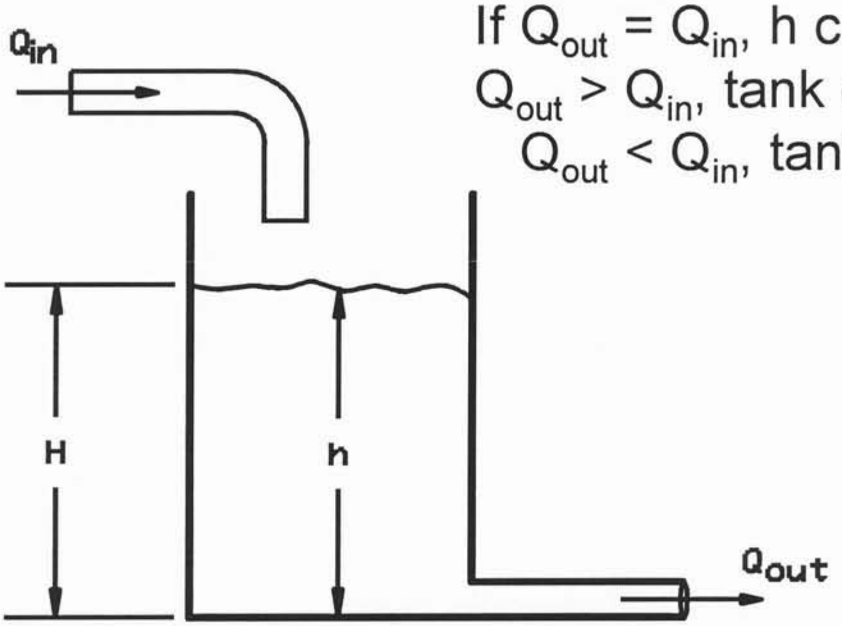
Process Control Principles

Fundamental control concepts

Regulate value of a quantity compared to a reference value. Reference called **setpoint**

Process - a collection of equipment and materials that takes inputs and has outputs

Example



Q_{out} depends on $\sqrt{h}$

If $Q_{out} = Q_{in}$, h constant

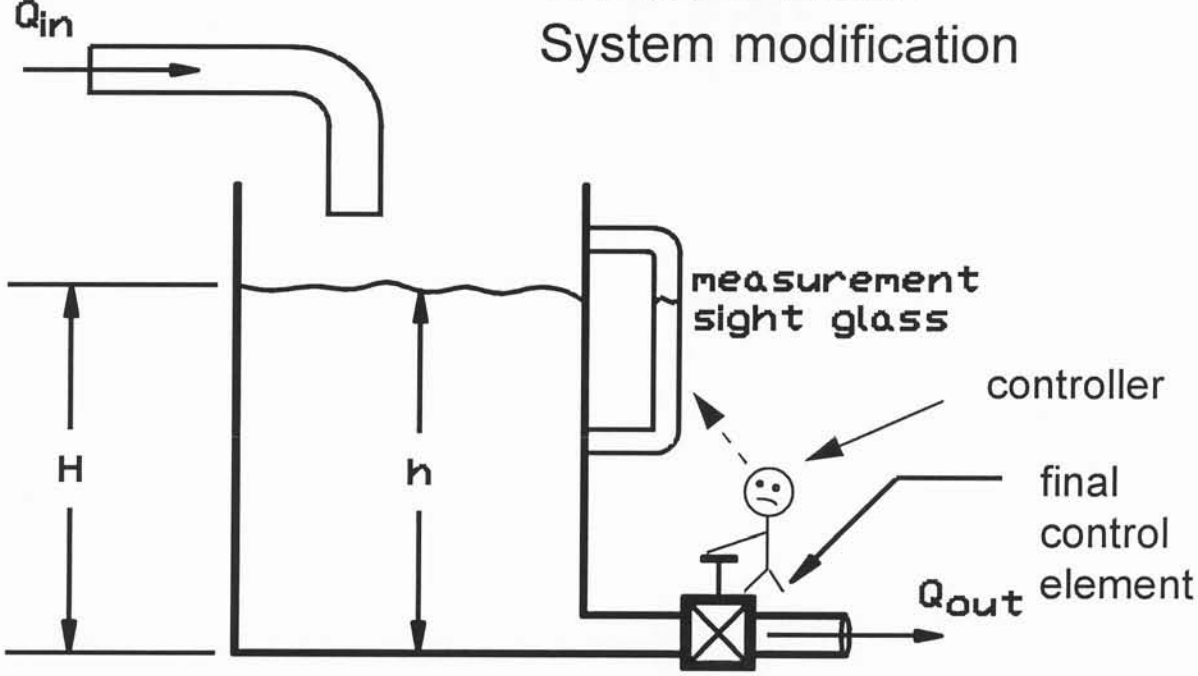
$Q_{out} > Q_{in}$, tank empties

$Q_{out} < Q_{in}$, tank overflows

No reference value given

Basic Control Elements

Measurement
Control decision
System modification

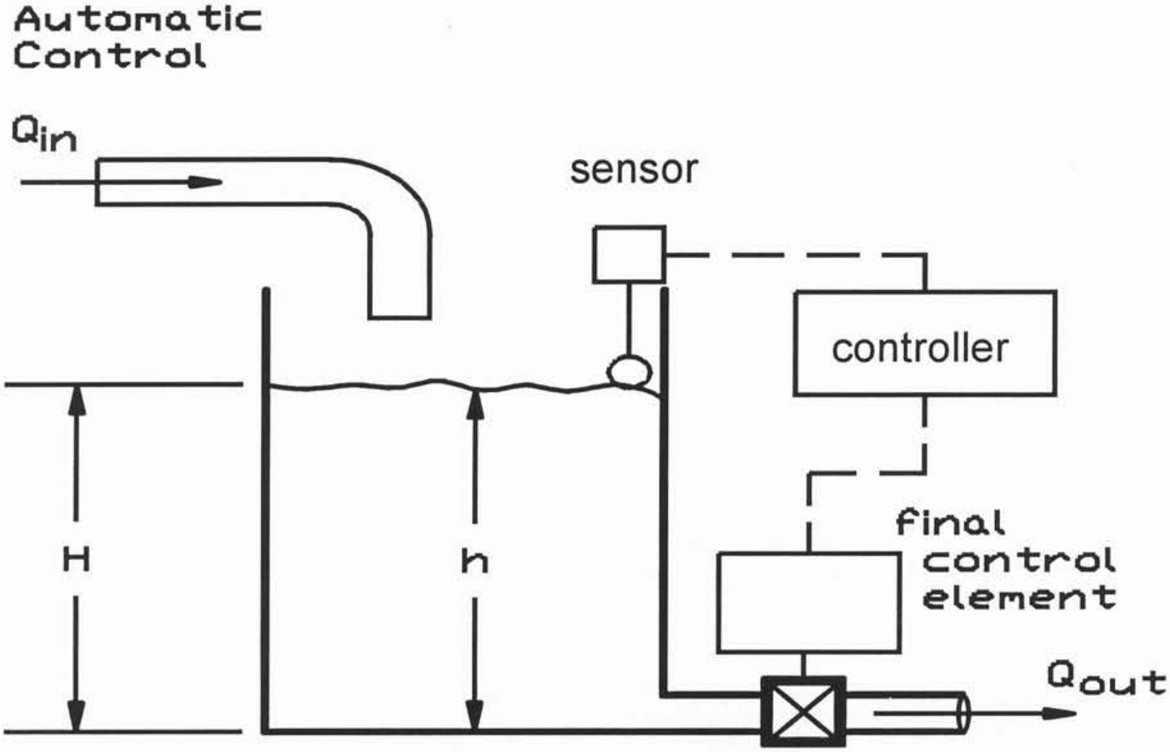


tank level, H, is reference (setpoint)
h is the control variable.

Human is the controller, adjusts
Q_{out} to maintain h as close as possible to H

Example of regulator action

Automatic Control Systems
Automation



Use sensors and analog or digital electronics to monitor system.

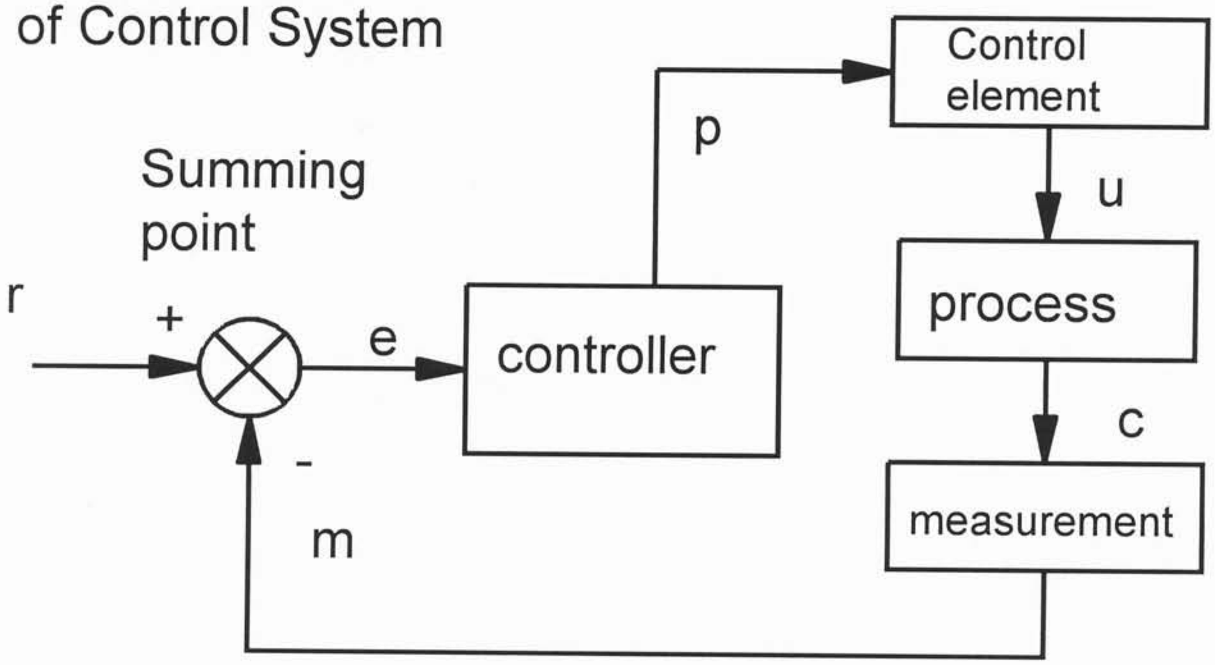
Elements of Automatic Control System

- **Process** - single or multiple variable
- **Measurement** - sensors / transducers
- **Error detection** - compare H to h
- **Controller** - generate corrective action
- **Final control element** - modify process

