\underbrace{bF^3 + b'F^4}_{H2})$$

Example: A Six-Input Majority Function



Decoding Circuits

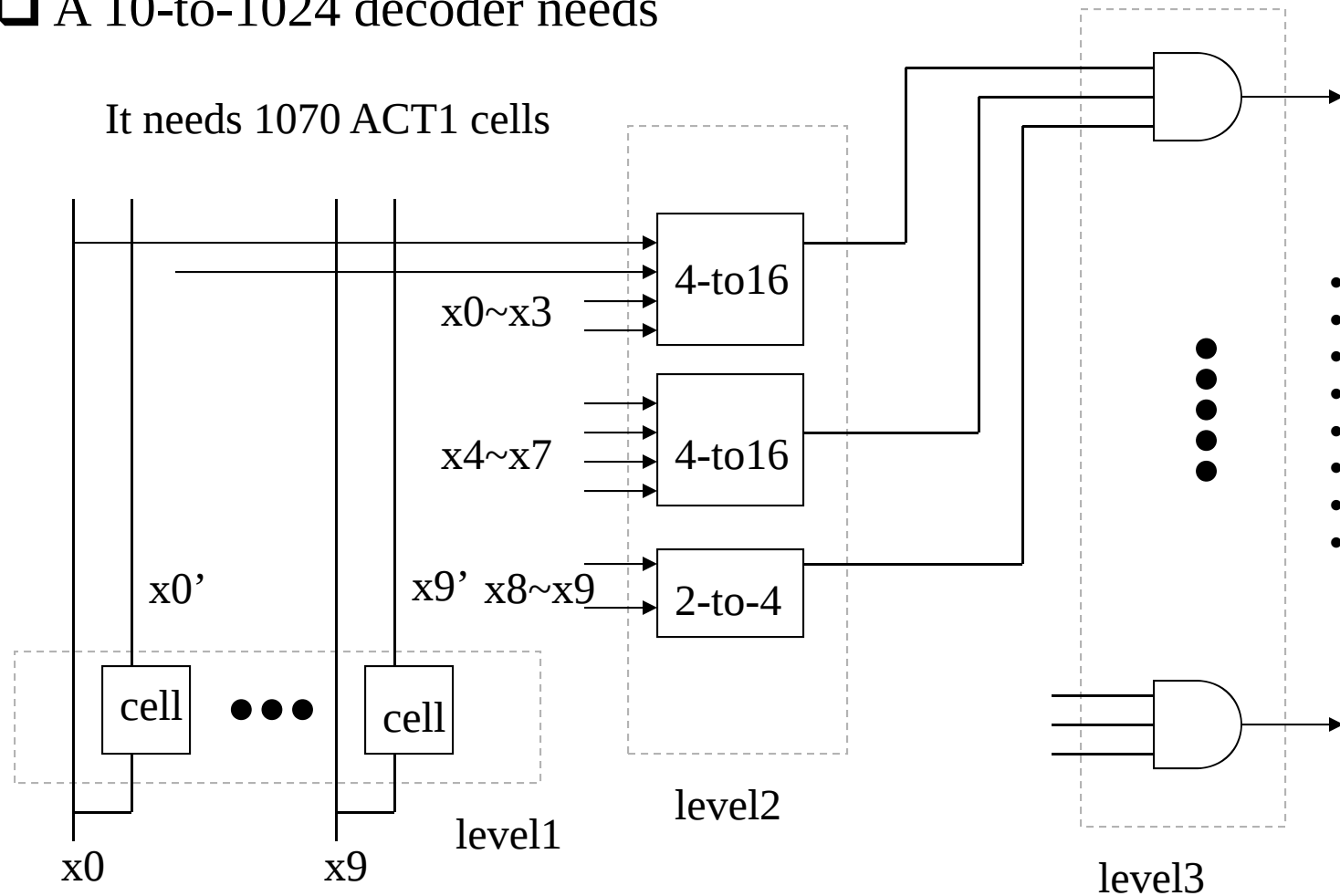
❑ 4-to-16 Decoding circuit



❑ Decoder circuits with more than four inputs need multi-level implementation.

