ET 438a
Homework Listing

Assignments	Lesson Number	Problems
Book Chapter 1		
1	1	1.5a, 1.5b 1-15a, 1-15b. 1-15c Bateson
2	1	1-27, 1-28, 1-29 Bateson
3	2	1.1, hw1.wp5* Johnson/handout
4	2	1.17, 1.24, 1.28, 1.33 Johnson
5	3	op_hw1.wp5
6	3	averhw.wp5
7	4	hw38a3a.wp5
8	4	hw38-3.wp5
Book Chapter 3		
9	5	3.5, 3.6, 3.8 Bateson
10	6	3.9d, 3.10a-d Bateson
11	6	3.11a-c, 3.12a-c Bateson
12	7	3.20d-e, 3.22a Bateson
13	8	3.28 hw38-5.wp5
Book Chapter 4		
14	8	4.1, 4.3, 4.5 Bateson
15	8	hw38-4.wp5
16	10	4.6a, b, d, g, i, m Bateson
17	10	4.7a, c, e, g, j Bateson
18	10	4.8, 4.9, 4.11, 4.15 Bateson
19	10	hw38-6a (problem 1 only)
20	10	hw38-6a.wp5
21	11	hw38-7.wp5
22	12	hw38-8.wp5
23	14	4.16, 10.16, 4.21 Bateson
Book Chapter 13		
24	14	dcmtrhw.wp5
25	16	13.14, 13.18 Bateson
26	17	13.20, 13.23, 13.34 Use Matlab
27	17	13.30, 13.32 Bateson
28	17	hw38a-12.wp5
29	18	intpros.wp5

Fall 2016
*Homeworks listed in this format are on handouts.

Book Chapter 14	Lesson Number	Problems
30	20	hw38a-11.wp5
31	20	14.6, 14.26 Bateson
32	21	hw38a-13.wp5
33	23	bodeny.wp5
34	23	14.27, 14.28, 14.31 Bateson

HW 1

Process variable: Any variable in the process. Process controllers often refer to the controlled variable as the process variable. (1.4)

Proportional mode: A control mode that produces a control action that is proportional to the error. (1.4)

Quarter amplitude decay: A criterion of good control that progressively reduces the amplitude of oscillation by a factor of 4. (1.11)

Residual error: The error that remains after all transient responses have faded out. This is sometimes referred to as offset. (1.10)

Saturation: The characteristic that limits the range of the output of a component. (1.6)

Sensor: An element that responds to a parameter to be measured and converts the response into a more usable form. (1.4)

Setpoint: The controller signal that defines the desired value of the controlled variable. (1.4)

Settling time: The time, following a disturbance, that is required for the transient response to fade out. (1.10)

Transfer function: The mathematical expression that establishes the relationship between the input and the output of a component. (1.2)

◆ EXERCISES

Section 1.1

1.1 Write a paragraph that describes how you act as a controller during a common activity such as taking a shower.

1.2 Identify the controlled variable and the manipulated variable in your answer to Exercise 1.1.

Section 1.2

1.3 Draw a block diagram of a typical home heating system with the following components:

(a) A household thermostat where the input signal is the temperature in the living room and the output signal is either on or off.

(b) A solenoid valve where the input is the on or off signal from the thermostat and the output is the flow of gas to the furnace.

(c) A household heating furnace where the input is gas flow from the solenoid valve and the output is heat to the rooms in the house.

(d) The inside of a house where the input is heat from the furnace and the output is the temperature in the living room. *Note:* The output of the living room is also the input to the thermostat, so your diagram should form a closed loop.

1.4 Name the two parts of the relationship between the input and the output of a component, name the function that establishes this relationship, and give an example of each part from your own experience.

1.5 Determine the gain, phase difference, and transfer function for each of the following input/output conditions. The components are linear.

(a)	Frequency:	0.004 Hz		
	Input amplitude:	10°C	phase:	15°
	Output amplitude:	0.447 mV	phase:	−30°
(b)	Frequency:	36 KHz		
	Input amplitude:	0.4 V	phase:	20°
	Output amplitude:	11.6 V	phase:	19°
(c)	Frequency:	0.5 Hz		
	Input amplitude:	4.4 V	phase:	32°
	Output amplitude:	600 rpm	phase:	0°
	(rpm means revolutions per minute)			
(d)	Frequency:	0.4 Hz		
	Input amplitude:	5%	phase:	10°
	Output amplitude:	50%	phase:	12°
(e)	Frequency:	0.05 Hz		
	Input amplitude:	3.5 psi	phase:	27°
	Output amplitude:	2.1 gpm	phase:	18°
	(psi means pounds per square inch)			
	(gpm means gallons per minute)			
(f)	Frequency:	1.6×10^{-4} Hz		
	Input amplitude:	3.2 gpm	phase:	0°
	Output amplitude:	8.4 inches	phase:	−58°

Section 1.3

1.6 Write a paragraph that describes an example of an open-loop control system taken from your own experiences.

Hw 1

1.7 List the advantages and disadvantages of open-loop control.

Section 1.4

1.8 Write a paragraph that describes an example of a closed-loop control system taken from your own experiences.

1.9 Assume that you are explaining feedback control to a friend who knows nothing about control systems. Explain the operations performed by a feedback control system. Name each component and each signal in the system and explain how the system works.

1.10 List the advantages and disadvantages of closed-loop control.

1.11 A flow measuring transmitter has a linear I/O graph similar to Figure 1.8. The input range is 0 to 10 liters per minute (L/min): the output range is 4 to 20 mA. Find the following:

(a) Transmitter gain. $H = C_m/C$
(b) The equation for output C_m in terms of input C
(c) Output C_m when input $C = 6$ L/min
(d) Input C when output $C_m = 4$ mA

1.12 A level-measuring transmitter has a linear I/O graph similar to Figure 1.8. The input range is 5.0 to 10.0 meters (m), and the output range is 3 to 15 psi. Find the following:

(a) Output C_m when input $C = 6.3$ m
(b) Input C when output $C_m = 9.1$ psi

1.13 A pressure-measuring transmitter has a linear I/O graph similar to Figure 1.8. The input range is 0 to 200 kilopascals (kPa), and the output range is 10 to 50 mA. Find the following:

(a) Output C_m when input C = 72 kPa
(b) Input C when output $C_m = 13.7$ mA

1.14 Derive Equation 1.2.

Section 1.5

1.15 Name the component identified by each of the following component codes (see Table D.1 in Appendix D).

(a) LI-112	(b) AIC-113	(c) PDT-201
(d) AIT-113	(e) PAH-201	(f) TDAL-320
(g) LRT-112	(h) ST-141	(i) SAL-141
(j) TI-320	(k) FS-106	(l) FQI-110
(m) SAH-141	(n) ZZI-117	(o) ZXI-117
(p) TAL-320		

1.16 A certain process consists of a kettle filled with liquid and heated by a gas flame. A thermocouple temperature transmitter measures the temperature of the liquid in the kettle. A control valve manipulates the flow of gas to the burner. The control system components are listed below. Name each component and sketch an instrumentation drawing for this system.

Component Code	Name	Input	Output
TT-201	______	Temperature	4–20 mA
TRC-201	______	4–20 mA	4–20 mA
TY-201	______	4–20 m	3–15 psi
TV-201	______	3–15 psi	cfm[a]

[a]Cfm stands for gas flow rate in cubic feet per minute.

Section 1.6

1.17 Test data from four components are given below. Each test consists of a complete traversal from an input value of 0 to an input value of 25. The data are listed in the order in which the traversal was made. Plot the data from each test on an input/output graph with the input on the horizontal axis and the output on the vertical axis. Use arrows to show the direction of traversal.

1.18 Identify all nonlinearities exhibited by each component in Exercise 1.17.

Component 1
Input
0 5 10 15 20 25 20 15 10 5 0
Output
0 16 38 62 84 100 84 62 38 16 0

Component 2
Input
0 5 10 15 20 25 20 15 10 5 0
Output
0 18 36 56 78 100 82 64 44 22 0

Component 3
Input
0 6 10 15 20 22 25 19 15 10 5 3 0
Output
10 10 30 55 80 90 90 90 70 45 20 10 10

Component 4
Input
0 6 10 15 20 22 25 19 15 10 5 3 0
Output
10 10 27 50 78 90 90 90 73 50 22 10 10

HW 2

1.16b. The first positive peak error measured 8°C. What should the second positive peak error measure to satisfy the quarter amplitude decay criteria?

1.25 Does the response in Figure 1.21 satisfy the quarter amplitude decay criteria?

1.26 In a chemical process, two components are blended together in a large mixer. The temperature of the mixture must be maintained between 100 and 112°C. If the temperature exceeds 114°C, the finished product will not satisfy the specifications. Which of the three criteria of good control should be used for the temperature control system?

Section 1.12

1.27 Reduce the block diagram in Figure 1.22a to the standard form shown in Figure 1.20b and to a single block similar to Figure 1.20c.

1.28 Repeat Exercise 1.27 for the block diagram in Figure 1.22b.

1.29 Repeat Exercise 1.27 for the block diagram in Figure 1.22c.

1.30 Repeat Exercise 1.27 for the block diagram in Figure 1.22c.

type of load change was applied to the air-conditioning system when 500 students arrive for an afternoon concert?

Section 1.9

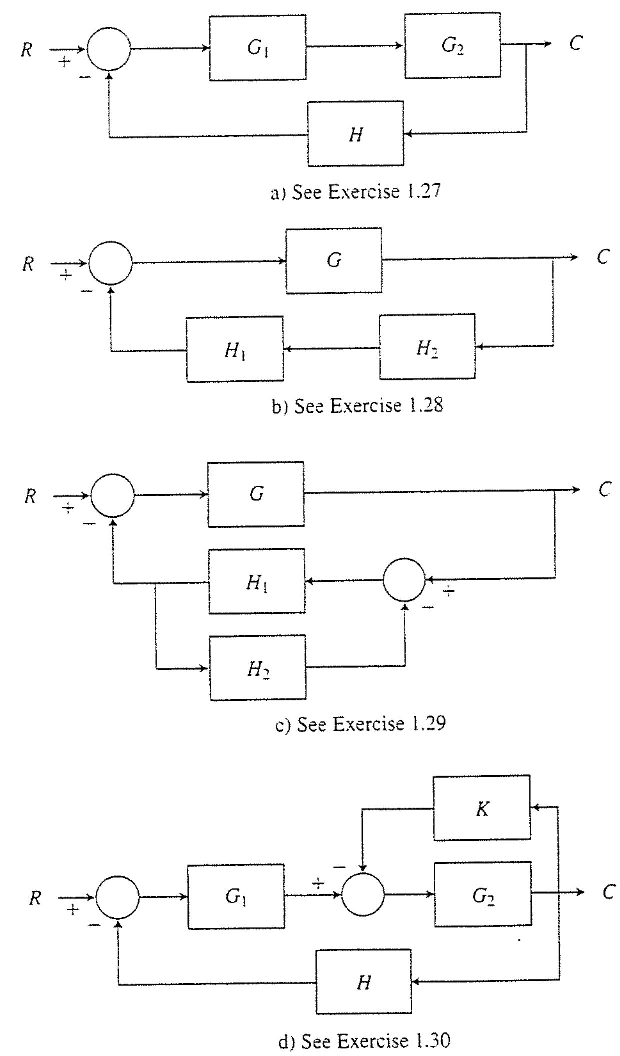
1.22 Which of the five types of damping shown in Figure 1.14 would be most desirable for the suspension of an automobile? Explain your answer by describing the ride of the automobile with each type of damping.

Section 1.10

1.23 A temperature control system produced the response shown in Figure 1.21. Determine the maximum error, the settling time, and the residual error from the response.

Section 1.11

1.24 A step change test of a temperature control system produced an underdamped response similar to Figure



◆ **Figure 1.22** Block diagrams used in Exercises 1.27, 1.28, 1.29, and 1.30.

Hw 3

values. Figure 1.5 shows a block diagram in which the elements of measurement, error detector, controller, and control element are connected to provide the required regulation.

2. Numerous criteria have been discussed that allow the evaluation of process-control loop performance, of which the settling time, peak error, and minimum area are the most indicative of loop characteristics.
3. Both analog and digital processing are used in process-control applications. The current trend is to make analog measurements of the controlled variable, digitize them, and used a digital controller for evaluation. The basic technique of digital encoding allows each bit of a binary word to correspond to a certain quantity of the measured variable. The arrangement of "0" and "1" states in the word then serves as the encoding.
4. The SI system of units forms the basis of computations in this text as well as in the process-control industry in general. However, it is still necessary to understand conversions to other systems, notably the English system (see Appendix 1).
5. A standard, adopted for analog process-control signals, is the 4–20 mA current range to represent the span of measurements of the dynamic variable.
6. The definitions of accuracy, resolution, and other terms used in process control are necessary and are similar to those in related fields.
7. The concept of transducer time response was introduced. The time constant becomes part of the dynamic properties of a transducer.
8. The use of significant figures is important to properly interpret measurements and conclusions drawn from measurements.
9. Statistics can help interpret the validity of measurements through the use of the arithmetic mean and the standard deviation.
10. P&ID drawings and symbols are the typical representation used to display process-control systems.

PROBLEMS

Section 1.2

1.1 Explain how the basic strategy of control is employed in a room air-conditioning system. What is the controlled variable? What is the manipulated variable? Is the system self-regulating?

1.2 Is the driving of an automobile best described as a servomechanism or a process-control system? Why?

Section 1.3

1.3 Construct a block diagram of a refrigerator control system. Define each block in terms of the refrigerator components. (If you do not know the components, look them up in an encyclopedia.)

Hw: 3

ET 438a
Continuous and Digital Control Systems

Homework

The output of an inverting OP AMP circuit must have an error of no more than ±3%. The input voltage has an error of ±1.75%. What is the maximum tolerance that the resistors in the circuit can have without exceeding the output error limit? Assume that the tolerance of the resistors will be the same.

Hw4

1.11 For the process-control system of Figure 1.13, suppose that the relays close at |1.5| volts and open at |1.1| volts. This means that as the voltage on the relay reaches ±1.5 volts, it closes, and does not open again until the voltage drops to 1.1 volts (i.e., there is a deadband). The amplifier has a gain of 10, the reference is 3 V, and the sensor outputs 150 mV/°C. Calculate the temperatures at which the heater turns on and off and at which the cooler turns on and off.

1.12 Show how the control system of Figure 1.6 would be modified to use **(a)** supervisory computer control and **(b)** DDC or computer control.

1.13 Think about how you adjust the water temperature coming from a single nozzle using the hot and cold hand valves in a kitchen sink. Construct the block diagram of an automatic system as follows: The desired water temperature is selected by the user, perhaps by a knob and LCD readout. One hand valve turns on the cold water. The hot water valve is automatically set to keep the temperature at the selected value. Describe what elements would be necessary to do this using **(a)** analog control and **(b)** computer control.

Section 1.6

1.14 What is your mass in kilograms? What is your height in meters?

1.15 Atmospheric pressure is about 14.7 lb/in^2 (psi). What is this pressure in pascals?

1.16 An accelerometer is used to measure the constant acceleration of a race car that covers a quarter mile in 7.2 s.

- **a.** Using $x = at^2/2$ to relate distance x, acceleration a, and time t, find the acceleration in ft/s^2.
- **b.** Express this acceleration in m/s^2.
- **c.** Find the car speed, v, in m/s at the end of the quarter mile using the relation $v^2 = 2ax$.
- **d.** Find the car energy in joules at the end of the quarter mile if it weighs 2000 lb, where the energy $W = mv^2/2$.

1.17 Suppose a liquid level from 5.5 to 8.6 m is linearly converted to pneumatic pressure from 3 to 15 psi. What pressure will result from a level of 7.2 m? What level does a pressure of 4.7 psi represent?

1.18 A controller output is a 4- to 20-mA signal that drives a valve to control flow. The relation between current and flow is $Q = 45[I - 2\text{ mA}]^{1/2}$ gal/min. What is the flow for 12 mA? What current produces a flow of 162 gal/min?

1.19 An instrument has an accuracy of ±0.5% FS and measures resistance from 0 to 1500 Ω. What is the uncertainty in an indicated measurement of 397 Ω?

1.20 A sensor has a transfer function of 0.5 mV/°C and an accuracy of ±1%. If the temperature is known to be 60°C, what can be said with absolute certainty about the output voltage?