Block Diagrams

Reduce complex systems to inputs and outputs

Block Diagram
of Control System



Summing point

r

+

e

-

m

controller

p

Control element

u

process

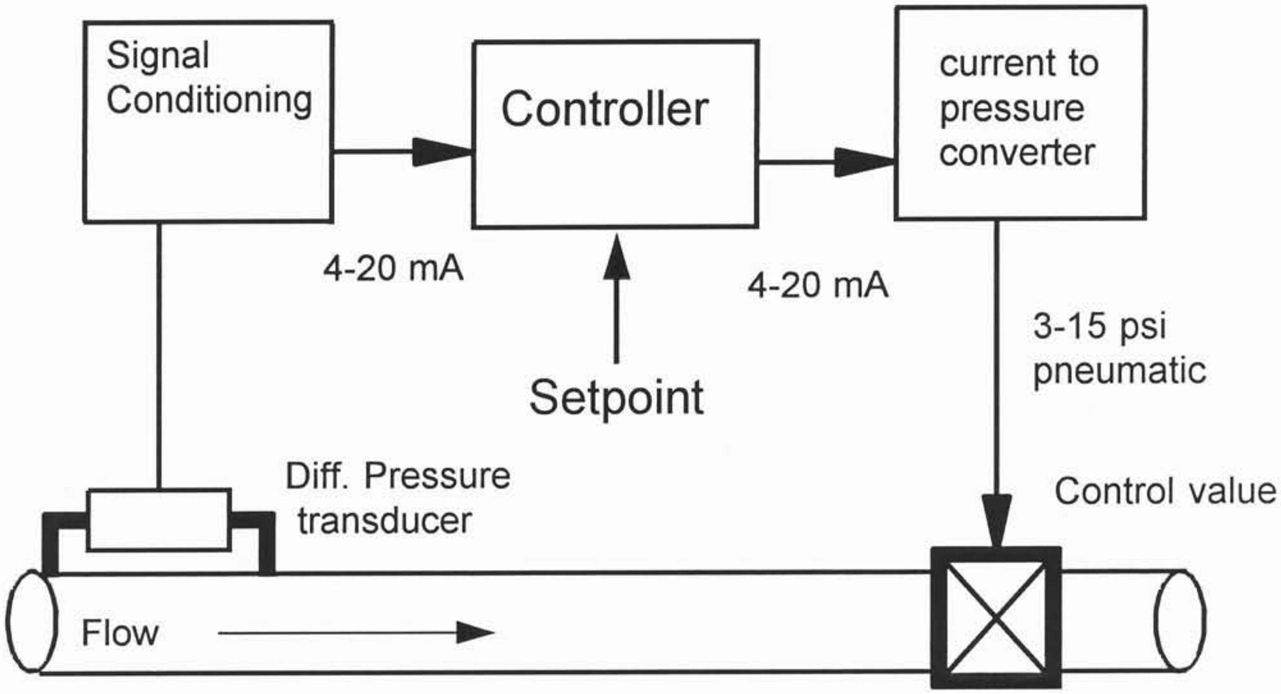
c

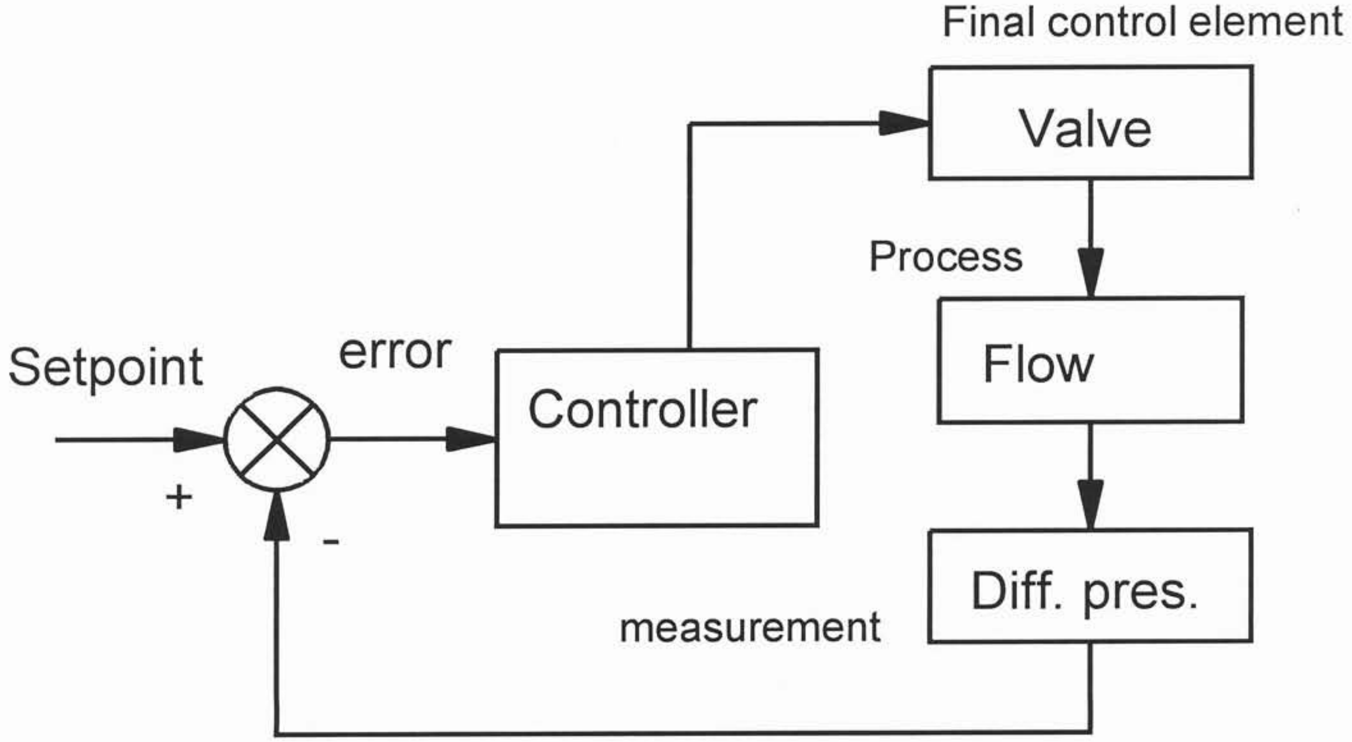
measurement

Signals form a loop   Control loop

r = reference setpoint value
c = control variable in the process
m = measurement of control variable
p = controller output
u = control element change
e = r - m   (due to signs on summer)

Practical System


Signal Conditioning
Controller
current to pressure converter
4-20 mA
Setpoint
4-20 mA
3-15 psi pneumatic
Diff. Pressure transducer
Control value
Flow


Final control element
Valve
Process
Setpoint
error
Flow
Controller
+
-
Diff. pres.
measurement

Control System Performance

System signal change over time so

$$e(t) = r - c(t)$$

Where $e(t)$ = error as a function of time
r = setpoint value
$c(t)$ = control variable as a function of time