Decoding Circuits

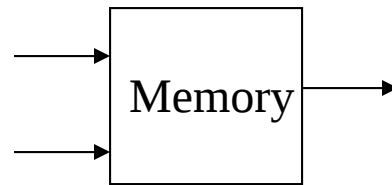
- ❑ A 10-to-1024 decoder needs



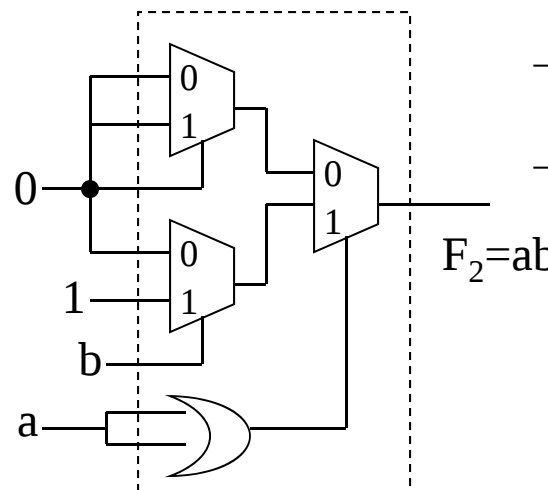
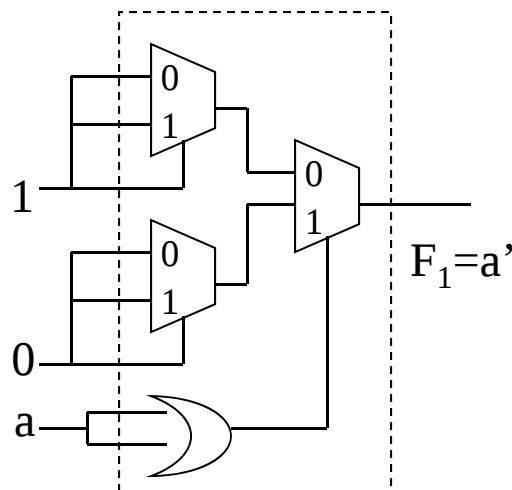
- ❑ Actel FPGAs also use dedicated decoding circuits.

Speed Considerations

- ❑ For a combinational circuit implemented by a **single** memory block, its delay depends on neither the function type nor the input transition patterns.



- ❑ For a combinational circuit implemented by mux-based FPGAs, its delay normally depends on the function type and the input transition patterns.