1.21 The sensor of Problem 1.20 is used with an amplifier with a gain of 15 ± 0.25 and displayed on a meter with a range of 0 to 2 volts at ±1.5% FS. What is the worst-case and rms uncertainty for the total measurement?

1.22. Using the nominal transfer function values, what is the maximum measurable temperature of the system in Problems 1.20 and 1.21?

1.23 A temperature sensor transfer function is 44.5 mV/°C. The output voltage is measured at 8.86 volts on a three-digit voltmeter. What can you say about the value of the temperature?

1.24 A level sensor inputs a range from 4.50 to 10.6 ft and outputs a pressure range from 3 to 15 psi. Find an equation such as Equation (1.6) between level and pressure. What is the pressure for the level of 9.2 ft?

1.25 Draw Figure 1.6a in the standard P&ID symbols.

Hw 4

Section 1.7

1.26 A temperature sensor has a static transfer function of 0.15 mV/°C and a time constant of 3.3 s. If a step change of 22°C to 50°C is applied at $t = 0$, find the output voltages at 0.5 s, 2.0 s, 3.3 s, and 9 s. What is the *indicated* temperature at these times?

1.27 A pressure sensor measures 44 psi just before a sudden change to 70 psi. The sensor measures 52 psi at a time 4.5 seconds after the change. What is the sensor time constant?

1.28 A photocell with a 35-ms time constant is used to measure light flashes. How long after a sudden dark to light flash before the cell output is 80% of the final value?

1.29 An alarm light goes ON when a pressure sensor voltage rises above 4.00 volts. The pressure sensor outputs 20 mV/kPa and has a time constant of 4.9 seconds. How long after the pressure rises suddenly from 100 kPa to 400 kPa does the light go ON?

1.30 A pressure sensor has a resistance that changes with pressure according to $R = (0.15\ \text{k}\Omega/\text{psi})p + 2.5\ \text{k}\Omega$. This resistance is then converted to a voltage with the transfer function

$$V = \frac{10R}{R + 10k}\ \text{volts.}$$

The sensor time constant is 350 ms. At $t = 0$ the pressure changes suddenly from 40 psi to 150 psi.

a. What is the voltage output at 0.5 seconds? What is the indicated pressure at this time?

b. At what time does the output reach 5.0 volts?

1.31 At $t = 0$ a temperature sensor was suddenly changed from 25°C to 100°C. The sensor outputs voltage given by the expression $V = (0.06\ \text{V/°C})\ [T - 20\text{°C}]$. The table below gives the voltages measured and the times. Determine the average time constant of the sensor.

t (seconds)	0	0.1	0.2	0.3	0.4	0.5
V (volts)	0.3	1.8	2.8	3.4	3.9	4.2

Section 1.8

1.32 A circuit design calls for a 1.5-kΩ resistor to have 4.7 volts across its terminals. What would be the expected current? The circuit is built and the resistance is measured at 1500 Ω and the voltage at 4.7 V. What is the current through the resistor?

1.33 Flow rate was monitored for a week, and the following values were recorded as gal/min: 10.1, 12.2, 9.7, 8.8, 11.4, 12.9, 10.2, 10.5, 9.8, 11.5, 10.3, 9.3, 7.7, 10.2, 10.0, 11.3. Find the mean and the standard deviation for these data.

1.34 A manufacturer specification sheet lists the transfer function of a pressure sensor as 45 ± 5% mV/kPa with a time constant of 4 ± 10% seconds. A highly accurate test system applies a step change of pressure from 20 kPa to 100 kPa.

a. What is the range of sensor voltage outputs initially and finally?

b. What range of voltages would be expected to be measured 2 seconds after the step change is applied?

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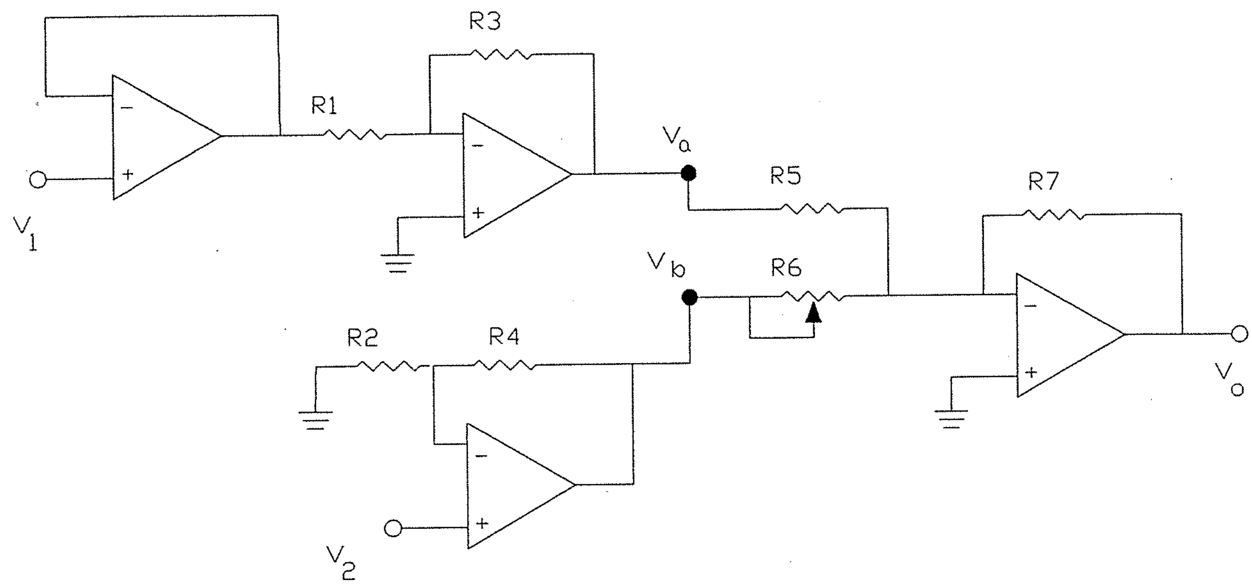
ET 438a
Continuous and Digital Control

Homework



For the circuit above the resistor values are:

R_1 = 470 kΩ R_2 = 10 kΩ R_3 = 100 kΩ
R_4 = 8.2 kΩ R_5 = 82 kΩ R_6 = set to 187 kΩ
R_7 = 270 kΩ

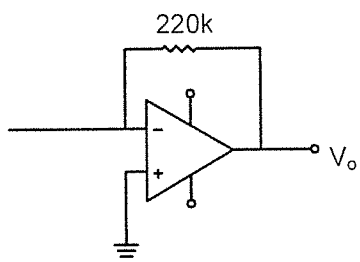
The values of input voltage are: V_1 = +3 Vdc
V_2 = -0.10 Vdc

Find the values of V_a, V_b and V_o for the circuit

HW. 6

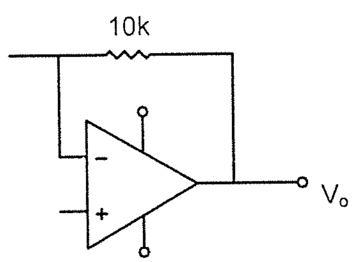
ET 438a
Continuous and Digital Control
OP AMP Averaging Circuits

1a.) Complete the schematic of the circuit started below so that it will give an output that is -3 times the average of 4 input voltages. Label the values of the resistors necessary to design the circuit on the completed schematic.



b.) For the circuit designed above, find the output value, V_o, if the four input voltages are 0.35, -.125, 0.67, -0.45 Vdc

2a.) Complete the circuit shown below to construct a 3 input non-inverting averaging circuit that has a gain of 2. The resistors available for this design are limited to 1 MΩ, 500 kΩ, 5 kΩ, and 2.5 kΩ. You can use any number of these values in the design. Design the circuit to have minimum loading effect on the devices attached to the input and use the minimum number of resistors.



b.) Compute the value of the second averaged input voltage, V_2 if the output is 6.78 Vdc and the other two inputs are 3.75 and 2.55 Vdc.

HW 7

ET 438a
Continuous and Digital Control

Homework

Find the final output gain formula fo a pressure transducer that has a measurement range of 10 to 100 psi. It has a linear gain of 25 mV/psi. The desired output range is 0-5 Vdc

Hw 8

ET 438a
Continuous and Digital Control

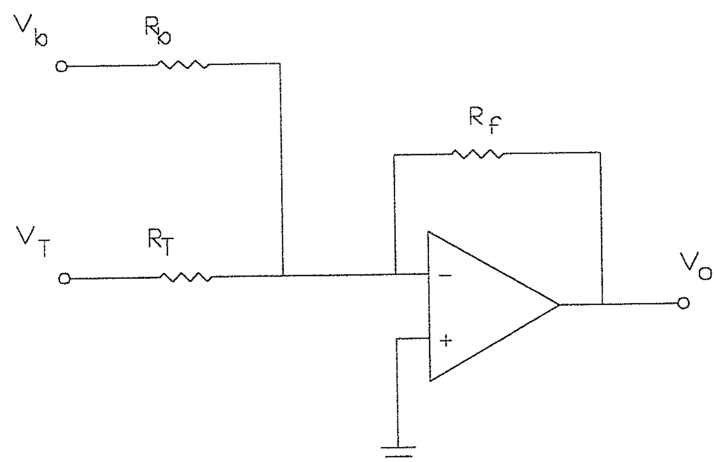
Homework

A linear flow transducer has an output voltage range of 0.25-2.5 Vdc over a range of flow of 0 - 50 gal/min (GPM).

a.) Find the equation that relates the flow, Q, to the transducer output voltage V_T.

b.) Find a linear equation that takes the output of the transducer V_T and scales it to a range of 0-10 Vdc.

c.) Find the values of R_b and R_t in the OP AMP circuit given below that will implement the linear equaiton from part b. The value of R_f = 330 kΩ and V_b = 2.0 Vdc.



Hw 9

◆ EXERCISES

Section 3.2

3.1 Ten components were tested to determine their electrical resistance by applying a voltage to each component and measuring the resulting current. The voltage on each component was slowly increased until the product of the voltage times the current was equal to 0.25 (the power dissipated in a component is equal to this product, so each component was dissipating $\frac{1}{4}$ watt when the measurement was made). Use Equation (3.3) to determine the static resistance of each component for the following voltage and current values.

(a) 1.04 V, 0.240 A
(b) 3.02 V, 0.0828 A
(c) 4.06 V, 0.0616 A
(d) 5.00 V, 0.0500 A
(e) 8.71 V, 0.0287 A
(f) 13.4 V, 0.0187 A
(g) 16.5 V, 0.0152 A
(h) 32.7 V, 0.00765 A
(i) 42.5 V, 0.00588 A
(j) 51.2 V, 0.00488 A

3.2 The ten components in Exercise 3.1 were tested a second time with the current reduced by 50%. Use Equation (3.3) to determine the static resistance of each component using the reduced voltage and current values listed below. Compare the reduced static resistance values with those from Exercise 3.1 and label each component as linear or nonlinear (a component is linear only if the two resistance values are equal).

(a) 0.52 V, 0.120 A
(b) 1.28 V, 0.0414 A
(c) 2.03 V, 0.0308 A
(d) 2.19 V, 0.0250 A
(e) 3.67 V, 0.0144 A
(f) 6.70 V, 0.00935 A
(g) 8.25 V, 0.0076 A
(h) 14.2 V, 0.00382 A
(i) 17.6 V, 0.00294 A
(j) 25.6 V, 0.00244 A

3.3 In Exercises 3.1 and 3.2, you computed the static resistance of 10 components using Equation (3.3). When we analyze and design control systems, we use the dynamic resistance at the operating point, not the static resistance. In this exercise, you will compute the dynamic resistances of the 10 components about the operating point values given in Exercise 3.1. Voltage and current values slightly above and below the operating point are listed below. Use Equation (3.4) to determine the dynamic resistance of each component.

	e_1	i_1	e_2	i_2
(a)	0.996 V	0.230 A	1.0825 V	0.250 A
(b)	2.831 V	0.0788 A	3.2135 V	0.0868 A
(c)	3.862 V	0.0586 A	4.257 V	0.0646 A
(d)	4.752 V	0.0480 A	5.252 V	0.0520 A
(e)	8.119 V	0.0272 A	9.322 V	0.0302 A
(f)	12.691 V	0.0177 A	14.125 V	0.0197 A
(g)	15.638 V	0.0144 A	17.376 V	0.0160 A
(h)	30.568 V	0.00725 A	34.880 V	0.00805 A
(i)	39.632 V	0.00558 A	45.443 V	0.00618 A
(j)	48.578 V	0.00463 A	53.824 V	0.00513 A

3.4 Compare the static and dynamic resistance values of the 10 components in Exercises 3.1, 3.2, and 3.3. Discuss the relationship between the static and dynamic resistances of linear and nonlinear components.

3.5 The following data were obtained in a test of a nonlinear electrical resistor.

Volts	0	5	10	15	20	25
Amperes	0	0.323	0.626	0.914	1.19	1.45

Plot a volt–ampere graph. Use the tangent line method to determine the resistance at 10 and 20 V.

3.6 An electric current, i, was applied to a capacitor for a duration of t seconds. This current pulse increased the voltage across the capacitor from e_1 to e_2 volts. Determine the capacitance, C, of the capacitor for each of the following values of i, t, e_1 and e_2.

	i	t	e_1	e_2
(a)	0.36 mA	0.25 s	0 V	16 V
(b)	0.22 mA	1.20 s	0 V	240 V
(c)	2.10 mA	0.84 s	50 V	120 V
(d)	1.50 mA	0.48 s	20 V	80 V
(e)	12.4 mA	0.09 s	0 V	90 V

HW9, HW10 ; HW11

3.7 A voltage pulse with an amplitude of e volts and a duration of t seconds was applied to an inductor. The current through the inductor increased from i_1 to i_2 amperes. Assume the resistance of the inductor was negligible. Determine the inductance, L, of the inductor for each of the following values of e, t, i_1 and i_2.

	e	t	i_1	i_2
(a)	12.6 V	4.0 ms	0 A	1.20 A
(b)	3.80 V	1.25 s	0 A	3.40 A
(c)	2.40 V	0.03 s	100 mA	200 mA
(d)	5.40 V	0.20 s	0.42 A	2.36 A
(e)	6.80 V	2.0 ms	0 A	0.64 A

HW9

3.8 Determine the dead-time delay in the transmission of electrical signals for the following values of distance, D, and velocity of propagation, v_p.

(a) 25 km, 2.8×10^8 m/s
(b) 2×10^5 km, 3.0×10^8 m/s
(c) 2000 ft, 2.4×10^8 m/s
(d) 12,000 miles, 2.2×10^8 m/s
(e) 200 m, 2.7×10^8 m/s

Section 3.3

3.9 Determine the Reynolds number, the liquid resistance, and the pressure drop for each of the following condition sets. The following abbreviations are used: tube for smooth tube, pipe for commercial pipe, Lpm for liters per minute, and gpm for gallons per minute.

	Fluid (15°C)	Type	Flow Rate	Inside Diameter	Length
(a)	Turpentine	tube	20 Lpm	3.8 cm	1600 m
(b)	Water	pipe	12 gpm	0.75 in.	100 ft
(c)	Ethyl alcohol	pipe	25 Lpm	1.9 cm	15 m
(d)	Gasoline	tube	36 gpm	1.25 in.	1200 ft
(e)	Glycerin	tube	2 gpm	1 in.	25 ft
(f)	Oil	pipe	10 Lpm	2.5 cm	30 m
(g)	Kerosene	tube	50 gpm	2 in.	5500 ft
(h)	Water	tube	36 Lpm	1.9 cm	26 m
(i)	Ethyl alcohol	pipe	12 gpm	0.625 in.	75 ft
(j)	Gasoline	pipe	11 Lpm	2.54 cm	16.000 m

HW10

HW10 **3.10** Determine the liquid capacitance for each of the following condition sets.