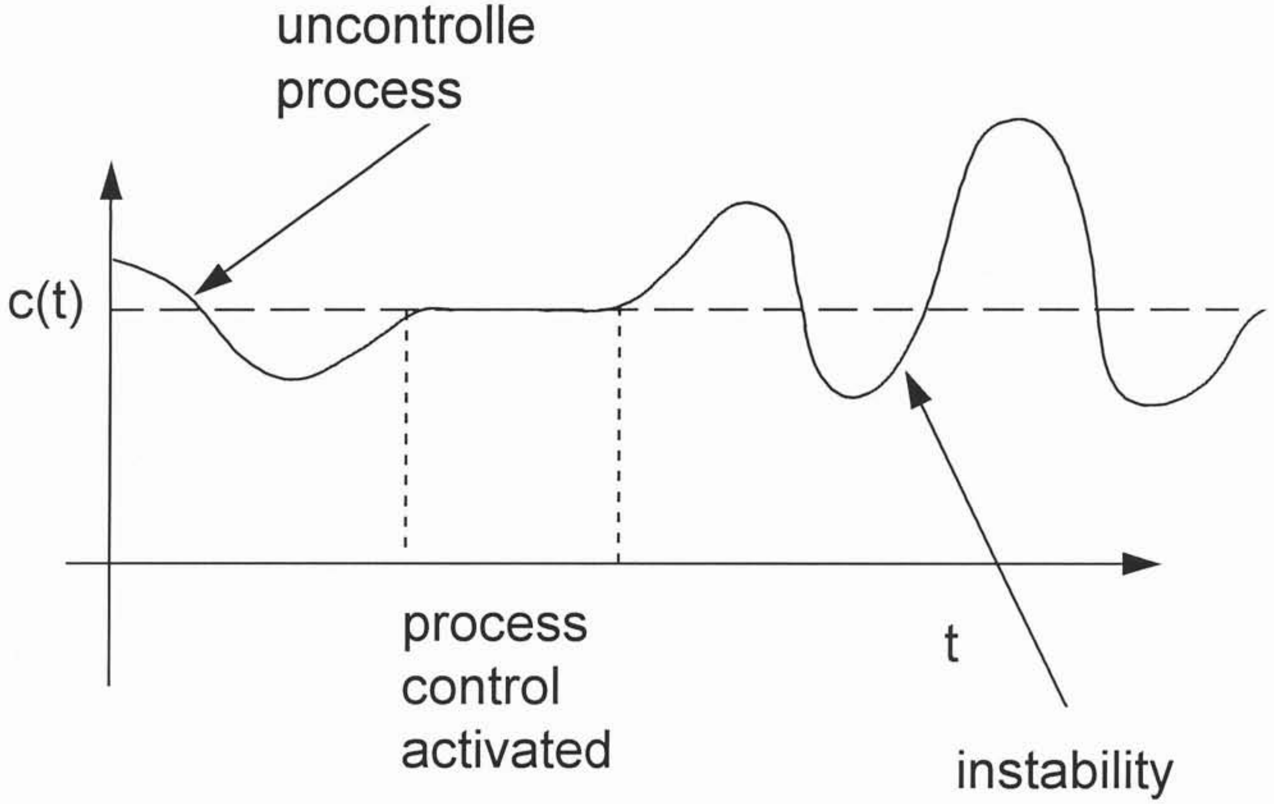
Control System Objectives

$e(t) = 0$ after changes or disturbances

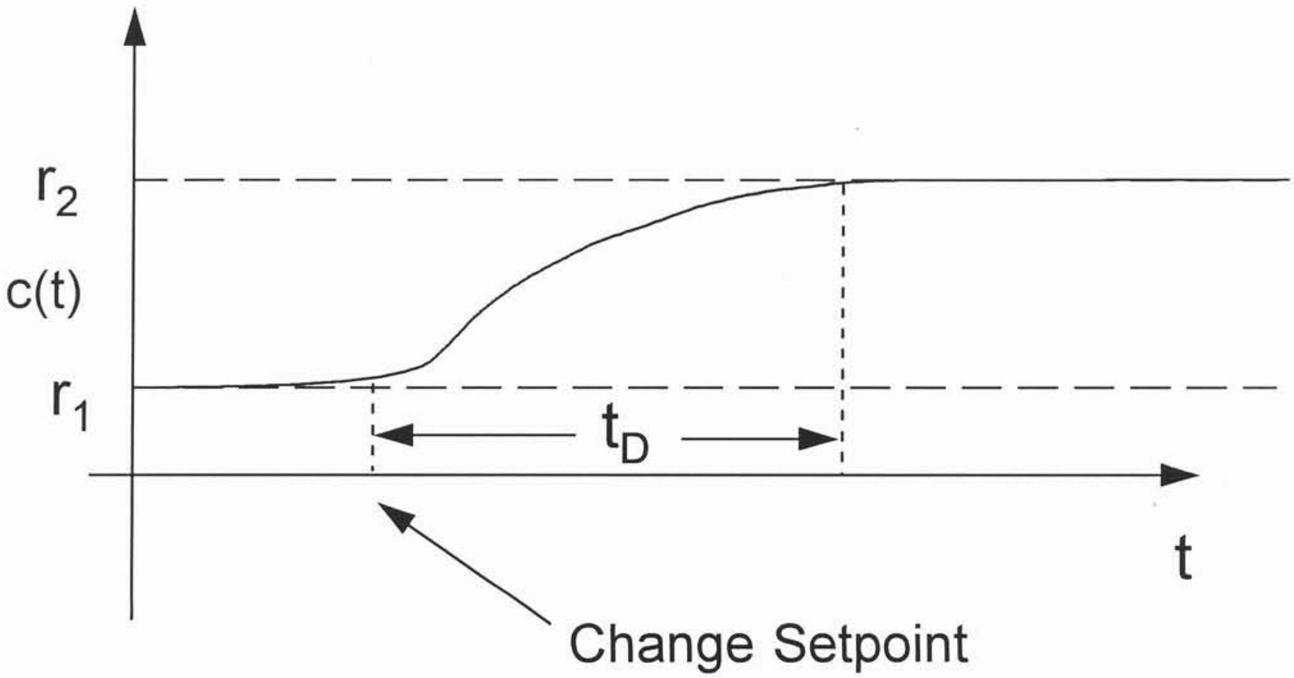
Stable $c(t)$ after changes or disturbances

Stability
Steady-state regulation - $e(t) = 0$ or within tolerances
Transient regulation - how does system perform under change