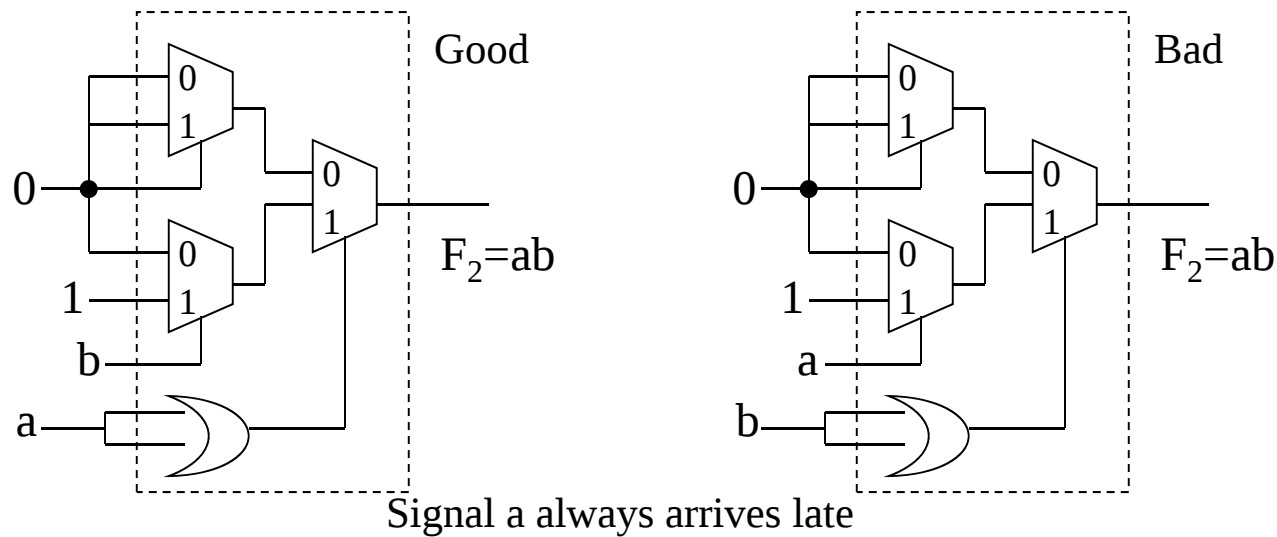
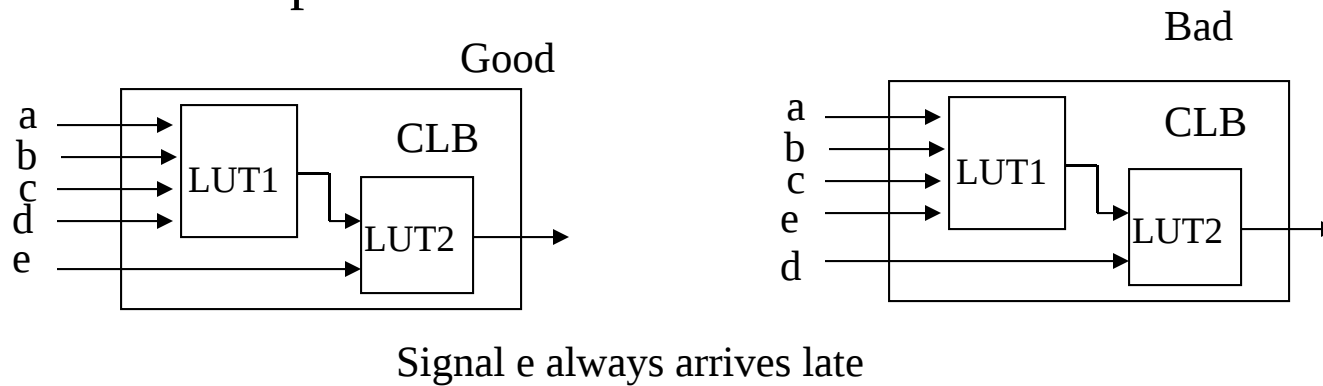


- The delay of F_1 is smaller than that of F_2
 - For F_2 , d_1 is the output delay when inputs switch from $a=1$ and $b=1$ to $a=0$ and $b=1$; d_2 is the delay when inputs switch from $a=1$ and $b=1$ to $a=1$ and $b=0$.
- $d_2 > d_1$**

Assume the delay of OR gate is smaller than that of MUX

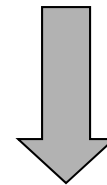
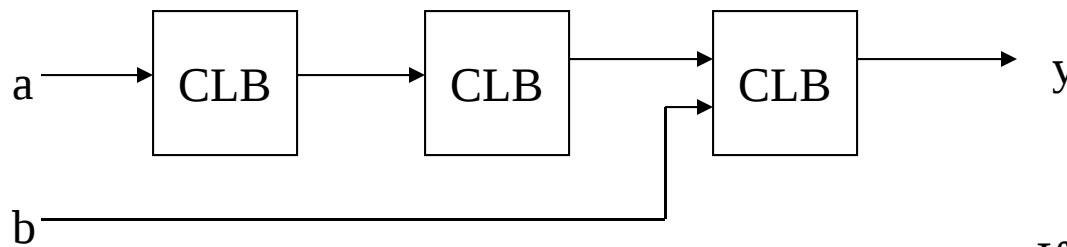
Speed Considerations

- ❑ To reduce propagation delay, always place signals which arrive late close to the output.



Other Considerations

- ❑ Always try to balance signal paths to avoid glitches



If possible, change circuit implementation into the following style