	Fluid (15°C)	Tank Diameter	Tank Height
(a)	Water	0.5 m	1.5 m
(b)	Kerosene	0.8 m	1.6 m
(c)	Ethyl alcohol	0.6 m	0.8 m
(d)	Turpentine	1.0 m	1.2 m
(e)	Oil	10.0 m	5.0 m
(f)	Gasoline	1.2 m	2.0 m

HW11

3.11 Determine the liquid inertance for each of the following condition sets.

	Fluid (15°C)	Pipe Diameter	Pipe Length
(a)	Water	1.27 cm	3.66 m
(b)	Kerosene	3.175 cm	6.1 m
(c)	Mercury	0.635 cm	0.25 m
(d)	Turpentine	1.905 cm	10.0 m
(e)	Oil	15.24 cm	1600 m
(f)	Gasoline	2.54 cm	40 m

HW11

3.12 Determine the dead-time delay for each of the following liquid flow condition sets.

	Flow Rate	Pipe Diameter	Pipe Length
(a)	6.33E − 4 m³/s	3.175 cm	15 m
(b)	9.12E − 4 m³/s	2.540 cm	20 m
(c)	3.18E − 4 m³/s	1.590 cm	8 m
(d)	17.10 Lpm	1.905 cm	4 m
(e)	5.12 Lpm	0.952 cm	5 m
(f)	10.64 Lpm	1.270 cm	10 m

Section 3.4

3.13 The following data sets were obtained for gas flow systems. Use Equation (3.26) to determine the value of the turbulent flow coefficient, K_g, for each data set.

	p_1 (kPa)	p_2 (kPa)	W (kg/s)
(a)	106	101	0.054
(b)	112	102	0.0049
(c)	108	104	0.031
(d)	96	88	0.012
(e)	102	96	0.12
(f)	80	73	0.023

Hw 12

3.14 Determine the gas flow resistance, R_g, for each data set in Exercise 3.13. Use Equation (3.27) and the values of K_g you obtained in that exercise.

3.15 Determine new values of p_2 for the data sets given in Exercise 3.13 with the values of W replaced by the new values given below. Use your computed values of K_g and the values of p_1 from Exercise 3.13.

(a) 0.032 kg/s (b) 0.028 kg/s (c) 0.016 kg/s
(d) 0.0074 kg/s (e) 0.071 kg/s (f) 0.014 kg/s

3.16 Use Equation (3.26) to compute the turbulent flow coefficient, K_g, of a smooth tube flow meter from the following data.

$p_1 = 102$ kPa $p_2 = 92$ kPa $W = 0.04$ kg/s

Then use your computed value of K_g to determine values of pressure drop, p, for the following gas flow rates, W_g: 0, 0.008, 0.016, 0.024, 0.032, and 0.04 kg/s.

Finally, plot your results in a calibration graph of mass flow rate (x-axis) vs pressure drop (y-axis).

3.17 The following data were obtained for a gas flow system. Plot the data and graphically determine the gas flow resistance, R_g, at flow rates of 0.002 and 0.004 kg/s. Check your results by using Equation (3.26) to compute K_g and Equation (3.27) to compute R_g.

W= gas flow rate, kg/s
p = pressure drop, kPa

W	0	0.001	0.002	0.003	0.004	0.005
p	0	0.4	1.6	3.6	6.4	10.0

3.18 In each data set below, a smooth tube is supplying the specified gas with an inlet pressure, p_1, of 120 kPa. Determine the pressure drop, p, gas flow resistance, R_g, and outlet pressure, p_2, for each data set.

	Gas (15° C)	Tube I.D. (cm)	Tube Length (m)	Flow Rate (kg/s)
(a)	Nitrogen	4.50	25.0	0.042
(b)	Carbon dioxide	2.54	8.0	0.032
(c)	Air	3.18	6.0	0.048
(d)	Helium	1.91	5.3	0.0055

3.19 Determine the gas flow capacitance for each of the following data sets.

	Gas	Temperature (°C)	Tank Volume (m°3)
(a)	Nitrogen	40	1.4
(b)	Carbon dioxide	20	2.5
(c)	Air	50	3.5
(d)	Helium	30	0.8

Section 3.5

3.20 Determine the unit thermal resistance, the total thermal resistance, the total heat flow, and the direction of the heat flow for each of the following composite wall sections (see Figure 3.7).

(a) Vertical walls:
$A = 35.7$ m^2 $T_i = 21$°C $T_o = -5$°C
Three inner layers:
(1) Wood (pine), 1.91 cm thick
(2) Insulation, 8.89 cm thick
(3) Wood (oak), 0.64 cm thick
Inside film: Still air, vertical surface, estimated $T_d = 4$°C
Outside film: Moving air, $v = 7$ m/s

(b) Ceiling:
$A = 13.4$ m^2 $T_i = 23$°C $T_o = -5$°C
Four inner layers:
(1) Asphalt, 0.32 cm thick
(2) Wood (pine), 1.27 cm thick
(3) Insulation, 15.24 cm thick
(4) Wood (pine), 1.27 cm thick
Inside film: Still air, horizontal surface facing down, estimated $T_d = 5$°C
Outside film: Moving air, $v = 2.75$ m/s

(c) Floor:
$A = 13.4$ m^2 $T_i = 20$°C $T_o = 10$°C
Three inner layers:
(1) Wood (oak), 0.95 cm thick
(2) Wood (pine), 1.27 cm thick
(3) Insulation, 10.16 cm thick
Inside film: Still air, horizontal surface facing up, estimated $T_d = 3$°C
Outside film: Still air, horizontal surface facing down, estimated $T_d = 6$°C

(d) Water-cooled oil tank:
$A = 1.8$ m^2 $T_i = 95$°C $T_o = 75$°C
One inner layer: Aluminum, 0.64 cm thick
Inside film: Still oil, estimated $T_d = 19$°C
Outside film: Still water, estimated $T_d = 1$°C

(e) Oil-heated water tank:
$A = 1.4$ m^2 $T_i = 20$°C $T_o = 90$°C

HW 12

One inner layer: Brass, 0.25 cm thick
Inside film: Still water, estimated $T_d = 4.5°C$
Outside film: Still oil, estimated $T_d = 16°C$

(f) Water flowing in a straight pipe:
$A = 0.29\ m^2$ $T_i = 10°C$ $T_o = 25°C$
One inner layer: Copper, 0.2 cm thick
Inside film: Turbulent water flow, $T_w = 10°C$, $d = 1.91$ cm, $v = 1.6$ m/s
Outside film: Still air, vertical surface, estimated $T_d = 6°C$

3.21 Cold water flows through a steel pipe that is exposed to warm air. Your job is to estimate how much heat flows through the pipe from the air to the water. The total surface area of the pipe is $\pi d_m L$, where d_m is the pipe mean diameter in meters and L is the pipe length in meters.

$d_m = 1.86$ cm $L = 20$ m $T_i = 5°C$ $T_o = 20°C$

One inner layer: Steel. 0.3 cm thick
Inside film: Turbulent water flow in straight pipes, $T_w = 5°C$, $v_w = 1.8$ m/s
Outside film: Still air, vertical surface

Estimate the value of T_d for the outside film and try several values for T_d between 1 and 15°C to see how sensitive your estimate is.

3.22 Determine the thermal capacitance for each of the following data sets.
(a) Ethyl alcohol, $V = 1.8\ m^3$
(b) Gasoline, $V = 0.2\ m^3$
(c) Water, $V = 50$ gallons
(d) Mercury, $V = 0.01$ L
(e) Oil, $V = 0.86\ m^3$
(f) Aluminum, $m = 4.25$ kg
(g) Steel, $m = 5.95$ kg
(h) Copper, $m = 1.28$ kg
(i) Lead, $m = 3.64$ kg
(j) Wood (pine), $m = 10.64$ kg

Section 3.6

3.23 A technician tested six linear dashpots to determine their mechanical resistances. Coulomb friction was found to be negligible. The force and velocity test results are listed below. Determine the mechanical resistance of each dashpot.

	Force (N)	Velocity (m/s)
(a)	135	1.6
(b)	127	1.1
(c)	143	1.8
(d)	201	2.3
(e)	80	1.5
(f)	152	0.9

3.24 Assume you tested six mechanical systems that combined a sliding load (Coulomb friction) with a dashpot (viscous friction). Your test results are listed below. Determine the mechanical resistance (R_m), the Coulomb friction force (F_c), and the equation for the applied force (F) in terms of the velocity (v).

	F_1 (N)	v_1 (m/s)	F_2 (N)	v_2 (m/s)
(a)	12.0	2.0	31.5	15.0
(b)	21.2	4.2	39.7	12.6
(c)	9.1	3.8	17.7	14.3
(d)	18.6	5.4	34.4	16.7
(e)	23.3	2.9	58.2	13.8
(f)	15.6	4.6	30.8	14.1

3.25 Determine the mechanical capacitance and the spring constant for springs with the following test results.

	Force (N)	Compression (cm)
(a)	1500	8.5
(b)	2100	10.2
(c)	960	7.6
(d)	3200	14.3
(e)	1810	12.9
(f)	1670	9.6

3.26 The lift portion of a mechanical cam has two sections, A and B. Section A accelerates the load (m, kg) at a constant rate from 0 m/s to v m/s in t_A s. Section B decelerates the load at a constant rate from v m/s to 0 m/s in t_B s. Determine the inertial forces in A and B for each of the following condition sets.

	m (kg)	v (m/s)	t_A (s)	t_B (s)
(a)	2.9	0.95	0.32	0.22
(b)	1.8	1.20	0.28	0.33
(c)	3.7	0.83	0.44	0.18
(d)	4.1	1.13	0.52	0.29
(e)	2.4	1.80	0.21	0.38
(f)	1.7	0.76	0.39	0.55

HW 13

3.27 Combine equations (3.22), (3.23), and (3.24) into Equation (3.21) to derive Equation (3.25).

3.28 The braking distance from 60 miles per hour (mph) is one measure of the brakes of an automobile. In this test, the brakes are applied while the car is traveling at a steady 60 mph. The braking distance is the distance from the point where the brakes were first applied to the point where the car comes to a complete stop. The average acceleration during braking is given by the following equation:

$$a = -\frac{V_o^2}{2D_b}$$

where V_o = initial velocity of the car

D_b = braking distance

Complete the following for a 2600-lb car that has a braking distance of 160 ft.

(a) Convert the initial velocity (60 mph) to feet per second and then to meters per second. Convert the mass (2600 lbm) to kilograms and the braking distance (160 ft) to meters. (The conversion factors are in Appendix B.)

(b) Compute the average acceleration in meters/second^2 and then use $f = ma$ to compute the average braking force in newtons.

(c) Convert the average braking force from newtons to pounds force (lbf).

(d) Compute the average acceleration in feet/second^2. Use the engineering fps system units (see Appendix B) to compute the average braking force in pounds. The fps system uses pounds to measure force (lbf) and pounds to measure mass (lbm). In this system, the relationship between force, mass, and acceleration is given by the following equation:

$$f = ma/g_c$$

The factor g_c is the acceleration due to gravity. The nominal value of g_c is 32.2.

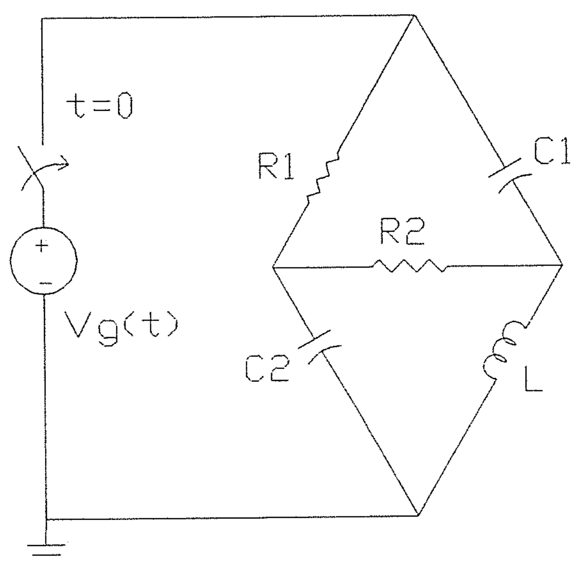
(e) Compare the average braking force obtained in (d) with the converted result obtained in (c).

HW 13

ET438a
Electric Circuit Modelling

Write the equations that completely describe the operation of the bridge circuit in the figure below using

a.) mesh equations



Hw 14

Integral: A mathematical expression for the accumulation of an amount (or quantity) of a variable. (4.2)
Integro-differential equation: An equation that has at least one derivative and one integral term. (4.2)
Inverse Laplace transform: A mathematical transformation that converts the solution of a differential equation in the frequency domain into a solution in the time domain. (4.4)
Laplace transform: A mathematical transformation that converts equations that involve derivatives and integrals in the time domain into equations that involve only algebraic terms in the frequency domain. (4.3)
Linear differential equation: A differential equation that has only first-degree terms (power = 1) in the dependent variable and its derivatives. (4.2)
Operational Laplace transform: The Laplace transform of a time-domain operation (e.g., addition, differentiation, integration, etc.). (4.3)
Order of a differential equation: The order of the highest derivative that appears in the equation. (4.2)
Partial fraction expansion: The result of breaking a ratio of polynomials into a sum of terms so that we can determine its inverse Laplace transform. (4.4)
Time constant (τ): A parameter that characterizes a group of components that are modeled by a linear first-order differential equation with constant coefficients. The unit of the time constant is seconds, which explains the name "time" constant. (4.2)
Time domain: A domain in which the equation that describes a component is a function of time. The equation defines the size of the output as a function of time and the input signal. (4.3)
Transfer function: The frequency-domain ratio of the output of a component over its input, with all initial conditions set to zero. (4.5)

◆ EXERCISES

Section 4.2

4.1 Equation (4.6) describes the relationship between the input flow rate (q_{in}) and the output level (h) of the self-regulating liquid tank shown in Figure 4.1. Determine the numerical values of τ and G for the following sets of parameter values, and write Equation (4.6) using the numerical values. Use a value of 9.81 m/s^2 for g.

(a) $R_L = 2.16 \times 10^6$ Pa · s/m^2, $A = 2.63$ m^2, $\rho = 1000$ kg/m^3
(b) $R_L = 5.76 \times 10^5$ Pa · s/m^2, $A = 0.35$ m^2, $\rho = 880$ kg/m^3

4.2 Equation (4.7) describes the relationship between the input flow rate (q_{in}) and the output level (h) of the nonregulating liquid tank shown in Figure 4.2. Determine the numerical value of A for the following tank diameters, and write Equation (4.7) using the numerical value of A.

(a) 1.5 m (b) 0.52 m

4.3 Equation (4.10) describes the relationship between the input voltage (e_{in}) and the output voltage (e_{out}) of the electrical circuit shown in Figure 4.3. Determine the numerical value of the time constant (τ) for the following sets of parameter values, and write Equation (4.10) using the numerical value of τ.

(a) $R_L = 22$ kΩ, $C_L = 47$ μF
(b) $R_L = 630$ kΩ, $C_L = 0.76$ μF

4.4 Equation (4.11) describes the relationship between the actual temperature (T_a) and the measured temperature (T_m) of the liquid-filled thermometer shown in Figure 4.4. Determine the numerical value of the time constant (τ) for the following sets of parameter values, and write Equation (4.11) using the numerical value of τ. Use natural convection in still water for the outside film coefficient.

	(a)	(b)
Liquid in bulb	mercury	ethyl alcohol
Bulb material	glass	glass
Bulb outside diameter, cm	1.94	0.86
Bulb outside length, cm	4.52	1.62
Bulb thickness, mm	1.4	1.1
Inside film coefficient, h_i, W/m^2 · K	4000	2210
T_m, °C	85	58
T_a, °C	20	8

Hw 12

4.5 Equation (4.17) describes the relationship between the inlet air pressure (p_{in}) and the position of the stem (x) of the control valve shown in Figure 4.5. Determine the numerical value of the mechanical capacitance of the valve spring for the following sets of parameters, and write Equation (4.17) using numerical values for A, m, R_m, and C_m. Hint: use Equation (3.50) to determine the mechanical capacitance: $C_m = \Delta x/\Delta F_a$ where Δx = the valve stroke (closed to open) and $\Delta F_a = \Delta p_{in} A$ change in applied force from closed to open.