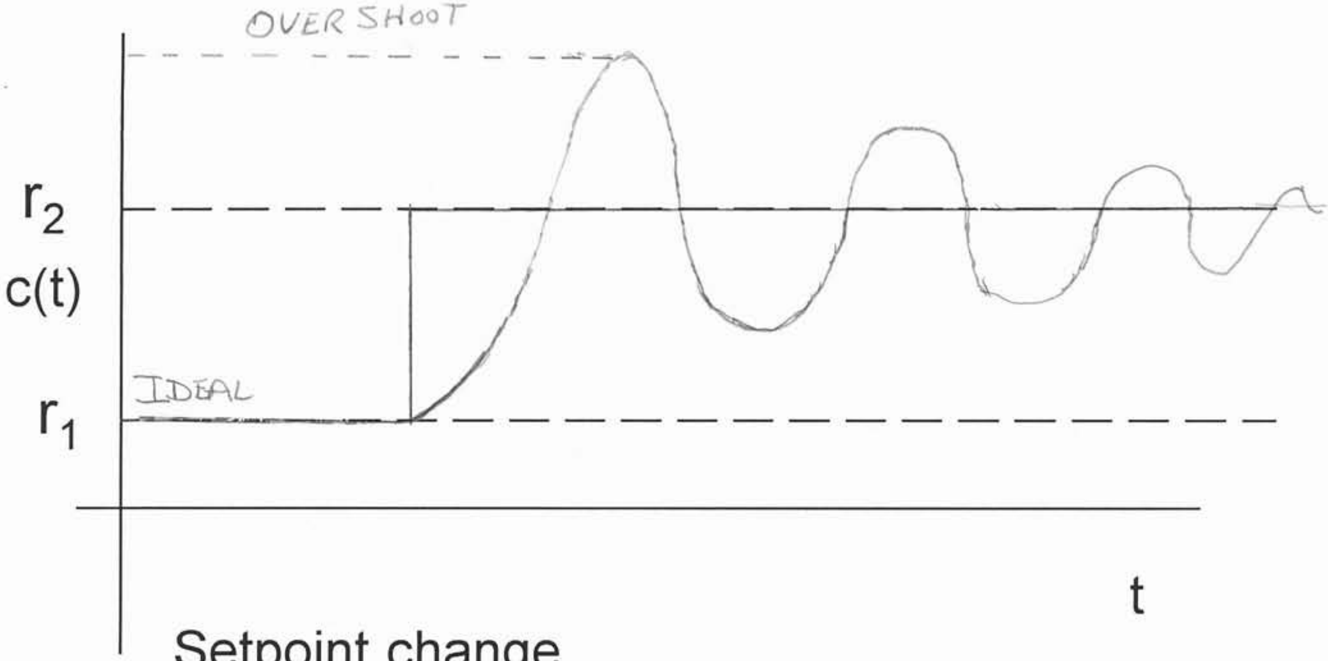
Types of system response



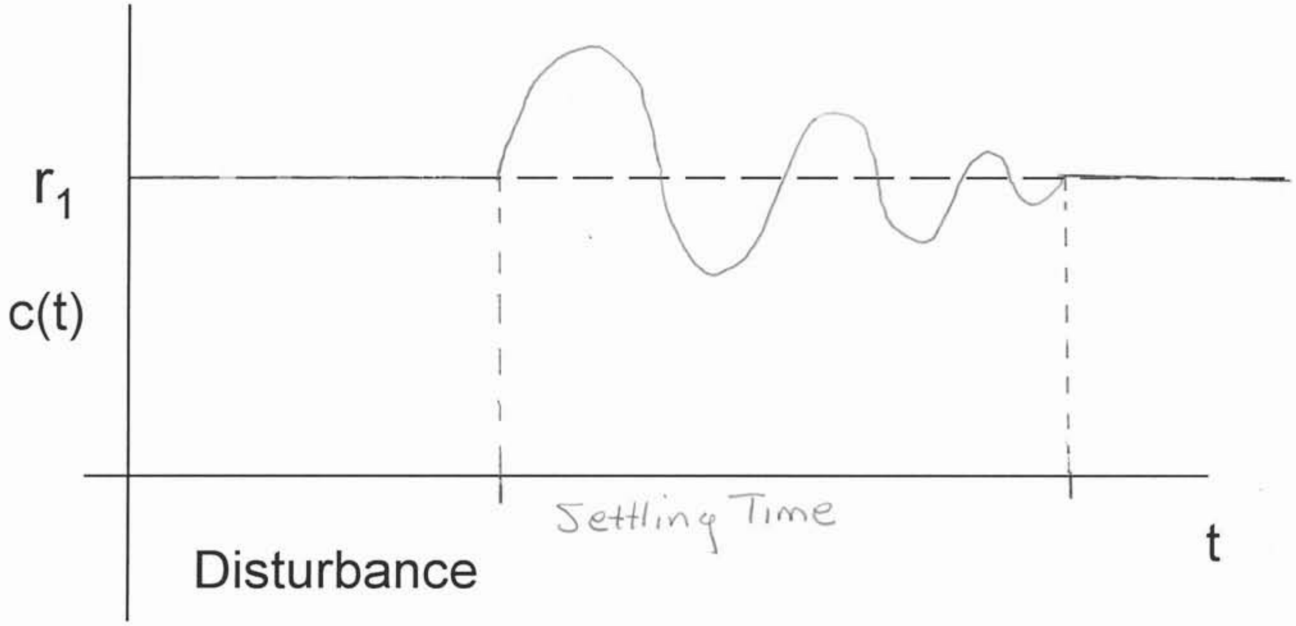
Damped Response



Transient Responses



Setpoint change



Disturbance

Analog Measurement Error

Error determines accuracy

Methods of determing accuracy

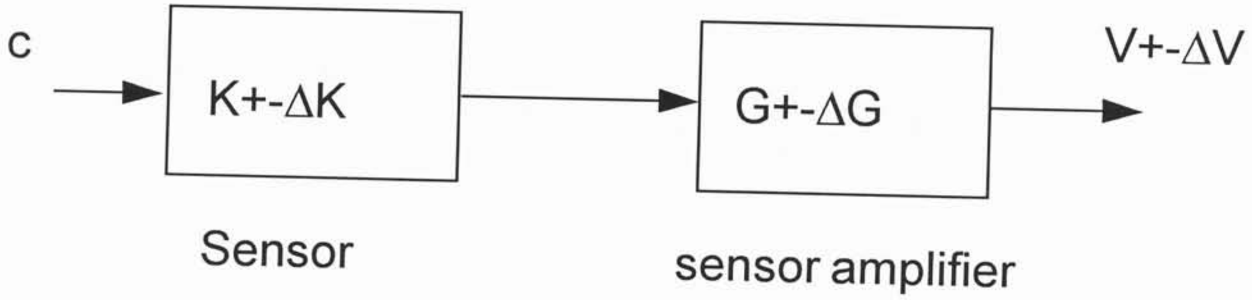
Measured Value: Reading +- value
100 psi +- 2 psi

Percent of Full Scale (FS): (FS Value) (%/100)
Meter accurate to +-5% of FS)
10 V scale
error = 10 V (+-5%/100) =+-0.5V

Percentage of Span (Span = max-min)
(Span)(%/100)
P measurement +-3% of span
20-50 psi
error = (50-20)(+-3%/100) = +-0.9 psi

Percentage of Reading
reading of 2 V +-2%
error = 2 V (+-2%/100) = +- 0.04 V

System Accuracy
Cummulative Error



K = sensor gain G = amplifier gain
V = sensor output voltage,
ΔG, ΔV, ΔK uncertainities in measurement

What is magnitude of ΔV?

With no error: $V = (K)(G)c$

input

output

With error $V\pm\Delta V = (K\pm\Delta K)(G\pm\Delta G)c$

Multiple out and simplify to get

$$\pm\frac{\Delta V}{V} = \pm\frac{\Delta G}{G} \pm \frac{\Delta K}{K}$$

Where $\pm\frac{\Delta V}{V}$ = normalized fractional uncertainity

$$\pm\frac{\Delta G}{G}, \pm\frac{\Delta K}{K}$$ = normalized fractional uncertainity

Combining the errors

Use RMS (RSS)

$$\left(\frac{\Delta V}{V}\right)_{Rms} = \pm\sqrt{\left(\frac{\Delta G}{G}\right)^2 + \left(\frac{\Delta K}{K}\right)^2}$$

Notes:

$\frac{\Delta G}{G}$ $\frac{\Delta K}{K}$ are fractional error. Divide % to get.

$\frac{\Delta V}{V}$ is fractional RMS error Multiply by 100 to get percent.

Works on all formulas that are only multiplication and division.

Example: inverting OP AMP

$$v_o = v_{in} \frac{-R_f}{R_{in}}$$

R_{in} & R_f 5% Tolerance

v_{in} Read with 2% accuracy

$$\frac{\Delta R_f}{R_f} = \frac{\Delta R_{in}}{R_{in}} = \frac{\pm 5\%}{100} = \pm 0.05$$

$$\frac{\Delta v_{in}}{v_{in}} = \frac{\pm 2\%}{100} = \pm 0.02$$

$$\left[\frac{\Delta V_o}{V_o}\right]_{Rms} = \pm\sqrt{\left(\frac{\Delta R_f}{R_f}\right)^2 + \left(\frac{\Delta R_{in}}{R_{in}}\right)^2 + \left(\frac{\Delta v_{in}}{v_{in}}\right)^2}$$

$$= \pm\sqrt{(0.05)^2 + (0.05)^2 + (0.02)^2}$$

$$= \pm 0.073 \quad \text{OR} \quad \underline{\underline{\pm 7.3\%}} \quad \underline{\underline{\text{ANS}}}$$

Sensor Characteristics

Sensitivity - Change in output for change in input. Equals slope in linear device

Hysteresis - output different for increasing or decreasing input

Resolution - Smallest measurement a sensor can make.

Linearity - how close to a line is the I/O relationship

$$c_m = m(c) + c_0$$

Where c = measured control variable
m = slope
c_0 = offset (y intercept)
c_m = sensor output

Example
Finding m and c_0 from data points

$$y - y_1 = m\left(x - x_1\right) \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Sensor has a linear resistance change of 100 to 195 ohms, as temperature changes from 20 - 120 C. Find the sensor I/O relationship

Define Points $(x_1, y_1) = (20°C, 100\Omega)$ x = input

$(x_2\ y_2) = (120°C, 195\Omega)$ y = output

Slope $m = \frac{195\Omega - 100\Omega}{120°C - 20°C} = \frac{95\Omega}{100°C} = 0.95\ \Omega/°C$

$y - 100\Omega = 0.95\Omega/°C(x - 20°C)$

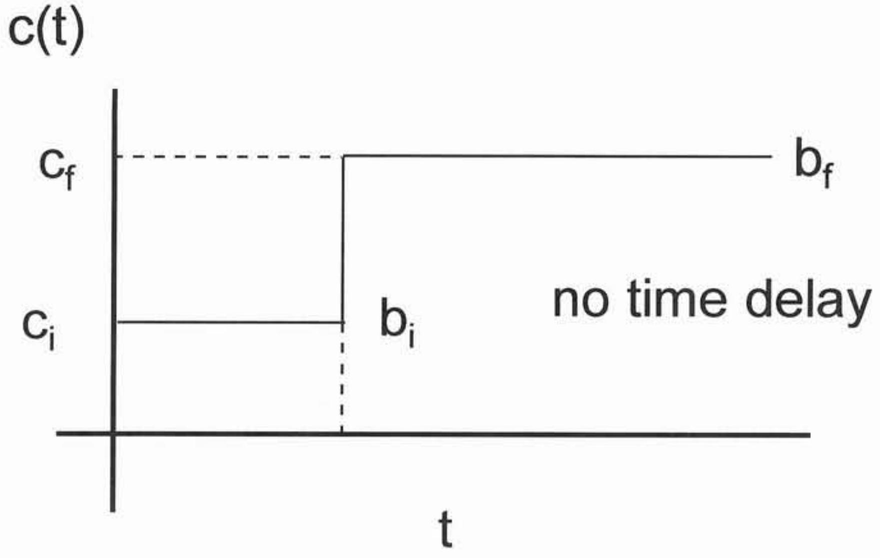
$y = 0.95x - 0.95(20) + 100$

$y = \underline{\underline{0.95x + 81\ \Omega}}/°C$ $\underline{\underline{ANS}}$

Plot to check points should be one line.

Sensor Response

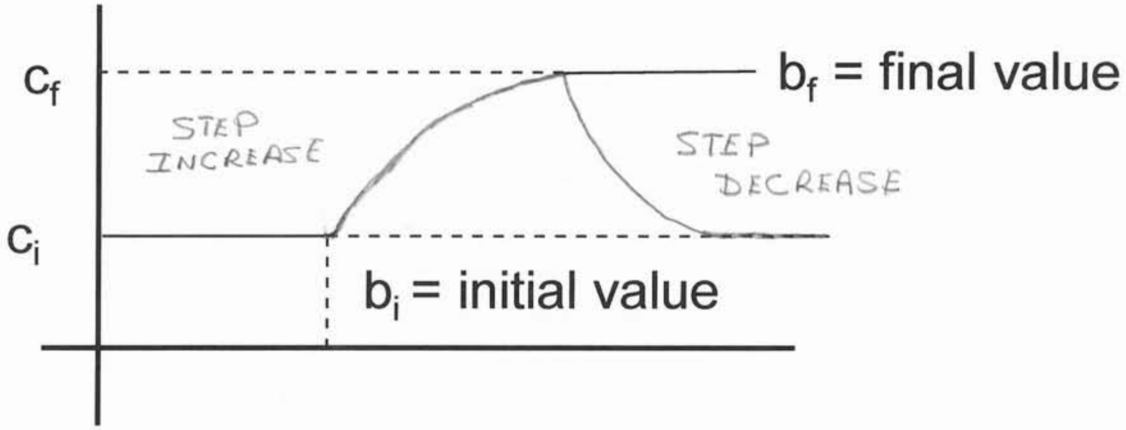
First Order Response - ideal



Step change in measured variable

Practical sensor response

c(t)   b(t) = sensor response function



Model ing 1st Order Respone

For step increase:

$$b(t) = b_i + (b_f - b_i) \cdot \left(1 - e^{-\frac{t}{\tau}}\right)$$

Where

b_f = final sensor value
b_i = initial sensor value
t = time
τ = time constant of sensor

For step decrease

$$b(t) = (b_i - b_f) \cdot e^{-\frac{t}{\tau}}$$

Example: Step increase

$b_f = 4.0$ V $b_i = 2.0$ V $\tau = 0.0025$ /s

Find time it takes for sensor to reach 90% of final value

$b(t) = 2 + (4-2)(1 - e^{-t/0.0025})$

$b(t) = 2 + 2(1 - e^{-t/0.0025})$

$b(t)$ at 90% of 4.0V = 3.6 V

$3.6 = 2 + 2(1 - e^{-t/0.0025})$

$\frac{3.6-2}{2} = 1 - e^{-t/0.0025}$

$\rightarrow$

$0.8 - 1 = -e^{-t/0.0025}$

$+0.2 = +e^{-t/0.0025}$

$\ln(+0.2) = \ln(e^{-t/0.0025})$

$-1.609 = -t/0.0025$

$1.609(0.0025) = t$

$0.004\,s = t$

$\underline{4\,mS = t}$ ANSW

Example: Step decrease

The sensor initial output is 1.0 V how long does it take to change to 0.2 V if the time constant of the sensor is 0.1/s.

$b_i = 1.0\text{V}$ $b_f = 0.0\text{ V}$ $\tau = 0.1/s$ $b(t) = (b_i - b_f)e^{-t/\tau}$

$b(t) = 0.2$ $0.2 = 1e^{-t/0.1}$

$\ln(0.2) = -t/0.1$

$-1.6094 = -t/0.1$

$1.6094(0.1) = t$

$\underline{\underline{0.1609\ S}} = \underline{\underline{t}}$ $\underline{\underline{\text{ANSW}}}$

Significant Digits in measurement and design

In meaurement: readable output of instruments
resolution of sensors and
transducers.

Calulation using meaurements: Trunicate calculator answers to match significant digits of measurements and readings,

Significant digit examples

Compute power based on the following measured values. Use correct number of significant digits.

I = 3.25 A   V = 117.8 V   P = V(I)

3.25 A	3 significant digits
117.8 V	4 significant digits

calculator 382.85 W

Truncate to 3 significant digits   P = **383 W**

Significant digits not factor in design calculations
Device values assumed to have no uncertainity.

Compute the current flow through a resistor that has a measured R of 1.234 kΩ and a voltage drop of 1.344 Vdc.

R = 1.234 kΩ	4 significant digits
V = 1.344 V	4 significant digits

I = (1.344)/(1.234x 10^3) = 1.089 mA 4 digits

Basic Statistics

Arithmatic Mean (Central tendency)

$$\bar{x} = \frac{\sum_{(i=1)}^{n} x_i}{n}$$

Where x_i = i th data measurement
n = total number of measurents taken
x = mean value

Variance (Measure of data spread from mean)

$$d_i = \left(x_i - \bar{x} \right)^2 \qquad \sigma^2 = \frac{\sum_{(i=1)}^{n} d_i}{n-1}$$

σ^2 = variance of data

Standard Deviation

σ = standard deviation

$$\sigma = \sqrt{\frac{\sum_{(i=1)}^{n} d_i}{n-1}}$$

Statistics Example

A 1000 ohm resistor is measured 10 times the follow readings where taken

Test #	Reading	Test #	Reading
1	1016	6	1011
2	986	7	997
3	981	8	1044
4	990	9	991
5	1001	10	966

Find the mean, variance and standard deviation of the tests What is the most likely value for the resistor to have?

$$\bar{X} = \frac{(1016+986+981+990+1001+1011+997+1044+991+966)}{10}$$

$$\bar{X} = 998.3\ \Omega$$

$d_1 = (1016-998.3)^2$ $\quad d_2 = (986-998.3)^2$

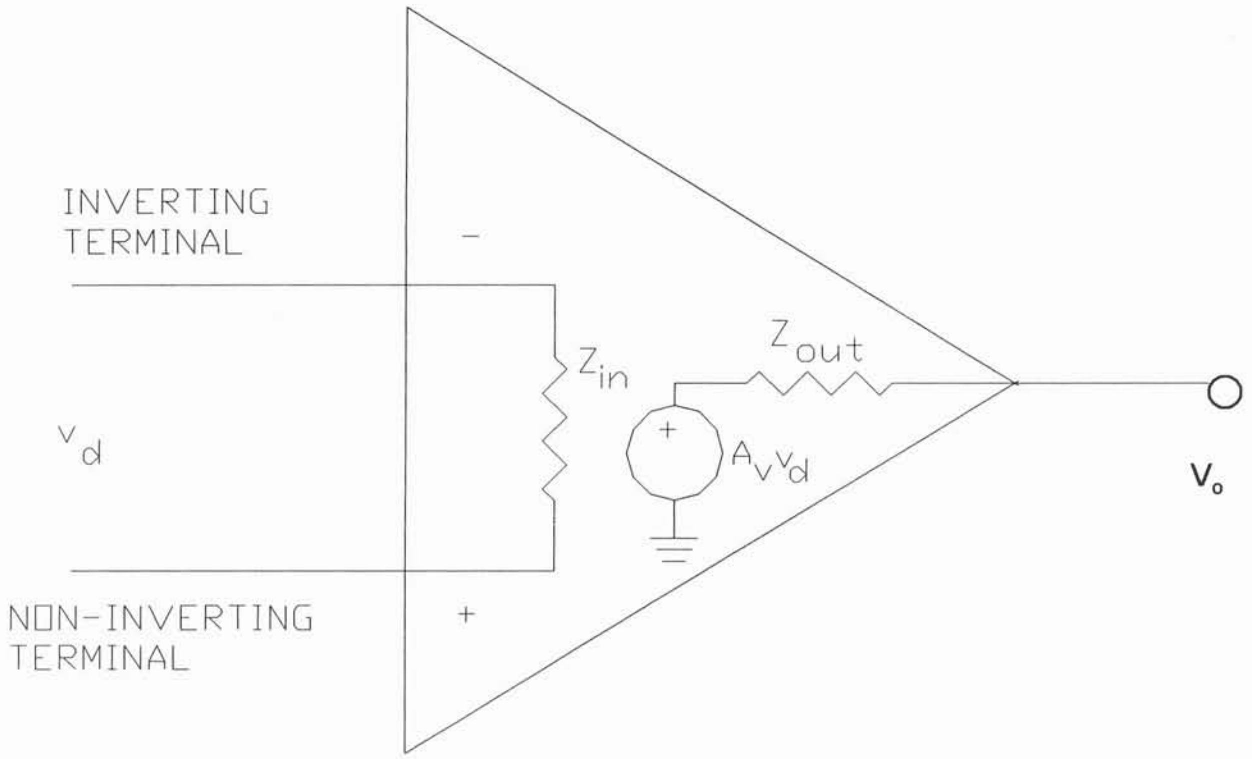
$d_1 = 313.29$ $\quad d_2 = 151.3$

$$\sigma^2 = \frac{\sum_{1}^{10} d_i}{10-1} = 465.3 \qquad \sigma = \sqrt{\sigma^2} = \sqrt{465.3} = 21.6$$

ALL VALUES SHOULD BE WITHIN ± 3σ

Most likely Value for R is $\bar{X}$ So 998.3Ω

Ideal OP AMP Characteristics

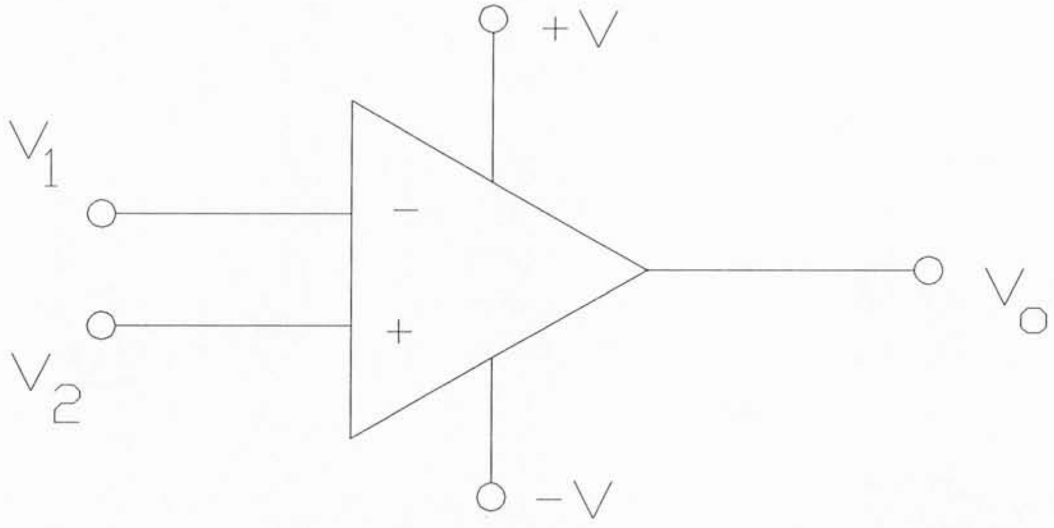


1.) infinite Z_{in}
2.) $Z_{out} = 0$ zero output impedance
3.) Bandwidth infinite Gain is constant for all frequencies
4.) With $V_d = 0$, $V_o = 0$ (No offset voltage)
5.) Infinite voltage gain (A_v)
6.) Instant recovery from saturation
7.) $I_{in} = 0$ (due to infinite Z_{in})

OPERATIONAL AMPLIFIERS
(OP AMP)

VOLTAGE AMPLIFIER DESIGNED FOR USE IN ANALOG COMPUTERS.