	(a)	(b)
A, m^2	0.0182	0.0142
m, kg	0.561	0.387
R_m, N · s/m	9.60	9.10
ΔP_{in}, kPa	82.7	82.7
Δx, m	0.025	0.020

Section 4.3

4.6 Determine the frequency-domain function, $F(s)$, for each of the following time-domain functions, $f(t)$:

(a) $f(t) = 7.8$
(b) $f(t) = 3.2 \cos 1000t$
(c) $f(t) = 120 \sin 25t$
(d) $f(t) = 18t$
(e) $f(t) = 16e^{-8t}$
(f) $f(t) = 9e^{-3t} \sin 100t$
(g) $f(t) = 8.2te^{-2.5t}$
(h) $f(t) = 5e^{-7t} \cos 50t$
(i) $f(t) = 45e^{-5(t-6)}$
(j) $f(t) = 2 \sin(t - 6)$
(k) $f(t) = 4.8e^{-5t} \cos(400t - 36°)$
(l) $f(t) = 8\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt}$,

where $\dfrac{dx(0)}{dt} = 8,\ x(0) = -4$

(m) $f(t) = 12 \int x\,dt + 17x$

4.7 Determine the time-domain function, $f(t)$, for each of the following frequency-domain functions, $F(s)$:

(a) $F(s) = \dfrac{6.7}{s^2}$

(b) $F(s) = \dfrac{25\omega}{s^2 + \omega^2}$

(c) $F(s) = \dfrac{45}{s + 72}$

(d) $F(s) = 345/s$

(e) $F(s) = \dfrac{650}{(s + 8)^2}$

(f) $F(s) = \dfrac{250\omega}{(s + 4)^2 + \omega^2}$

(g) $F(s) = \dfrac{82}{s(5s + 1)}$

(h) $F(s) = \dfrac{16(s + 5)}{(s + 5)^2 + \omega^2}$

(i) $F(s) = \dfrac{28s}{s^2 + \omega^2}$

(j) $F(s) = \dfrac{64\angle 48°}{s + 8 - j16} + \dfrac{64\angle -48°}{s + 8 + j16}$

Section 4.4

4.8 Complete the partial fraction expansion and find the inverse Laplace transformation of each of the following functions:

(a) $\dfrac{4(s + 5)(s + 7)}{s(s + 3)(s + 6)}$ (b) $\dfrac{2(s + 5)}{(s + 1)^2}$

(c) $\dfrac{s + 2}{s^2 + 2s + 4}$

Section 4.5

4.9 The dead-time process shown in Figure 4.9 is described by the following equation:

$$f_o(t) = f_i(t - t_d)$$

where f_o = output signal, kg/s
f_i = input signal, kg/s
t_d = dead-time lag, s

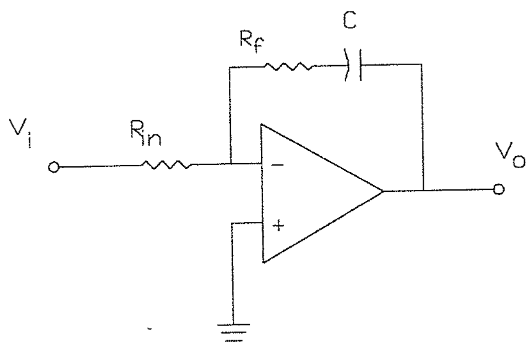
Determine the transfer function, $F_o(s)/F_i(s)$, from the preceding equation if the dead time, t_d, is 245 s.

4.10 Determine the transfer function, $I(s)/\theta(s)$, for the temperature transmitter described by the following differential equation:

HW 15

ET 438a
Continuous and Digital Control

Use the generalized gain formula and the Laplace impedance concept to find the output voltage v_o in terms of the input v_i for the circuit below. Assume that v_i and v_o are arbitrary functions of time.



HW16, HW17, HW18

4.5 Equation (4.17) describes the relationship between the inlet air pressure (p_{in}) and the position of the stem (x) of the control valve shown in Figure 4.5. Determine the numerical value of the mechanical capacitance of the valve spring for the following sets of parameters, and write Equation (4.17) using numerical values for A, m, R_m, and C_m. Hint: use Equation (3.50) to determine the mechanical capacitance: $C_m = \Delta x/\Delta F_a$ where Δx = the valve stroke (closed to open) and $\Delta F_a = \Delta p_{in} A$ change in applied force from closed to open.

	(a)	(b)
A, m^2	0.0182	0.0142
m, kg	0.561	0.387
R_m, N · s/m	9.60	9.10
ΔP_{in}, kPa	82.7	82.7
Δx, m	0.025	0.020

Section 4.3

HW16

4.6 Determine the frequency-domain function, $F(s)$, for each of the following time-domain functions, $f(t)$:

(a) $f(t) = 7.8$
(b) $f(t) = 3.2 \cos 1000t$
(c) $f(t) = 120 \sin 25t$
(d) $f(t) = 18t$
(e) $f(t) = 16e^{-8t}$
(f) $f(t) = 9e^{-3t} \sin 100t$
(g) $f(t) = 8.2te^{-2.5t}$
(h) $f(t) = 5e^{-7t}\cos 50t$
(i) $f(t) = 45e^{-5(t-6)}$
(j) $f(t) = 2 \sin(t - 6)$
(k) $f(t) = 4.8e^{-5t}\cos(400t - 36°)$
(l) $f(t) = 8\dfrac{d^2x}{dt^2} + 5\dfrac{dx}{dt}$,

where $\dfrac{dx(0)}{dt} = 8$, $x(0) = -4$

(m) $f(t) = 12 \int x\,dt + 17x$

HW17 4.7 Determine the time-domain function, $f(t)$, for each of the following frequency-domain functions, $F(s)$:

(a) $F(s) = \dfrac{6.7}{s^2}$

(b) $F(s) = \dfrac{25\omega}{s^2 + \omega^2}$

(c) $F(s) = \dfrac{45}{s + 72}$

(d) $F(s) = 345/s$

(e) $F(s) = \dfrac{650}{(s + 8)^2}$

(f) $F(s) = \dfrac{250\omega}{(s + 4)^2 + \omega^2}$

(g) $F(s) = \dfrac{82}{s(5s + 1)}$

(h) $F(s) = \dfrac{16(s + 5)}{(s + 5)^2 + \omega^2}$

(i) $F(s) = \dfrac{28s}{s^2 + \omega^2}$

(j) $F(s) = \dfrac{64\angle 48°}{s + 8 - j16} + \dfrac{64\angle -48°}{s + 8 + j16}$

Section 4.4

HW18

4.8 Complete the partial fraction expansion and find the inverse Laplace transformation of each of the following functions:

(a) $\dfrac{4(s + 5)(s + 7)}{s(s + 3)(s + 6)}$ (b) $\dfrac{2(s + 5)}{(s + 1)^2}$

(c) $\dfrac{s + 2}{s^2 + 2s + 4}$

Section 4.5

4.9 The dead-time process shown in Figure 4.9 is described by the following equation:

$$f_o(t) = f_i(t - t_d)$$

where f_o = output signal, kg/s
f_i = input signal, kg/s
t_d = dead-time lag, s

Determine the transfer function, $F_o(s)/F_i(s)$, from the preceding equation if the dead time, t_d, is 245 s.

4.10 Determine the transfer function, $I(s)/\theta(s)$, for the temperature transmitter described by the following differential equation:

HW 18

$$8.6\frac{di}{dt} + i = 0.1\theta$$

where i = output current signal, mA
θ = input temperature signal, °C

4.11 Determine the transfer function, $X(s)/I(s)$ for a process-control valve/electropneumatic converter described by the following differential equation.

$$0.0001\frac{d^2x}{dt^2} + 0.02\frac{dx}{dt} + x = 0.3i$$

where x = valve stem position, in.
i = current input signal to the converter, mA

4.12 Determine the transfer function, $\theta(s)/X(s)$, for a tubular heat exchanger similar to the one shown in Figure 2.3 and described by the following differential equation.

$$25\frac{d^2\theta}{dt^2} + 26\frac{d\theta}{dt} + \theta = 125x$$

where x = valve position, in.
θ = temperature of the fluid leaving the heat exchanger, °C

4.13 A manufacturing plant uses a liquid surge tank to feed a positive-displacement pump. The pump supplies a constant flow rate of liquid to a continuous heat exchanger. Determine the transfer function, $H(s)/Q(s)$, if the surge tank is described by the following equation:

$$h(t) = 0.5 \int q(t)\, dt$$

where $h(t)$ = level of liquid in the surge tank, m
$q(t)$ = difference between the input flow rate and the output flow rate, m^3
t = time, s

4.14 The spring-mass-damping system shown in Figure 3.8 is described by the following differential equation:

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = f$$

where m = mass, kg
R = dashpot resistance, N/(m/s)
K = spring constant, N/m
x = position of the mass, m
f = external force applied to the mass, N

Determine the transfer function, $X(s)/F(s)$, if

m = 3.2 kg
R = 2.0 N/(m/s)
K = 800 N/m

4.15 A PID controller is described by the following equation:

$$v = Pe + PD\frac{de}{dt} + PI\int e\, dt + v_o$$

where e = error, % of full scale (F.S.)
v = controller output, % of F.S.
v_o = controller output at t = 0 s, % of F.S.
P = proportional gain setting (dimensionless)
D = derivative action time constant, s
I = integral action rate, 1/s

Determine the transfer function, $V(s)/E(s)$, if P is 3.6, D is 0.008 s, I is 0.455 s, and v_o is 0.

4.16 An armature-controlled dc motor is sometimes used in speed and position control systems (see Figures 2.6 and 2.15). The dc motor operation is described by the following equations:

$$e = Ri + L\frac{di}{dt} + K_e\omega$$

$$i = \frac{q}{K_t}$$

$$q = J\frac{d\omega}{dt} + b\omega$$

where e = armature voltage, V
i = armature current, A
ω = motor speed, rad/s
q = motor torque, N · m
J = moment of inertia of the load, kg · m^2
b = damping resistance of the load, N · m/ (rad/s)
R = armature resistance, Ω
L = armature inductance, H
K_e = back emf constant of the motor, V/(rad/s)
K_t = torque constant of the motor, N · m/A

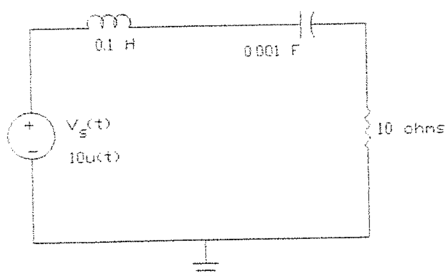
A small permanent-magnet dc motor has the following parameter values:

$$J = 8 \times 10^{-4}\ \text{kg} \cdot \text{m}^2$$

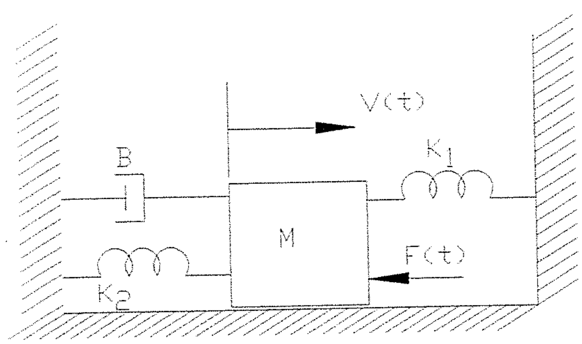
Hw 19, 20

ET 438a
Continuous and Digital Control
Laplace Tranform of Dynamic Systems

Hw 19 1.) Write the integro-differential equation that describes the loop current response of the circuit below and then convert it to an algebraic equation using the Laplace transform.



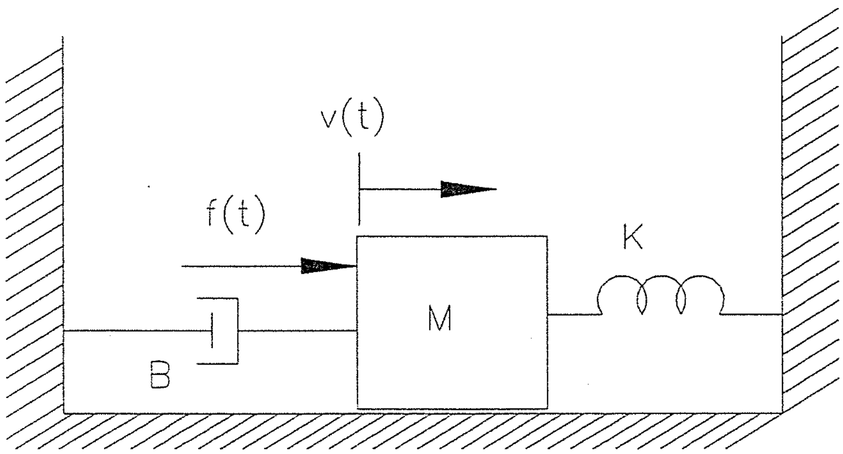
Hw 20 2.) Write the Laplace formula that describes the velocity response of the system below to the force F(t) = 5sin(0.1t)



Hw 2.1

ET 438a
Solving Differential Equations With Laplace Transforms

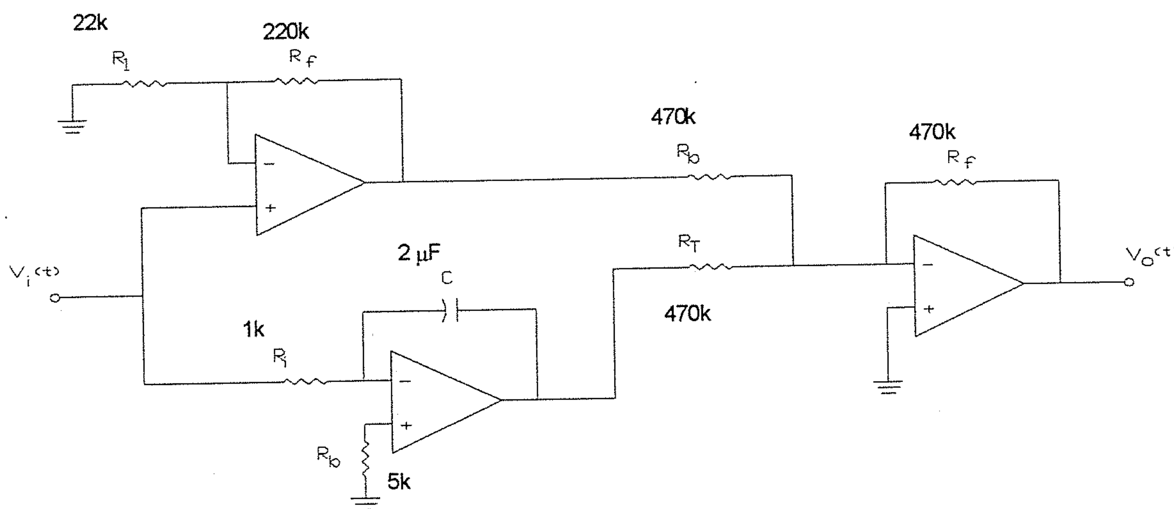
Using the Laplace transform method, find the function v(t) if the system below is disturbed by a force f(t) = 2.5 N-m at t = 0. Assume that there is no friction between the mass and the surface. The spring constant is 1.5 N/m. The block has a mass of 0.5 kg, and the viscous damping constant is 0.25 N-s/m.

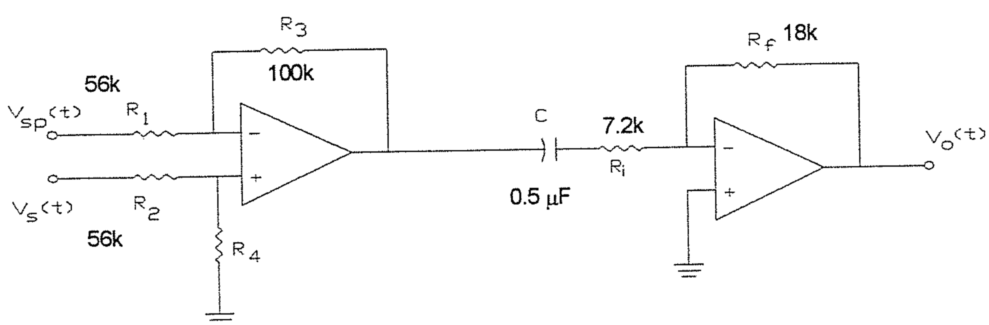


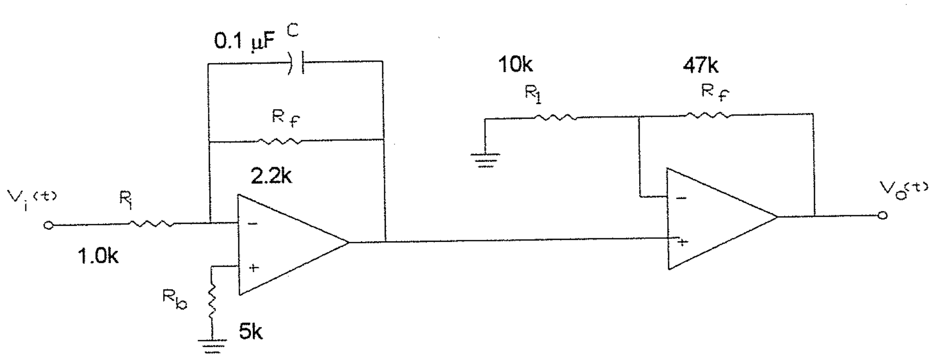
Hw 22

ET 438a
Transfer Functions and Block Diagrams

Find the transfer function and draw a block diagram for each of the circuits shown below.







Hw 23

$$8.6\frac{di}{dt} + i = 0.1\theta$$

where i = output current signal, mA
θ = input temperature signal, °C

4.11 Determine the transfer function, $X(s)/I(s)$ for a process-control valve/electropneumatic converter described by the following differential equation.

$$0.0001\frac{d^2x}{dt^2} + 0.02\frac{dx}{dt} + x = 0.3i$$

where x = valve stem position, in.
i = current input signal to the converter, mA

4.12 Determine the transfer function, $\theta(s)/X(s)$, for a tubular heat exchanger similar to the one shown in Figure 2.3 and described by the following differential equation.

$$25\frac{d^2\theta}{dt^2} + 26\frac{d\theta}{dt} + \theta = 125x$$

where x = valve position, in.
θ = temperature of the fluid leaving the heat exchanger, °C

4.13 A manufacturing plant uses a liquid surge tank to feed a positive-displacement pump. The pump supplies a constant flow rate of liquid to a continuous heat exchanger. Determine the transfer function, $H(s)/Q(s)$, if the surge tank is described by the following equation:

$$h(t) = 0.5 \int q(t)\,dt$$

where $h(t)$ = level of liquid in the surge tank, m
$q(t)$ = difference between the input flow rate and the output flow rate, m^3
t = time, s

4.14 The spring-mass-damping system shown in Figure 3.8 is described by the following differential equation:

$$m\frac{d^2x}{dt^2} + R\frac{dx}{dt} + Kx = f$$

where m = mass, kg
R = dashpot resistance, N/(m/s)
K = spring constant, N/m
x = position of the mass, m
f = external force applied to the mass, N

Determine the transfer function, $X(s)/F(s)$, if

m = 3.2 kg
R = 2.0 N/(m/s)
K = 800 N/m

PID controller is described by the following equation:

$$v = Pe + PD\frac{de}{dt} + PI \int e\,dt + v_o$$

where e = error, % of full scale (F.S.)
v = controller output, % of F.S.
v_o = controller output at $t = 0$ s, % of F.S.
P = proportional gain setting (dimensionless)
D = derivative action time constant, s
I = integral action rate, 1/s

Determine the transfer function, $V(s)/E(s)$, if P is 3.6, D is 0.008 s, I is 0.455 s, and v_o is 0.

4.16 An armature-controlled dc motor is sometimes used in speed and position control systems (see Figures 2.6 and 2.15). The dc motor operation is described by the following equations:

$$e = Ri + L\frac{di}{dt} + K_e\omega$$

$$i = \frac{q}{K_t}$$

$$q = J\frac{d\omega}{dt} + b\omega$$

where e = armature voltage, V
i = armature current, A
ω = motor speed, rad/s
q = motor torque, N · m
J = moment of inertia of the load, kg · m^2
b = damping resistance of the load, N · m/ (rad/s)
R = armature resistance, Ω
L = armature inductance, H
K_e = back emf constant of the motor, V/(rad/s)
K_t = torque constant of the motor, N · m/A

A small permanent-magnet dc motor has the following parameter values:

$$J = 8 \times 10^{-4}\ \text{kg} \cdot \text{m}^2$$

$$b = 3 \times 10^{-4}\ \text{N} \cdot \text{m/(rad/s)}$$
$$R = 1.2\ \Omega$$
$$L = 0.020\ \text{H}$$
$$K_e = 5 \times 10^{-2}\ \text{V/(rad/s)}$$
$$K_t = 0.043\ \text{N} \cdot \text{m/A}$$

Substitute these parameters into the preceding equations to obtain the exact differential equations of the dc motor. Determine the transfer function, $\Omega(s)/E(s)$, by transforming all three equations into frequency-domain algebraic equations. Use algebraic operations to obtain the ratio of Ω/E, which is the desired transfer function.

4.17 A PD controller is described by the following equation:

$$v = Pe + PD\frac{de}{dt} - \alpha D\frac{dv}{dt} + v_o$$

where
- e = error, % of full scale (F.S.)
- v = controller output, % of F.S.
- v_o = controller output at $t = 0$ s, % of F.S.
- P = proportional gain setting (dimensionless)
- D = derivative action time constant, s
- α = derivative limiter coefficient

Determine the transfer function, $V(s)/E(s)$ if P is 2, D is 0.025 s, α is 0.1 and v_o is 0.

4.18 The PD controller in Exercise 4.17 is one example of a control system component capable of a sudden change in output value. To show the sudden change, determine the time-domain response of the output $v(t)$ to the following step change in the input $e(t)$. Refer to Example 4.16 for further guidance.

$$e(t) = 0\ \%\ \text{F.S.}, \quad t < 0\ \text{s}$$
$$= 10\ \%\ \text{F.S.}, \quad t \geq 0\ \text{s}$$

Section 4.6

4.19 **(a)** Use the initial and final value theorems to determine the initial and final values of $v(t)$ for the step response of the PD controller in Exercise 4.18.
(b) Use the time-domain equations you developed in Exercise 4.18 to determine the initial and final values of $v(t)$.
(c) Compare the values from part a with the values from part b.

4.20 A temperature transmitter is calibrated so that the output is 0 mA when the input temperature is 0°C and 10 mA when the input temperature is 100°C. The temperature probe was held in ice water for a long time before time $t = 0$ s when it was suddenly plunged into water boiling at 100°C. The output current, $I(s)$, that resulted from this sudden temperature change is given by the following equation:

$$I(s) = \frac{10.0}{s(1 + 25s)},\ \text{mA}$$

Determine the initial and final values of the output current, $i(t)$. Do your initial and final values make sense? Explain your answer.

4.21 The armature-controlled dc motor in Exercise 4.16 was not moving at $t = 0$ s when the armature voltage was suddenly increased from 0 to 50 V. The motor speed, $\Omega(s)$, that resulted from this sudden voltage change is given by the following equation:

$$\Omega(s) = \frac{50}{s(0.0584 + 0.225s + 0.00037s^2)},\ \text{rad/s}$$

Determine the initial and final values of the motor speed. Do your initial and final values make sense? Explain your answer.

4.22 The liquid surge tank in Exercise 4.13 was empty at $t = 0$ s when the input flow rate, q_{in}, was suddenly changed from 0 to 0.001 m^3/s. Also at $t = 0$ s, the output flow rate, q_{out}, changed from 0 to 0.00037 m^3/s. The level of the liquid in the tank, $H(s)$, is given by the following equation:

$$H(s) = \left[\frac{q_{in} - q_{out}}{s}\right]\left[\frac{0.5}{s}\right],\ \text{m}$$

Determine the initial and final values of the liquid level. Does the initial value make sense? Explain the meaning of the final value. Given that the tank has a capacity of 0.5 m^3, how long will it take to fill the tank at the flow rates given above?

Section 4.7

4.23 Use the program BODE to generate frequency-response data from the transfer functions obtained in Exercises 4.10, 4.11, 4.12, 4.14, 4.15, 4.16, and 4.17. Construct Bode diagrams from the data.

Hw 23

$N_{max} = 5000$ rpm

$R = 0.8\ \Omega$

Determine the following:

(a) The maximum output torque
(b) The maximum power output
(c) The maximum armature voltage
(d) The no-load motor speed when $e = E_{max}$

10.15 The motor in Exercise 10.7 is operated at 3000 rpm with a load torque of 0.15 N · m. Determine the following:

(a) The armature voltage
(b) The armature speed if the torque increases to 0.30 N · m and the armature voltage is not changed

10.16 A dc motor has the following specifications:

Type: dc permanent magnet motor	
Model No. ICST-6	
Maximum operating speed, ω_{max} (rad/s)	524
Maximum armature current, I_{max} (A)	2.4
Voltage constant, K_E (V · s/rad)	0.15
Torque constant, K_T (N · m/A)	0.15
Friction torque, T_f (N · m)	0.03
Armature resistance, R (Ω)	0.8
Armature inductance, L (H)	0.045
Armature moment of inertia, J_m (N · m · s²/rad)	10.2E − 4
Armature viscous friction, B_m (N · m · s/rad)	1.2E − 4

Determine the following velocity and position transfer functions.

(a) The voltage-driven motor
(b) The simplified voltage-driven motor with τ_e neglected
(c) The current-driven motor
(d) The voltage-driven motor with the following speed-reducer and load specifications:

$$N_1 = 16 \text{ teeth}$$
$$N_2 = 48 \text{ teeth}$$
$$J_L = 7.6\text{E} - 3 \text{ N} \cdot \text{m} \cdot \text{s}^2/\text{rad}$$
$$B_L = 0.20\text{E} - 4 \text{ N} \cdot \text{m} \cdot \text{s/rad}$$

10.17 The following specifications were obtained from a manufacturer's catalog:*

Type: dc ceramic permanent magnet motor	
Model No. SD 12-20	
Rated voltage, E_r (V)	90
No-load speed, N_o (rpm)	1386
Maximum continuous current, I_{max} (A)	2.3
Continuous stall torque, T_{max} (lb · in.)	12
Friction torque, T_f (lb · in.)	1.8
Voltage constant, K_E (V/krpm)	65
Torque constant, K_T (lb · in./A)	5.49
Armature resistance, R (Ω)	11.30
Armature inductance, L (mH)	30.0
Armature moment of inertia, J_m (lb · in. · s²/rad)	0.007
Armature viscous friction, B_m (lb · in. · s/rad)	0.003

Convert the motor parameters to SI units and determine the full and simplified velocity transfer functions.

10.18 Repeat Exercise 10.17, with the following manufacturer's specifications:†

Type: dc rare-earth permanent magnet motor	
Model No. SX 12-30	
Rated voltage, E_r (V)	90
No-load speed, N_o (rpm)	1828
Maximum continuous current, I_{max} (A)	2.1
Continuous stall torque, T_{max} (lb · in.)	8.9
Friction torque, T_f (lb · in.)	1.59
Voltage constant, K_E (V/krpm)	49
Torque constant, K_T (lb · in./A)	4.16
Armature resistance, R (Ω)	5.10
Armature inductance, L (mH)	9.00
Armature moment of inertia, J_m (lb · in. · s²/rad)	0.0035
Armature viscous friction, B_m (lb · in. · s/rad)	0.004

10.19 The voltage-driven motor, speed reducer, and load are shown in Figure 10.14. The transfer function for this system is defined by Equation (10.29) which uses the total inertia (J_T) defined by Equation (10.26). The inertia of the two gears was ignored in Equation (10.26). Revise Equation (10.26) so that it includes the inertia of the gears.

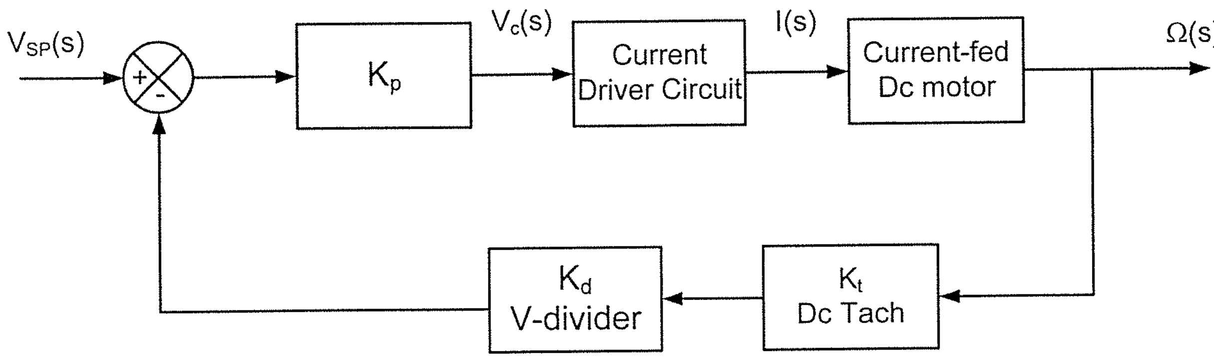
*D.C. Servomotors, Baldor Electric Company, Fort Smith, AR, Form #DB-210-A, 1986.

†Rare Earth D.C. Servomotors, Baldor Electric Company, Fort Smith, AR, Form #DB-220-A, undated.

HW27

ET 438A
Dc Motor Control Homework

A proportional controller is applied to a current-fed permanent magnet dc motor. The objective is to maintain a constant speed on the output. The speed can be changed by adjusting the set point voltage V_{sp}. The block diagram for this system is shown below.



A dc tachometer measures the motor speed. The tachometer sends an output to a scaling circuit to bring the voltage into the set point input range.

The parameters defining the feedback circuit are:

K_T = 0.064 V-s/rad
K_d = 0.1 V/V

The controller drives a transistor circuit that is the current source for the dc motor. The current source takes the voltage from the controller and produces a current based on the following function

$$i(t)=(5.5\times10^{-4}V_c(t))h_{fe} \ \ \text{A/V}$$

$$h_{fe} = 100$$

The dc motor and load parameters are:

K_e = 0.064 V-s/rad
K_t = 0.064 N-m/A
L_a = 0.10 H
R_a = 22.6 ohms
J_m = 1.1e-4 N-m-s^2/rad
B_m = 9.204e-6 N-m-s/rad
J_L = 8.2e-4 N-m-s^2/rad
B_L =0.15e-5 N-m-s/rad

HW23

The load is directly coupled to the motor.

For the system described above:

1.) find the closed loop transfer functions when K_p = 1, 10, 100. Assume that the offset of the proportional controller is zero. This is required for all transfer function models.

2.) Find the poles and zeros of the transfer functions found in 1. Evaluate the poles and determine if the system is stable of each of the gain values given in 1.

3.) Use Matlab's control tool box functions to generate bode plots for each of the transfer functions given in 1. Turn in the graphs with the other calculations.

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ET 438a

Using Matlab to Generate Bode and Step Responses

1.) Start any of the computers in D106 and find the student version of Matlab under the program menu.
2.) Start Matlab. After Matlab loads, click on the file menu and select New m-File. This will start a text editor for making Matlab script files.
3.) Enter the following lines of code and save the file.

```
clear all;
close all;
n=input('enter the numerator coefficients: ');
d=input('enter the denominator coefficients: ');
kp=input('enter the value of Kp: ');
sys=tf(n,d);
sys
bode(sys);
grid on;
G=kp./n;
sys=sys*G
sys
figure;
step(sys);
```

4.) From the Matlab prompt, EDU>>, type the filename that you give to this script file. This will run the script file. You can also use the run script command and browse for the script file.
5.) Follow the program prompts. Enter the numerator coefficients in decreasing powers of s (e.g. s^2, s^1, s^0).
6.) Enter the denominator coefficients in decreasing powers of s.
7.) The program will automatically produce the Bode and step responses of the defined transfer function.
8.) Cut and paste the graphs into Word for printing or print the results directly from Matlab. Use the copy figure command to make copies of the Matlab figures.

Hw 25

◆ EXERCISES

Section 13.1

13.1 Describe the three operations performed by a feedback control system.

13.2 Name the five common modes of control.

13.3 Name the two parts of a controller.

13.4 Name three methods of implementing a controller.

13.5 Describe how you would determine if there has been a load change in a control system that is maintaining the controlled variable at a constant set point value (e.g., a regulator system).

Section 13.2

13.6 Describe the operation of each of the five modes of control.

13.7 Describe three ways to reduce the amplitude of the oscillation in a two-position control system.

13.8 Describe the conditions for which you would select a two-position control mode.

13.9 The two-position controller in Example 13.1 is modified so that the valve moves between two partially open positions instead of between ON and OFF. The inlet flow rate, m, is 0.001 m^3/s when the valve is in the minimum flow position and 0.003 m^3/s when the valve is in the maximum flow position. The rest of the conditions are the same as in Example 13.1. Determine the amplitude and the period of oscillation of the level, h. Compare your results with the results in Example 13.1. What was given up to reduce the amplitude of the oscillation?