Symbol for basic OP AMP



Two Inputs V_1 Inverting input
V_2 Non-inverting input

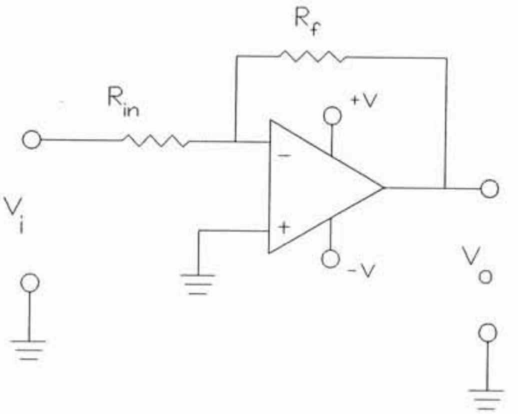
$V_1>0$ $V_0<0$
$V_2>0$ $V_0>0$

Requires bipolar power supplies

OP AMPS ARE DIRECT COUPLED (dc) AMPLIFIERS AND CAN AMPLIFY dc & ac SIGNALS SIMULTANEOUSLY

Basic Amplifier Circuits

Inverting Voltage Amplifier

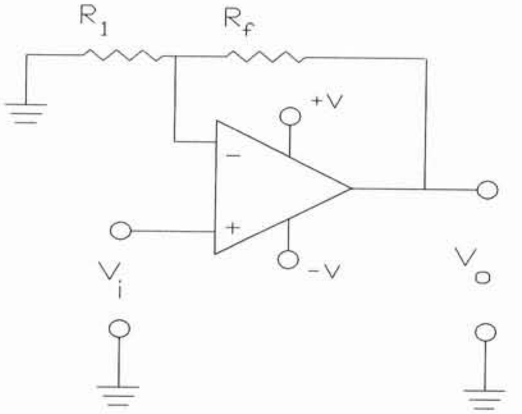


$$A_V = \frac{-R_f}{R_{in}}$$

$$V_o = V_i\left(\frac{-R_f}{R_{in}}\right)$$

V_o Limited by +- V
Large A_v causes V_o = V (saturation)

Non-Inverting Voltage Amplifier



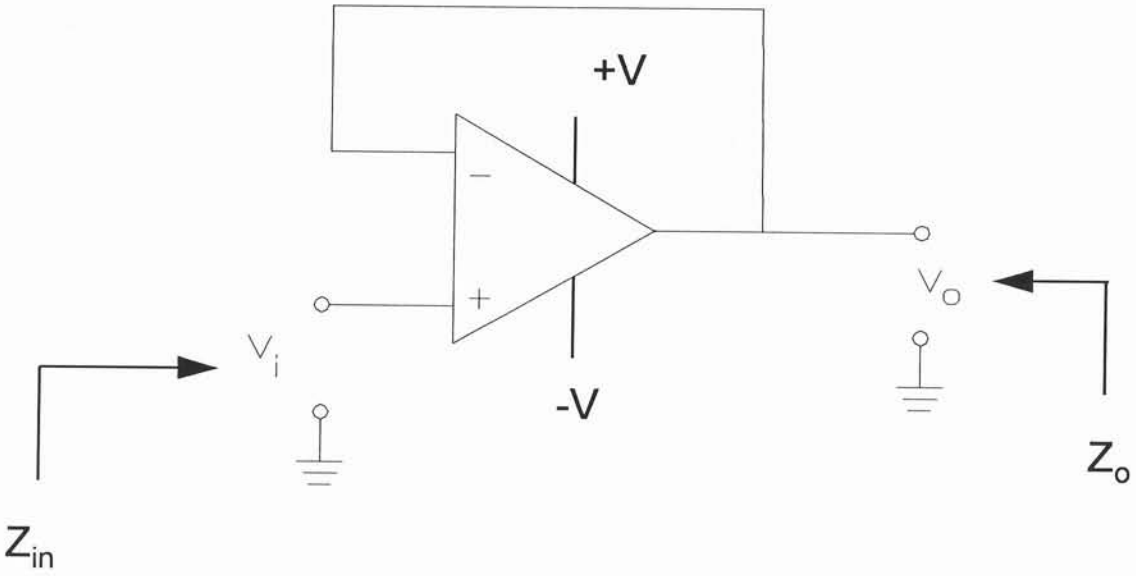
$$A_V = \left(1 + \frac{R_f}{R_{in}}\right)$$

$$V_o = V_i\left(1 + \frac{R_f}{R_{in}}\right)$$

R_{in} = R_{in} of OP AMP
A_v has minimum value of 1

Other useful OP AMP circuits

Impedance buffers (Voltage follower)
Used where high Z_{in} needed

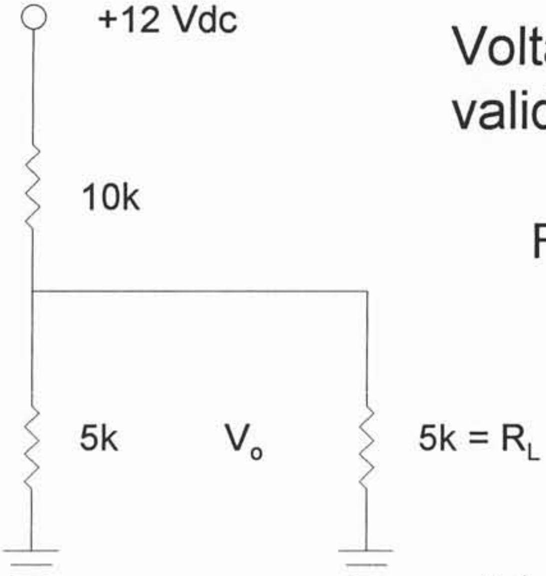


Characteristics

<u>Practical Circuit</u> (LM741)	<u>Ideal</u>
$A_v = 1$	$A_v = 1$
Z_{in} = 1 MW	Z_{in} = infinite
Z_o = 10 W	$Z_o = 0$

Circuit causes minimum loading on previous stage

Example: Buffered voltage divider circuit



Voltage divider formula only valid for infinite load resistance

Find V_o under load

NO LOAD V_o

$$V_o = \frac{5k\Omega}{10k\Omega + 5k\Omega}(12V)$$

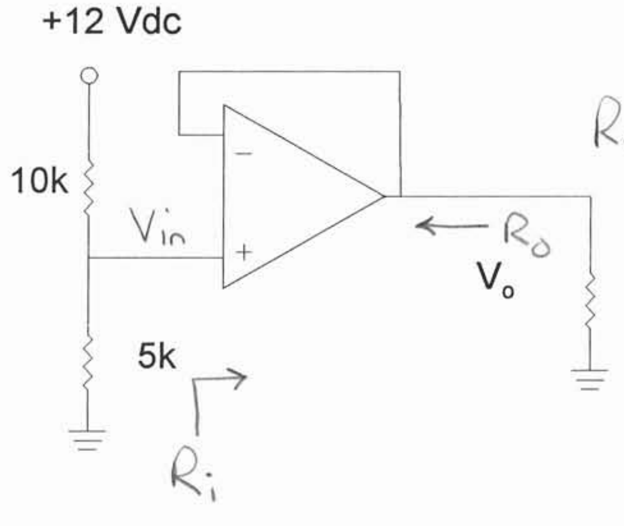
$V_o = \underline{4\,Vdc}$

WITH LOAD

$R_L \| 5k\Omega = 5k\Omega \| 5k\Omega = 2.5k\Omega$

$$V_o = \frac{2.5K}{10K + 2.5K}(12V) = \underline{2.4V}$$

Add impedance buffer



ASSUME LM741 $R_i = 1\,M\Omega$

$R_o = 10\Omega$

$A_V = 1$ SO $V_{in} = V_o$

$R_{eq} = R_i \| 5k\Omega = 1M\Omega \| 5k\Omega$

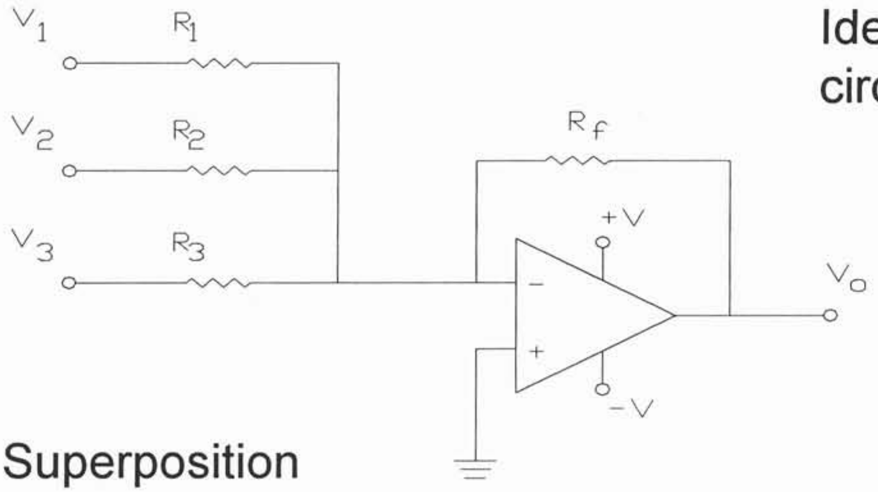
$$R_{eq} = \frac{1M\Omega(5k\Omega)}{1M\Omega + 5k\Omega} = 4975\Omega$$

$$V_{in} = \frac{4975\Omega}{10k\Omega + 4975\Omega}(12V) = \underline{3.987V}$$

$V_{in} = V_o = \underline{3.987V}$

Electronic Adding and Subtracting

Inverting Summing Amplifier



Ideal circuit

Using Superposition

Gain v_1 $\frac{-R_f}{R_1}$ Gain v_2 $\frac{-R_f}{R_2}$ Gain v_3 $\frac{-R_f}{R_3}$

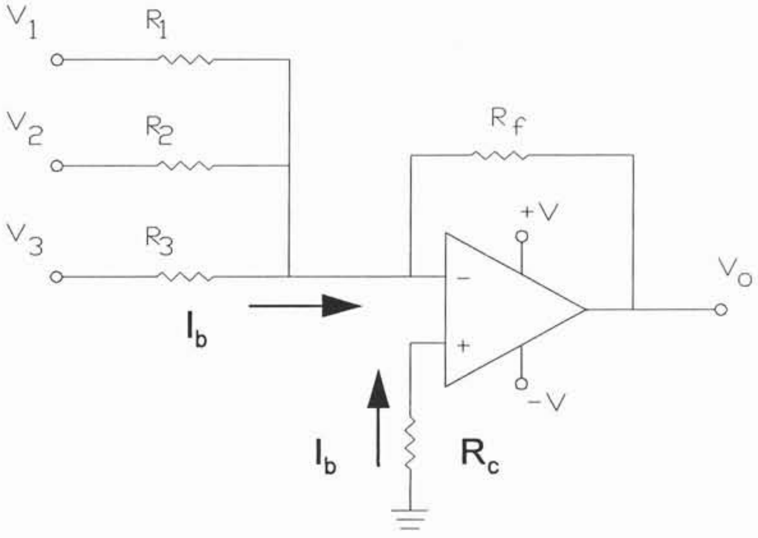
Total output $$v_0 = -R_f\left(\frac{v_1}{R_1}+\frac{v_2}{R_2}+\frac{v_3}{R_3}\right)$$

Output is inverted sum of v_1, v_2, and v_2

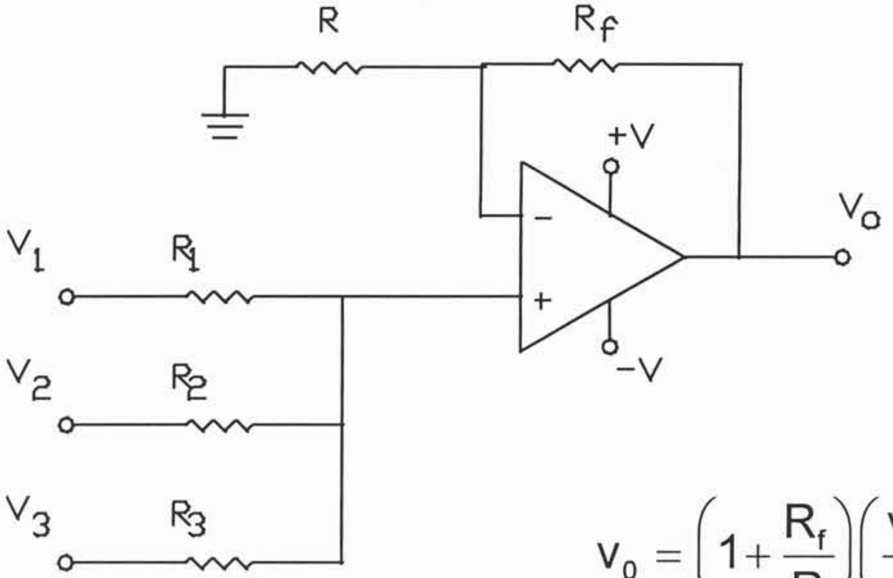
Improved circuit (non-ideal OP AMP)

bias compensation R

$R_c = R_1 \parallel R_2 \parallel R_3 \parallel R_f$



Non-inverting Summing Amp



$R_1 = R_2 = R_3$

$$v_0 = \left(1 + \frac{R_f}{R}\right)\left(\frac{v_1 + v_2 + v_3}{3}\right)$$

For any number, n, inputs.

$$v_0 = \left(1 + \frac{R_f}{R}\right)\left(\frac{v_1 + v_2 + \ldots\ldots + v_n}{n}\right)$$

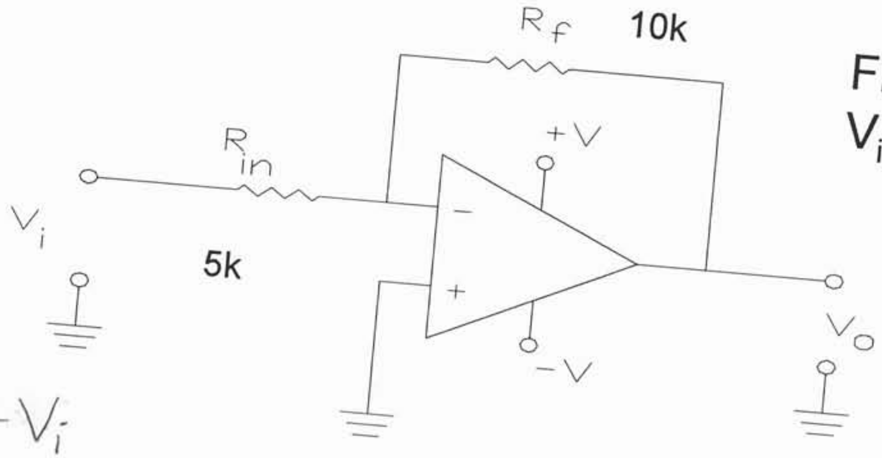
Assuming R_1=R_2=R_3=...R_n

Example: n=3, R_1 = R_2 = R_3=56k R_f = 9k R = 1k
V_1 = 0.5 Vdc V_2 = 0.37 Vdc V_3 = 0.8 Vdc Find V_o

$$v_0 = \left(1 + \frac{R_f}{R}\right)\left(\frac{v_1 + v_2 + v_3}{3}\right)$$

$$v_0 = \left(1 + \frac{9k}{1k}\right)\left(\frac{0.5 + 0.37 + 0.8}{3}\right) = 5.5667$$

Inverting Amp Examples



Find V_o with
$V_{in} = 2$ Vdc

$$V_o = \frac{-R_f}{R_{in}} V_i$$

$$V_o = \frac{-10\,k\Omega}{5\,k\Omega} V_i \longrightarrow V_o = -2V_i \quad V_o = -2(2\,V) = \underline{\underline{-4\,Vdc}}\ \underline{\underline{ANS}}$$

Let $R_f = 5\,k\Omega$, find A_V & V_o $V_i = 2\,Vdc$

$$A_V = \frac{-R_f}{R_{in}} = \frac{-5\,k\Omega}{5\,k\Omega} = \underline{\underline{-1}}$$ INVERTING VOLTAGE FOLLOWER

$$V_{in} = 2\,Vdc \quad V_o = A_V V_i = -1(2) = \underline{\underline{-2\,Vdc}}\ \underline{\underline{ANS}}$$

Let $R_f = 2.5\,k\Omega$ Find A_V & V_o $V_i = 2\,Vdc$

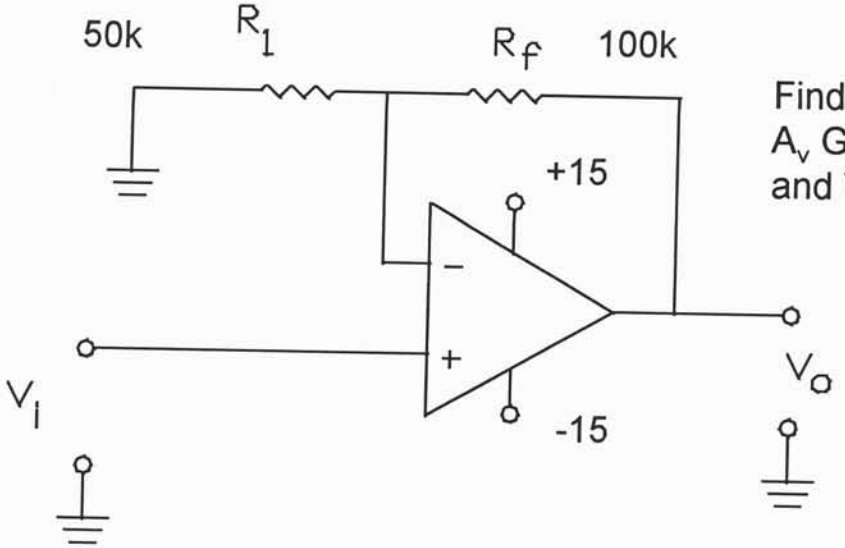
$$A_V = \frac{-R_f}{R_{in}} = \frac{-2.5\,k\Omega}{5\,k\Omega} = -0.5$$ Reduces INPUT VOLTAGE

$$V_o = A_V V_i = -0.5(2\,vdc) = \underline{\underline{-1\,vdc}}$$

OUTPUT SMALLER THAN INPUT

Like Dividing V_i by 2

Non-Inverting Amp Examples



Find V_o and A_v Given values of R and V_i = -1 Vdc

$$V_o = V_i\left(1+\frac{R_f}{R_1}\right) = \left(1+\frac{100\,k\Omega}{50\,k\Omega}\right)V_i$$

$$V_o = -1(3) = \underline{\underline{-3\ \text{Vdc}}}\ \underline{\underline{\text{ANS}}}$$ NO SIGN CHANGE

$$A_V = \left(1+\frac{R_f}{R_1}\right) = \left(1+\frac{100k}{50k}\right) = \underline{\underline{3}}\ \underline{\underline{\text{ANS}}}$$

NOTE: R_{in} of OP AMP IS ∞ AMP WILL NOT REDUCE OUTPUT OF PREVIOUS STAGE

V_i Rises to 6 Vdc, what is V_o?