13.10 Describe the conditions for which you would select a floating control mode.

13.11 The solid flow rate control system shown in Figure 2.19 uses a gate to control the level of material on the belt. A single-speed reversible motor is used to drive the cam that positions the gate. If the solid feed rate is below a predetermined value, the controller drives the gate up at a constant rate. If the feed rate is above a second predetermined value, the controller drives the gate down at a constant rate. Between the two predetermined values, the gate is motionless. Identify the mode of control used in this system.

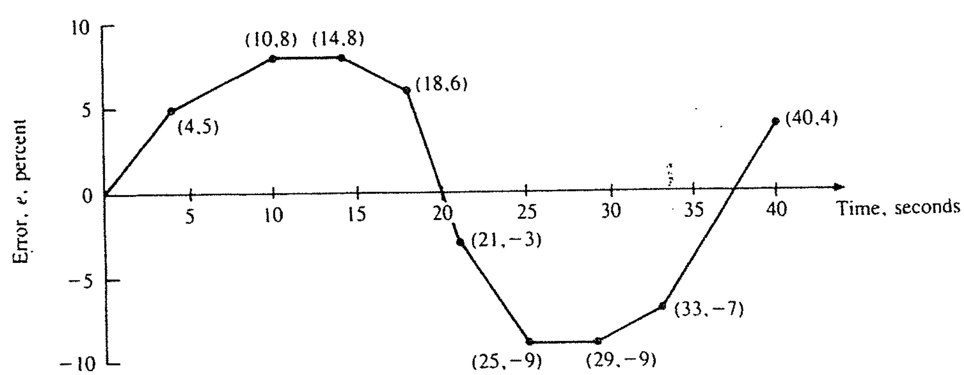
13.12 Sketch the input/output graph for each of the following proportional controller conditions:
(a) $P = 8$, $v_0 = 40\%$
(b) $P = 0.25$, $v_0 = 55\%$
(c) $P = 0.5$, $v_0 = 25\%$

13.13 Describe the conditions for which you would select a proportional control mode.

13.14 Determine the proportional offset required to maintain $v - v_0$ at 12% for proportional controllers with each of the following gain values:
(a) $P = 0.2$
(b) $P = 0.6$
(c) $P = 1.2$

13.15 Describe the conditions for which you would select a PI control mode.

13.16 A PI controller has a gain of 0.5, an integral action rate of 0.0125 s^{-1}, and a value of v_0 of 25%. The graph of the error signal is given in Figure 13.32. Determine the value of the controller output at $t = 15$ s and $t = 30$ s.



◆ **Figure 13.32** Error graph used in Exercises 13.16 through 13.8.

HW 25 HW 26 HW 27

13.17 Describe the conditions for which you would select a PD control mode.

HW 25 **13.18** A PD controller has the following parameter values: $P = 0.5$, $D = 12$ s, $\alpha = 0$, and $v_0 = 42\%$. The graph of the error signal is given in Figure 13.32. Determine the value of the controller output at **(a)** $t = 15$ s and **(b)** $t = 30$ s.

13.19 Describe the conditions for which you would select a PID control mode.

HW 26 **13.20** A PID controller has the following parameter values: $P = 0.5$, $I = 0.0125\ s^{-1}$, $D = 12$ s, $\alpha = 0$, and $v_0 = 55\%$. The graph of the error signal is given in Figure 13.32. Determine the value of the controller output at **(a)** $t = 15$ s and **(b)** $t = 30$ s.

13.21 Equation (13.21) defines the transfer function for a PID controller. Use program BODE to print a Bode Data Table for each of the following sets of controller parameters. Parameter set a sets a basis for comparison, and parameter sets b, c, and d each reduce one control mode by 50%. Highlight the gain and phase values in the Bode Data Tables for parameter sets b, c, and d that differ from the corresponding gain and phase values in the table for parameter set a. The highlighted values mark the values changed by the reductions in the P, I, and D control modes.

(a) $P = 4.0$, $I = 0.14\ s^{-1}$, $D = 0.50$ s, $\alpha = 0.1$
(b) $P = 2.0$, $I = 0.14\ s^{-1}$, $D = 0.50$ s, $\alpha = 0.1$
(c) $P = 4.0$, $I = 0.07\ s^{-1}$, $D = 0.50$ s, $\alpha = 0.1$
(d) $P = 4.0$, $I = 0.14\ s^{-1}$, $D = 0.25$ s, $\alpha = 0.1$

The conditions in Exercise 13.21 were selected to show how each control mode (P, I, D) affects the frequency response of the controller.

(a) Review your Bode Data Table for parameter set b, and comment on the effect of the 50% reduction in the P control mode.

(b) Review your Bode Data Table for parameter set c, and comment on the effect of the 50% reduction in the I control mode.

(c) Review your Bode Data Table for parameter set d, and comment on the effect of the 50% reduction in the D control mode.

(d) Draw a Bode diagram and plot the gain and phase angle for parameter set b, using solid lines. Then plot the gain and phase angle for parameter set a, using dashed lines where the values differ from set b.

(e) Repeat Exercise 13.22d with parameter set b replaced by parameter set c.

(f) Repeat Exercise 13.22d with parameter set b replaced by parameter set d.

HW 26 **13.23** A certain process has a small capacitance. Sudden, moderate load changes are expected, and a small offset error can be tolerated. Recommend the control mode or combination of modes most suitable for controlling this process.

13.24 A process has a large capacitance and no dead-time lag. The anticipated load changes are relatively small. Recommend the control mode or combination of modes most suitable for controlling this process.

13.25 A dc motor is used to control the speed of a pump. A tachometer–generator is used as the speed sensor. The load changes are insignificant, and there is no dead-time lag in the process. Recommend the control mode or combination of modes most suitable for controlling the motor speed.

13.26 A liquid flow process is fast and the flow rate signal has many "noise spikes." Large load changes are quite common, and there is very little dead-time lag. Recommend the control mode or combination of modes most suitable for controlling the flow rate. Why is the derivative mode usually avoided in liquid flow controllers?

13.27 An electric heater is used to control the temperature of a plastic extruder. The process is a first-order lag with almost no dead-time lag. The time constant is very large. Under steady operation, the load changes are insignificant. Recommend the control mode or combination of modes most suitable for controlling the temperature of the extruder.

13.28 A dryer is a slow process with a very large dead-time lag. Sudden load changes are common, and proportional offset is undesirable. Recommend the control mode or combination of modes most suitable for controlling the dryer.

13.29 A PI controller defined by Equation 13.11 has a gain that increases as frequency decreases. At very low frequencies, the gain becomes very large. In practical PI controllers, the maximum gain is limited. In the following transfer function of a PI controller, the term b in the denominator limits the gain to a maximum value of $1/b$.

$$\frac{V}{E} = P\left(\frac{I + s}{bI + s}\right)$$

Use the program BODE to generate two sets of frequency data for parameter sets 1 and 2 below (see Example 4.19). Plot both sets of data on a single Bode diagram and comment on the comparison of the effect of term b.

Set 1: $P = 1$, $I = 0.1\ s^{-1}$, $b = 0$
Set 2: $P = 1$, $I = 0.1\ s^{-1}$, $b = 0.01$

Section 13.3

HW 27

13.30 An analog proportional mode controller (Figure 13.19) uses a value of 100 kΩ for R_f. Determine the value of R_1 for each of the following controller gains:

Hw 26 Hw 27

(a) $P = 0.25$
(b) $P = 1.75$
(c) $P = 8.6$

13.31 Determine the values of R_1 and R_i for an analog PI controller (Figure 13.20) with $P = 6.4$ and $I = 0.00541$ s^{-1}. Use a 1000-μF capacitor for C_i.

Hw 27 **13.32** Determine the value of R_I, R_d, and R_f for an analog PD controller (Figure 13.21) with $P = 3.6$, $D = 12$ s, and $\alpha = 0.1$. Use a 100-μF capacitor for C_d.

13.33 Determine the values of R_1, R_i, R_d, and C_i for an analog PID controller, as shown in Figure 13.22a. The controller parameters are $P = 0.65$, $I = 1/70$ s^{-1}, $D = 0.32$ s, $\alpha = 0.1$. Use a 1-μF capacitor for C_d.

Hw 26 **13.34** Use the program BODE to generate frequncy-response data for (a) the noninteracting PID controller [Equation (13.21)], and (b) the interacting PID controller [Equation (13.23)]. Plot Bode diagrams for both controllers. Comment on the difference between the two Bode diagrams. The controller has the following parameters: $P = 4.0$, $I = 1/7$, $D = 0.5$, and $\alpha = 0.1$.

Section 13.4

13.35 A digital controller is started up with the setpoint (SP) at 20%, and the controlled variable at 0%. The first 24 samples are listed below. For each sample, compute the value of the error, e_n, the summation of errors, S_n, and the change in the error signal, Δe_n, as defined by the following equations.

$$e_n = SP - C_{mn} \qquad S_n = \sum_{i=1}^{i=n} e_i \qquad \Delta e_n = e_n - e_{n-1}$$

Put your result in a table with five columns and the following heading:

n	$C_{mn}(\%)$	$e_n(\%)$	$S_n(\%)$	$\Delta e_n(\%)$

First 24 Samples of the Digital Controller

n	$C_{mn}(\%)$	n	$C_{mn}(\%)$	n	$C_{mn}(\%)$
1	0.0	9	18.5	17	19.8
2	3.1	10	20.0	18	20.0
3	5.9	11	21.4	19	20.1
4	8.5	12	22.0	20	20.2
5	10.9	13	21.2	21	20.1
6	13.1	14	20.0	22	20.0
7	15.1	15	19.8	23	20.0
8	16.9	16	19.5	24	20.0

13.36 The digital controller in Exercise 13.35 uses the positional algorithm defined by Equation (13.24).

$$v_n = P_{mode} + I_{mode} + D_{mode}$$

where $P_{mode} = Pe_n$
$I_{mode} = PI\Delta t S_n$
$D_{mode} = PD\Delta e_n/\Delta t$

The PID modes and sample time are set to the following values:

$$P = 2 \qquad I = 0.1\ s^{-1} \qquad D = 1\ s \qquad \Delta t = 2\ s$$

Compute P_{mode}, I_{mode}, D_{mode}, and V_n for each sample in Exercise 13.36. Put your results in a table with five columns and the following heading.

n	P_{mode} (%)	I_{mode} (%)	D_{mode} (%)	V_n (%)

13.37 Construct the following graphs of your results from Exercise 13.36: C_{mn} vs. n, P_{mode} vs. n, I_{mode} vs. n, and D_{mode} vs. n.

13.38 Draw a PID flow diagram similar to Figure 13.23 with the following change: Replace the integral mode operation with the following operations:

(a) Add the nth sample error, e_n, to a new variable called the unused sum, S:

$$S = S + e_n$$

(b) Divide S by I_{DIV} to obtain a quotient, Q, and a remainder, R.

$$I_{DIV} = \frac{1}{PI\ \Delta t}$$

$$Q = S \text{ div } I_{DIV}$$

$$R = S \text{ mod } I_{DIV}$$

(c) Add Q to the past integral mode action, I_{n-1}, to form the current integral mode action, I_n.

$$I_n = I_{n-1} + Q$$

(d) Set the unused sum, S, equal to the remainder, R:

$$S = R$$

Explain the purpose of these operations.

13.39 Draw a PID flow diagram similar to Figure 13.23 with the following change: Replace the derivative mode operation in the PID flow diagram (Figure 13.23) with a

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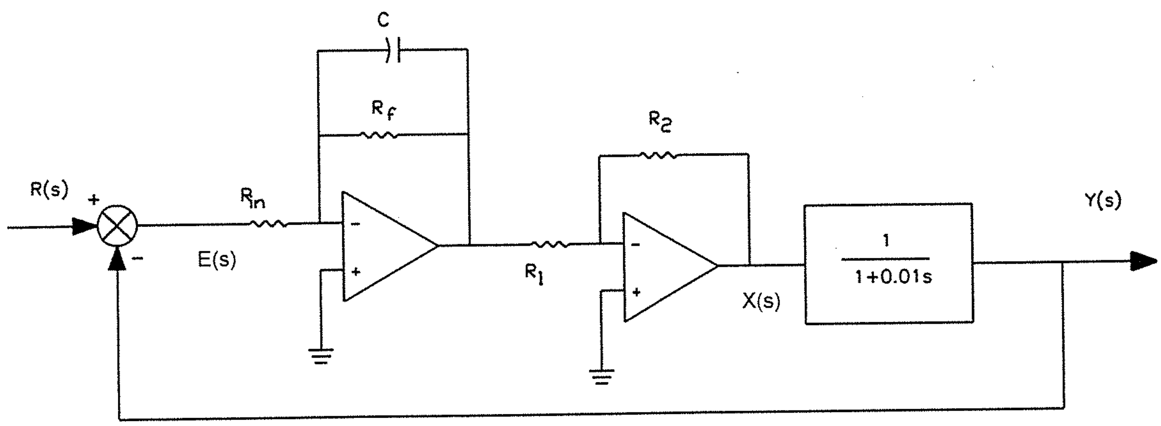
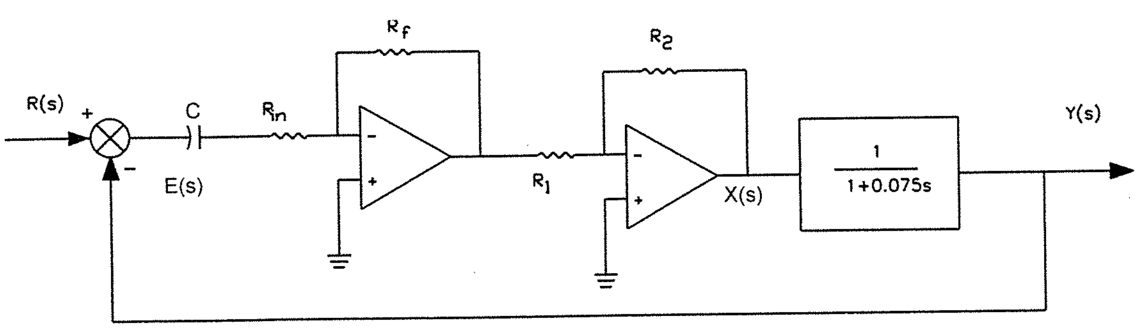
ET438a
Practical Integrator and differentiator Circuits
Homework

1. Practical integrator circuits in control
a) Find the transfer function, X(s)/E(s), of the practical integrator circuit in the control system diagram below.
b.) Find the closed loop transfer function of the system Y(s)/R(s).
R_{in} = 10kΩ R_f = 50 kΩ C = 1uF R_1= 100kΩ R_2 = 100kΩ



2. Practical differentiator circuits in control
a.) Find the transfer function, X(s)/E(s) of the practical differentiator circuit including the inverting amplifier.
b.) Find the closed loop transfer function of the the system Y(s)/R(s).



R_f = 10kΩ R_{in} = 12kΩ C = 2.5 uF R_1 = 100kΩ R_2 = 220kΩ

Hw 29

ET 438a
Continuous and Digital Control
Integral Process Homework

A tank is drained by a pump that removes liquid at a fixed rate of 0.027 m^3/sec. The full scale flow rate of the pump is 0.075 m^3/sec. The tank has a diameter of 5 meters and is 7.5 meters high. This height is considered the full scale level that the tank can hold. The current level (t=0) in the tank is 35% of it full scale value. The input flow rate into the tank varies due to system demand and is modeled with the following equation:

$$q_{in}(t)=0.0097\sin(0.25t)+0.022 \text{ m}^3/\text{sec}$$

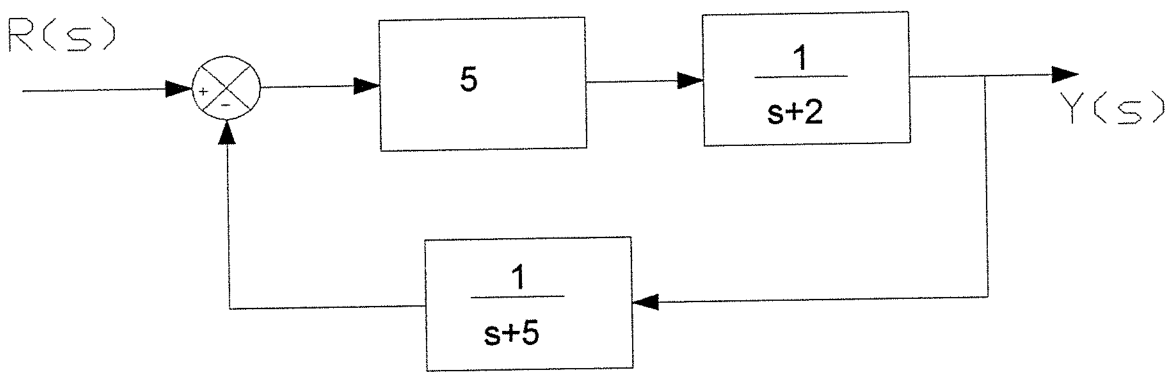
Assuming that q_{out} is constant find: 1.) percent q_{in} in terms of full scale pump flow, 2.) the integral time constant for the system, Ti, 3.) a function that gives the tank height as a function of time, h(t), with the initial height of liquid level set at t=0. 4.) Find the height of liquid in the tank after 5 minutes has elapsed from t=0.

Hw: 30

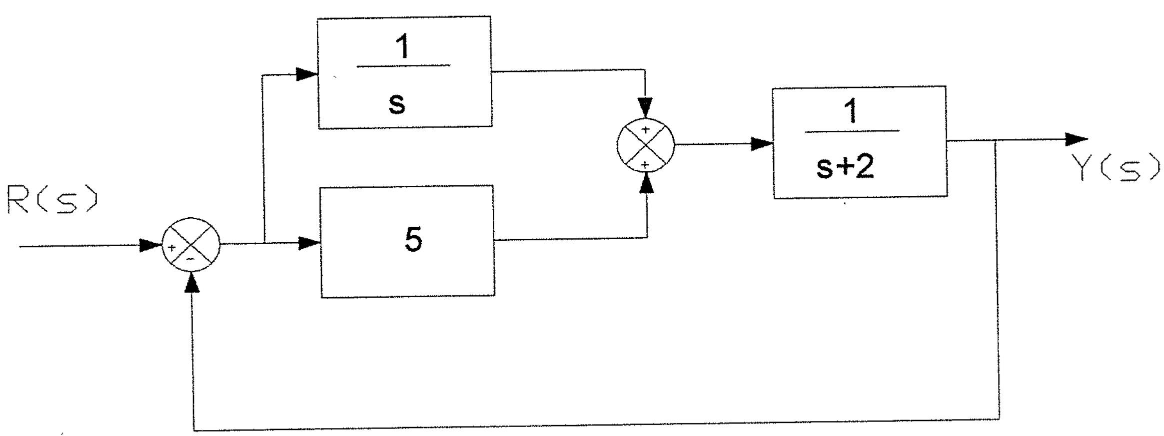
ET 438a
Continuous and Digital Control
Second Order System Homework

Find the closed-loop transfer functions for each of the block diagram systems given below. Determine the natural frequnency ω_n, and the damping factor ζ of each system

a.)



b.)



Hw 31

(b) The input flow rate varies and the output flow rate is constant.
(c) Both the input and output flow rates are constant, but they are not equal.
(d) The input flow rate is equal to the output flow rate.
(e) The amount of stored material or energy is constant.

14.4 The input signal (inflow minus outflow) to an integral process is suddenly changed from 0% to 10% and then held at 10%. The process output was at 20% when the step change occurred. Determine the output at the following times (measured from the instant the step change occurred). The process has an integral action time constant of 40 s.
(a) 20 s
(b) 40 s
(c) 80 s
(d) 120 s

14.5 The input signal (inflow minus outflow) to an integral process varies sinusoidally at a radian frequency, ω_1, of 0.025 rad/s. The output signal varies sinusoidally at the same frequency with an amplitude of 10%. Determine the amplitude of the output for each of the following input signal radian frequencies, assuming the input signal amplitude is unchanged.
(a) $\omega_2 = 0.25$ rad/s
(b) $\omega_2 = 0.10$ rad/s
(c) $\omega_2 = 0.010$ rad/s
(d) $\omega_2 = 0.005$ rad/s

14.6 A liquid level integral process (Figure 14.2a) has the following parameters and conditions:

Tank height = 5.2 m
Tank diameter = 1.8 m
FS_{in} = 0.02 m3/s
FS_{out} = 5 m
$h^*(t_o)$ = 1.7 m = 34% of FS
q^*_{out} = 0.009 m^3/s = 45% of FS
q^*_{in} = 0.011 m^3/s = 55% of FS

Determine the time-domain equation, the transfer function, the integral action time constant, and the level at time t_o + 50 s.

14.7 Construct a Bode diagram of the integral process in Exercise 14.1. The input signal is expressed as a percentage of FS_{in}, and the output signal is expressed as a percentage of FS_{out}. Use the Bode diagram to determine the output amplitude produced by a sinusoidal input at each frequency given below. The input signal has an amplitude of 10% of FS_{in}. Express the output amplitude both as a percentage of FS_{out}, and in meters. Indicate any output that is limited by FS_{out}.

Frequency (rad/s)
1.57×10^{-5}
1.57×10^{-4}
1.57×10^{-3}
1.57×10^{-2}
1.57×10^{-1}
1.57×10^{0}

Section 14.3

14.8 A liquid level first-order lag process has an output flow rate, q_{out}, of 2 gpm when the tank level, $h(t)$, is at 40% of full scale. Determine the output flow rate for each of the following values of the level, $h(t)$.
(a) 20%
(b) 80%
(c) 10%
(d) 100%

14.9 The input signal of a first-order lag process is suddenly changed from 10% to 30% of full scale. The process output was at 10% of full scale when the step change occurred. The process has a time constant of 20 seconds. Two hundred seconds after the step change, the output was 30% of full scale. Determine the output at each of the following times after the step change.
(a) 20 s
(b) 40 s
(c) 60 s
(d) 80 s
(e) 100 s

14.10 A first-order lag process has a time constant, τ, of 10 s, and a steady-state gain, G, of 1. Use the Bode diagram in Figure 14.7 to determine the gain, m, in decibels, and the phase angle, β, in degrees for each of the following values of the radian frequency.
(a) 0.001 rad/s
(b) 0.01 rad/s
(c) 0.1 rad/s
(d) 1.0 rad/s
(e) 10.0 rad/s

14.11 The gain values in Exercise 14.10 are expressed in decibels. If g is the ordinary gain value, and m is the decibel gain value, then the following equations convert between dB and ordinary gain.

$$m = 10 \log_{10} g \qquad g = 10^{m/20}$$

Convert the decibel gain values, m, that you obtained in Exercise 14.10 into ordinary gain values, g.

HW31 ... HW34

program BODE to print two Bode Data Tables to check the graphical results you obtained in Exercises 14.23 and 14.24. Comment on the comparison between the values you obtained from the graph in Figure 14.13 and the values you obtained from program BODE.

HW33 **14.26** A spring-mass-damping system consists of a 25-kg mass, a spring capacitance of 6.9×10^{-4} m/N, and a damping resistance of 42 N · s/m.
Determine the time-domain equation, the transfer function, the reso-nant frequency, ω_0, the damping ratio, ζ, and whether the process is overdamped, underdamped, or critically damped.

14.27 Construct the Bode diagram of the spring-mass-damping system in Example 14.7. Determine the amplitude and phase angle of the output produced by each of the following input signals: HW34

(a) $f = 10\cos(t + 0°)$N
(b) $f = 10\cos(10t + 0°)$N
(c) $f = 10\cos(100t + 0°)$N

HW34 **14.28** An electrical series *RLC* circuit consists of a 0.04-H inductor, a 4-μF capacitor, and a 100-Ω resistor. Verify that the system is underdamped. The damping of the circuit may be increased by adding a second resistor in series with the 100-Ω resistor. Determine the value of the second resistor that will result in a critically damped circuit. Determine the time-domain equation and the transfer function of the critically damped circuit.

14.29 A liquid noninteracting two-capacity system has time constants τ_1 and τ_2 of 300 s and 1200 s. The liquid is water, and the value of R_2 is 2.2×10^6 Pa · s/m^3. The full-scale ranges are $FS_{in} = 1.0 \times 10^{-2}$ m^3/s and $FS_{out} = 4$ m. Determine the time-domain equation and transfer function.

14.30 An electrical interacting second-order circuit has the following component values:

$$R_1 = 100\ \Omega$$
$$R_2 = 300\ \Omega$$
$$R_L = 50\ \Omega$$
$$C_1 = 0.1\ \mu F$$
$$C_2 = 0.8\ \mu F$$

Determine τ_1, τ_2, the time-domain equation, and the transfer function.

HW34 **4.31** An armature-controlled dc motor has the following characteristics:

$$B = 2.0 \times 10^{-3}\ N \cdot m \cdot s/rad$$
$$J = 3.2 \times 10^{-3}\ kg \cdot m^2$$
$$K_e = 0.22\ V \cdot s/rad$$
$$K_t = 0.22\ N \cdot m/A$$
$$L = 0.075\ H$$
$$R = 1.2\ \Omega$$

Determine the mechanical time constant, the electrical time constant, the time-domain equation, the transfer function, the resonant frequency, ω_0, the damping ratio, ζ, and the type of damping (i.e., overdamped, critically damped, or underdamped).

14.32 Construct the Bode diagram of the dc motor second-order system in Example 14.11.

14.33 The response of a control system is very much like the response of a second-order system. A closed-loop system may be overdamped, critically damped, or underdamped. The damping coefficient, α, determines the type of damping present in a system. In Equation (14.26), the equation of the second-order system is expressed in terms of A_1 and A_2. The equation can also be expressed in terms of ω_0 and α.

$$\frac{1}{\omega_0^2}\frac{d^2y}{dt^2} + 2\left(\frac{\alpha}{\omega_0^2}\right)\frac{dy}{dt} + y = Gx$$

Notice that α is part of the coefficient of the first derivative of the output (dy/dt). The first derivative is the rate of change (or velocity) of the output. Consider a dc motor position control system that is underdamped. A speed sensor measures the rate of change of the output (i.e., dy/dt). Explain how the signal from the speed sensor could be used to increase the damping coefficient of the system.

Section 14.5

14.34 Determine the dead-time lag for each of the following distance and velocity parameters.

(a) D = 15 ft, v = 4 ft/s
(b) D = 34 ft, v = 5.5 ft/s
(c) D = 4.6 m, v = 1.2 m/s
(d) D = 9.8 m, v = 2.4 m/s

14.35 A dead-time process similar to Figure 14.15a consists of an 8.2-m-long belt conveyor with a belt velocity of 0.44 m/s. Determine the dead-time lag, the time-domain equation, and the transfer function.

14.36 Construct a Bode diagram for a dead-time lag process with a dead time of 250 s.

Section 14.6

14.37 Name and describe the two parameters that characterize a first-order lag plus dead-time lag process.

HW32

ET 438a

Continuous and Digital Control

Open-loop, Close-loop and Error Ratio Calculations

1.) Draw the block diagrams for the each of the system transfer functions given below. Assume that the input signal is X(s) and the output is Y(s). All systems have negative feedback.

2.) Compute the open-loop gain, the closed loop gain, and the error ratio for each of the system transfer functions given below

a.)

$$G(s)=\frac{2s+1}{s^2+4s+1} \qquad H(s)=0.5$$

b.)

$$G(s)=\frac{5}{s^3+3s^2+2s+10} \qquad H(s)=5$$

c.)

$$G(s)=\frac{1}{(s+2)(s+4)} \qquad H(s)=\frac{0.5}{1+0.02s}$$

HW 33

ET 438a
Continuous and Digital Control
Bode Plot and Nyquist Plots

1.) Find the gain and phase margins for each of the systems described by the following Bode plots. State if the system is stable, marginally stable or unstable by evaluating the gain and phase margins. Give a justification for your choice. Hand in the marked-up plots as part of the assignment
2.) For the attached Nyquist diagrams, plot the stability point, the gain margin point and the phase margin point. Determine if the systems represented by the plots are stable, unstable or marginally stable. Give a justification for your choice. Hand in the marked up plot as part of the assignment.

For Bode Plots
A.) Open Loop Transfer Function

$$\frac{2500}{s(s+5)(s+50)}$$

B.) Open Loop Transfer Function

$$\frac{250}{s^3+6.5s^2+8s+2.5}$$

C.) Open Loop Transfer Function

$$\frac{80}{s(s+5)(s+15)}$$

For Nyquist Plots
A.) Open Loop Transfer Function

$$\frac{17.5}{0.7s+1}e^{-0.08s}$$

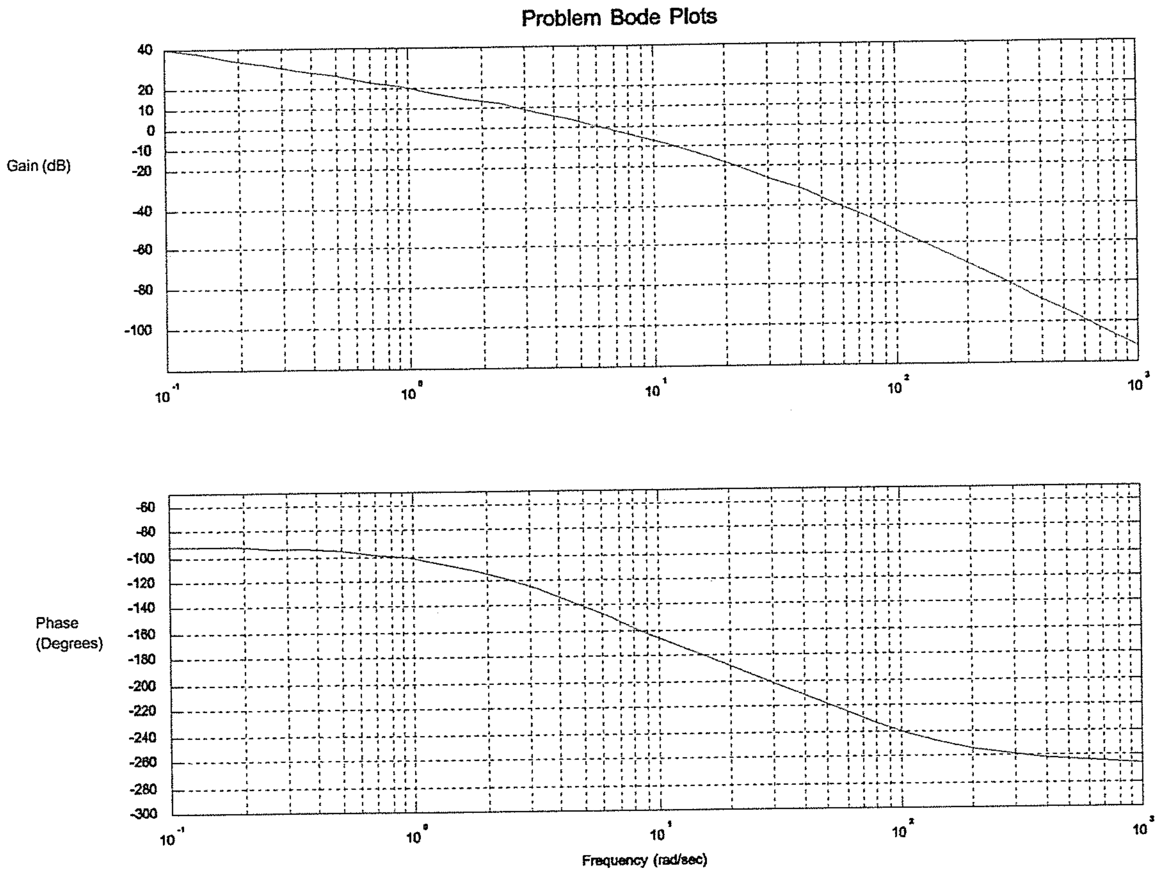
B.) Open Loop Transfer Function

$$\frac{17.5}{0.7s+1}e^{-0.01s}$$

C.) Open Loop Transfer Function

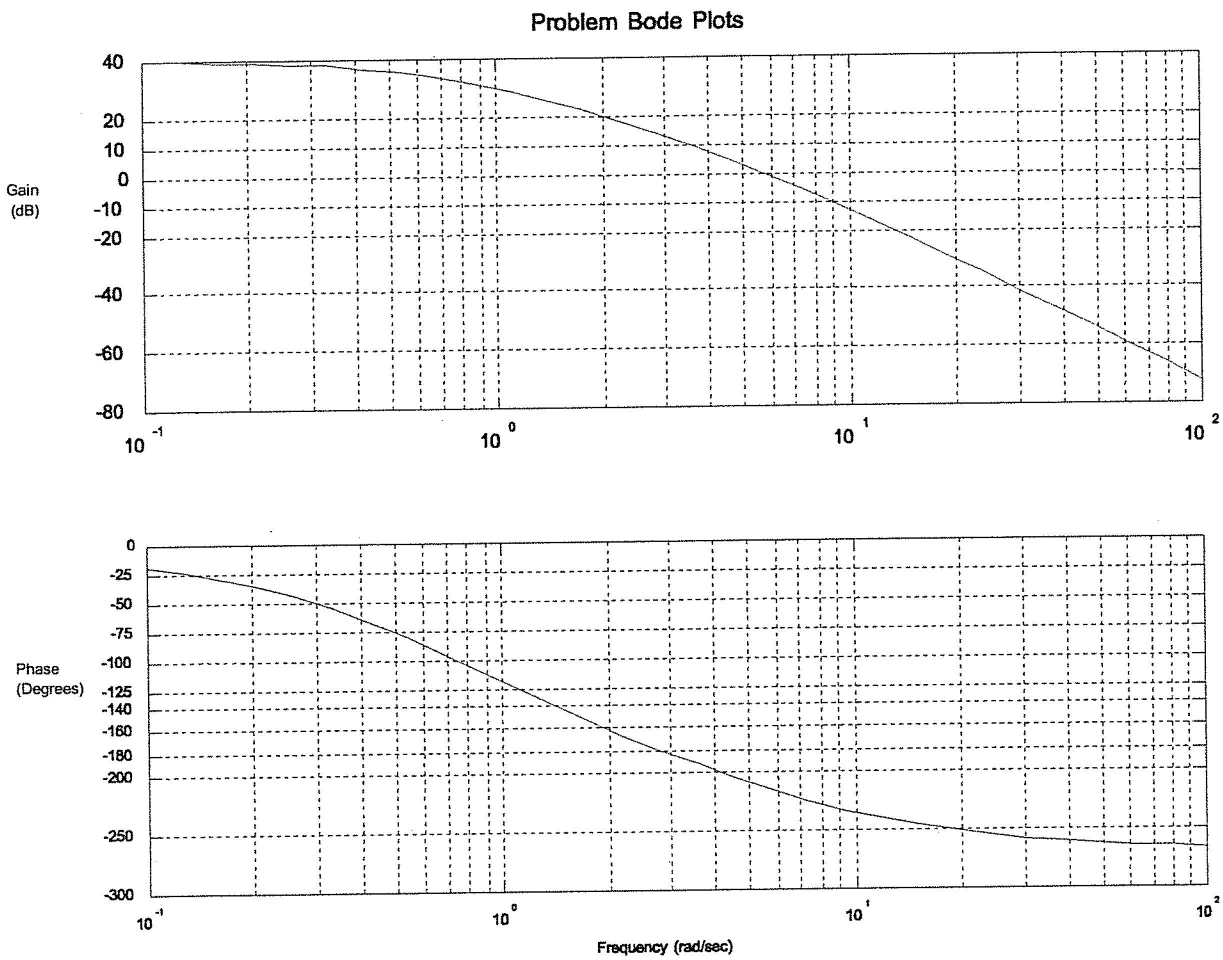
$$\frac{80.5}{0.7s+1}e^{-0.01s}$$

Hw 33



Control System A

Hw 33



Control System B

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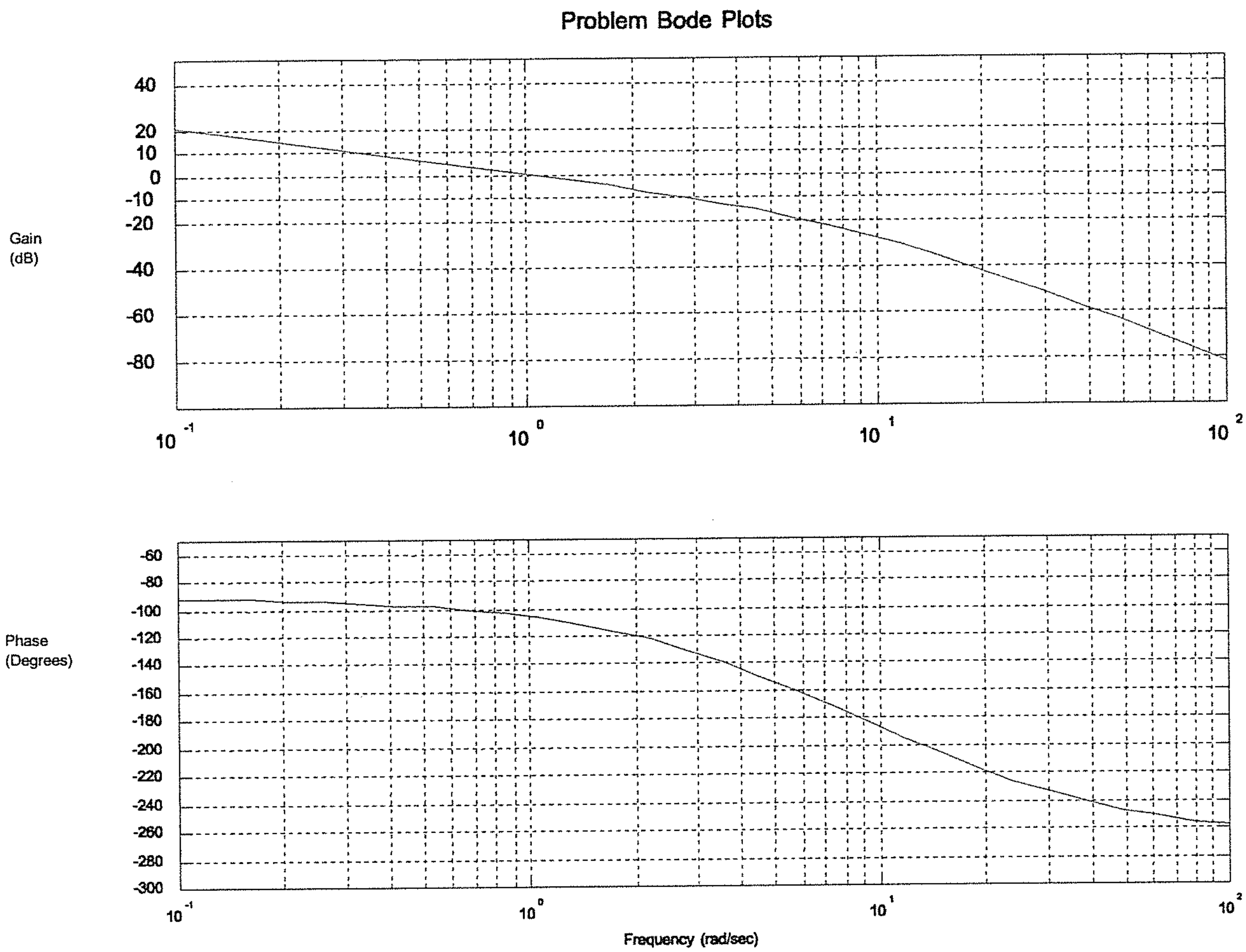
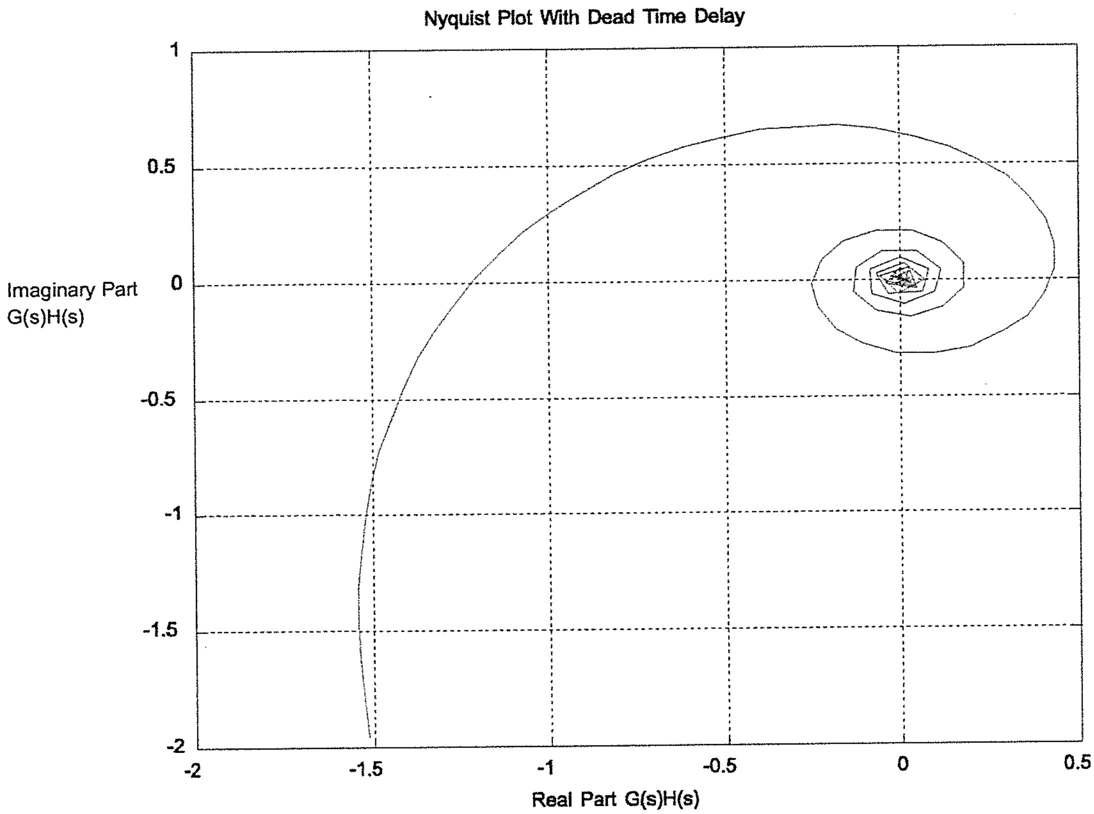
Hw 33



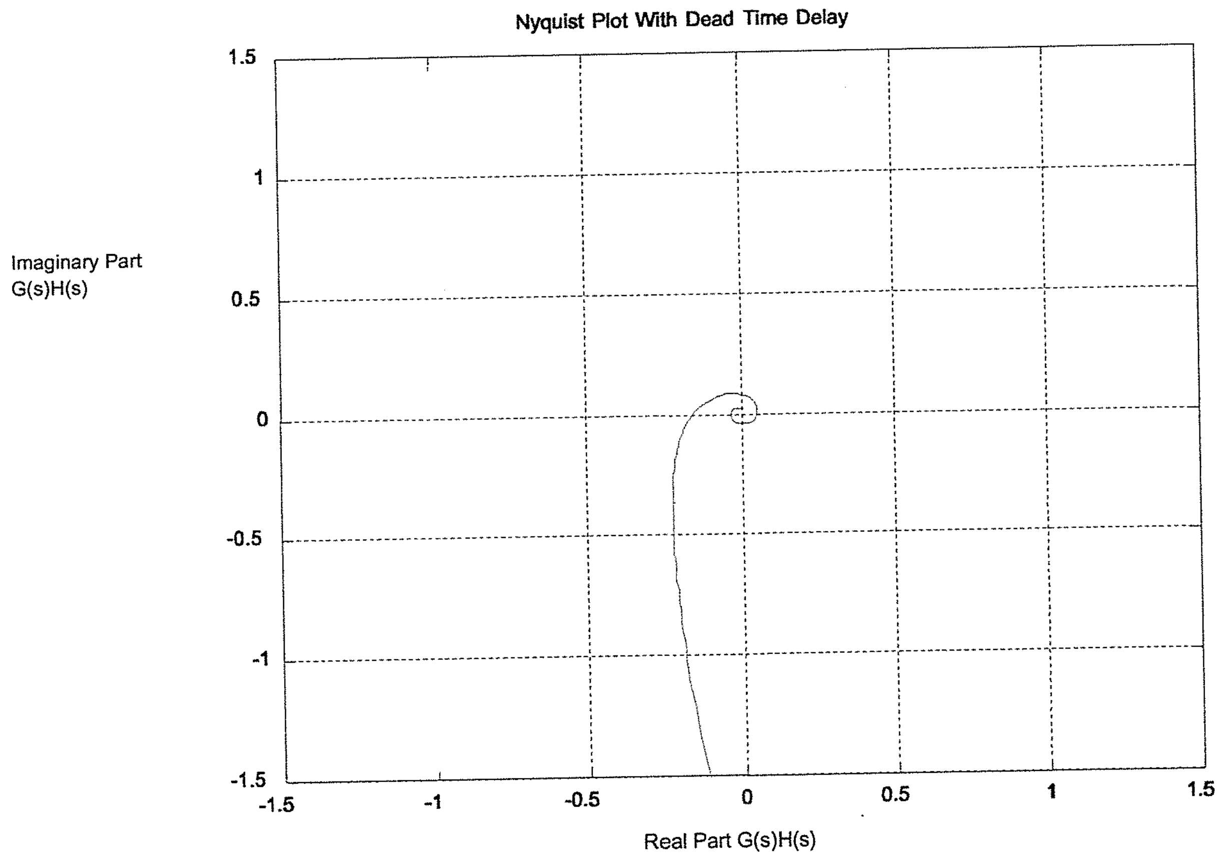
Control System C

Hw 33



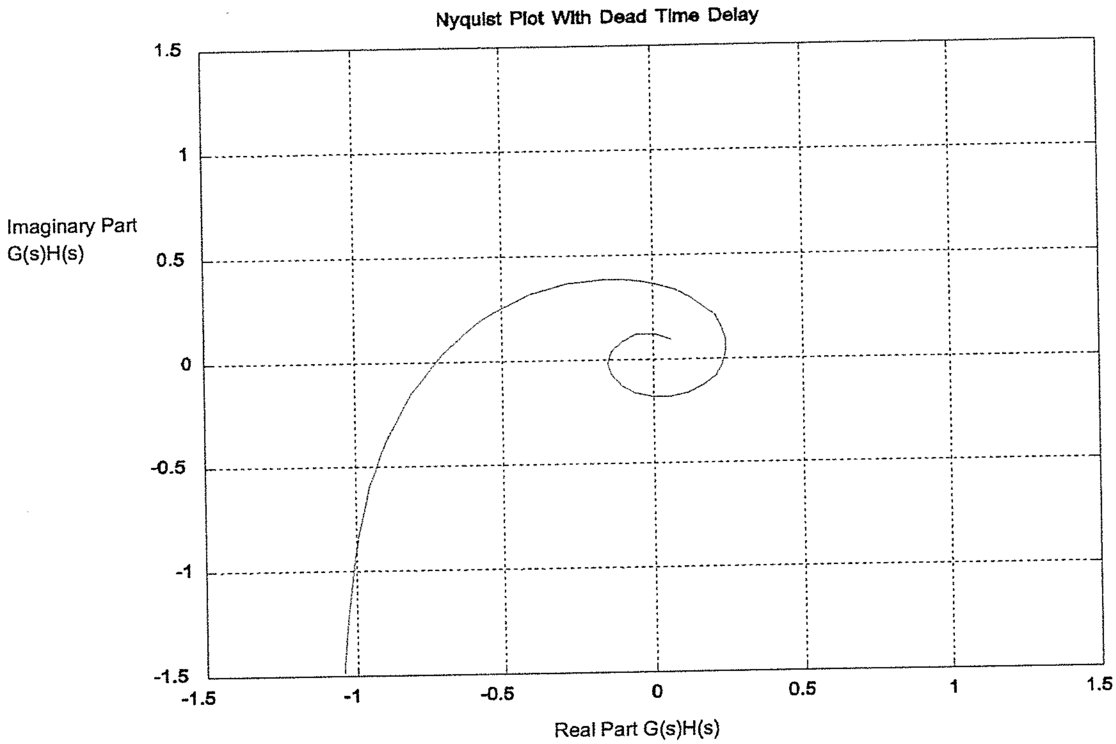
Nyquist Plot A

Hw 33



Nyquist Plot B

Nw 33



Nyquist Plot C