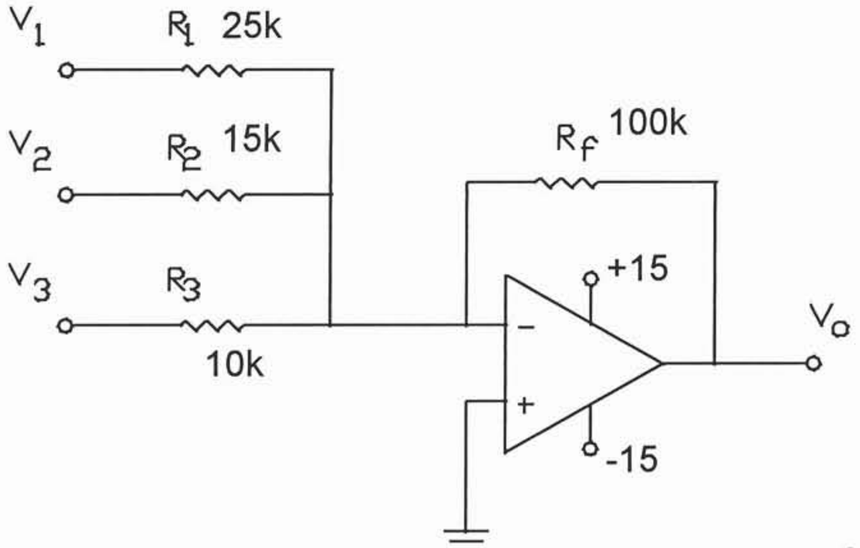
$$V_o = A_V V_i = 3(6) = 18\ \text{Vdc}$$

CAN'T HAPPEN! AMP SATURATES AT 13-15 Vdc

Power Supply Limited

Ac signals distorted (Clipping)

Summing Amplifier Example



Find V_o Given
v_1 = 0.2 Vdc
v_2 = 0.1 Vdc
v_3 = -0.1 Vdc

$$V_o = -R_f\left(\frac{V_1}{R_1}+\frac{V_2}{R_2}+\frac{V_3}{R_3}\right) = \frac{-R_f}{R_1}V_1 + \frac{-R_f}{R_2}V_2 + \frac{-R_f}{R_3}V_3$$

$$V_o = \frac{-100k\Omega}{25k\Omega}(0.2V) + \frac{-100k\Omega}{15k\Omega}(0.1V) + \frac{-100k\Omega}{10k\Omega}(-0.1V)$$

$$V_o = -4(0.2V) + -6.667(0.1V) + -10(-0.1V)$$

GAINS

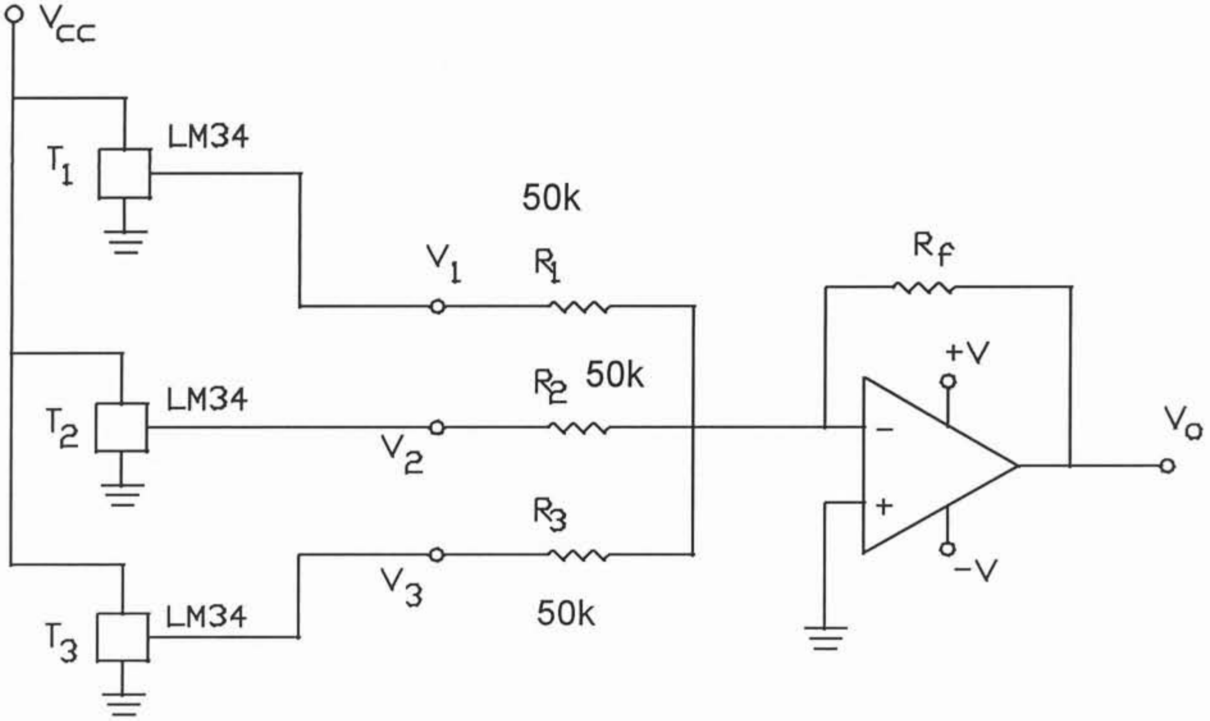
$$V_o = -0.8 - 0.6667 + 1.0\text{ Vdc} = -0.4667\text{ Vdc}$$

APPLICATIONS

R_1, R_2 R_3 POTENTIOMETERS CREATES
AUDIO MIXER

WHEN $R_1 = R_2 = R_3$ V_o = AVERAGE OF $V_1, V_2 V_3$

Averaging the Output of 3 Temperature Sensors



LM34 - temperature sensors. Gain = 10 mV/F
T_1 = 50 F T_2= 45 F T_3= 50 F

Average the temperature using a gain of -1 and -5. Find the value of R_f and V_o for each value of Gain

TO AVERAGE $R_1 = R_2 = R_3 = 50\,k\Omega$

$$V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$

SINCE $R_1 = R_2 = R_3$

$$V_o = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_1} + \frac{V_3}{R_1}\right)$$

Averaging Example continued

$$V_o = \frac{-3R_f}{R_1}(V_1+V_2+V_3) \quad \text{for 3 inputs}$$

for n inputs

$$V_o = \frac{-nR_f}{R_1}(V_1+V_2+V_3+\ldots+V_n)$$

for a gain of -1 find Relationship between R_1 and R_f

$$\frac{-nR_f}{R_1} = -1 \Rightarrow \boxed{R_f = \frac{R_1}{n}}$$

for any gain value A_V $\quad \frac{-nR_f}{R_1} = A_V \Rightarrow \boxed{R_f = \frac{A_V R_1}{n}}$

find R_f and V_o $\quad A_V = -1$

$T_1 = 50°F \quad V_1 = 10\,mV/F\,(T_1) = 10\,mV/F\,(50°F) = \underline{0.5\,V}$

$T_2 = 45°F \quad V_2 = 10\,mV/F\,(T_2) = 10\,mV/F\,(45°F) = \underline{0.45\,V}$

$T_3 = 40°F \quad V_3 = 10\,mV/F\,(T_3) = 10\,mV/F\,(40°F) = \underline{0.40\,V}$

$R_1 = 50k\Omega \quad R_f = \frac{A_V R_1}{n} = \frac{1(50k\Omega)}{3} = \underline{16.67k\Omega}$ ANS

$$V_o = \frac{-nR_f}{R_1}(V_1+V_2+V_3) = -\frac{(16.67k\Omega)}{50k\Omega}(0.5+0.45+0.4)$$

$V_o = \underline{-0.45\,V}$ $\quad$ AVERAGE $= \frac{0.5+0.45+0.4}{3} = 0.45$

Checks

Averaging Example continued

Part 2 Find R_f and V_o for $A_v = -5$

Find Value of R_f

$$R_f = \frac{A_V R_1}{n} \quad n = 3 \quad R_1 = 50\,k\Omega \quad A_V = 5$$

$$R_f = \frac{5(50\,k\Omega)}{3} = \underline{\underline{83.33\,k\Omega}}\ \text{ANS}$$

$$V_o = -\frac{R_f}{R_1}\left(V_1 + V_2 + V_3\right) = -\frac{83.33\,k\Omega}{50\,k\Omega}\left(0.5 + 0.45 + 0.4\right)$$

$$V_o = -1.667(1.35V) = \underline{\underline{-2.25V}}\ \underline{\underline{\text{ANS}}}$$

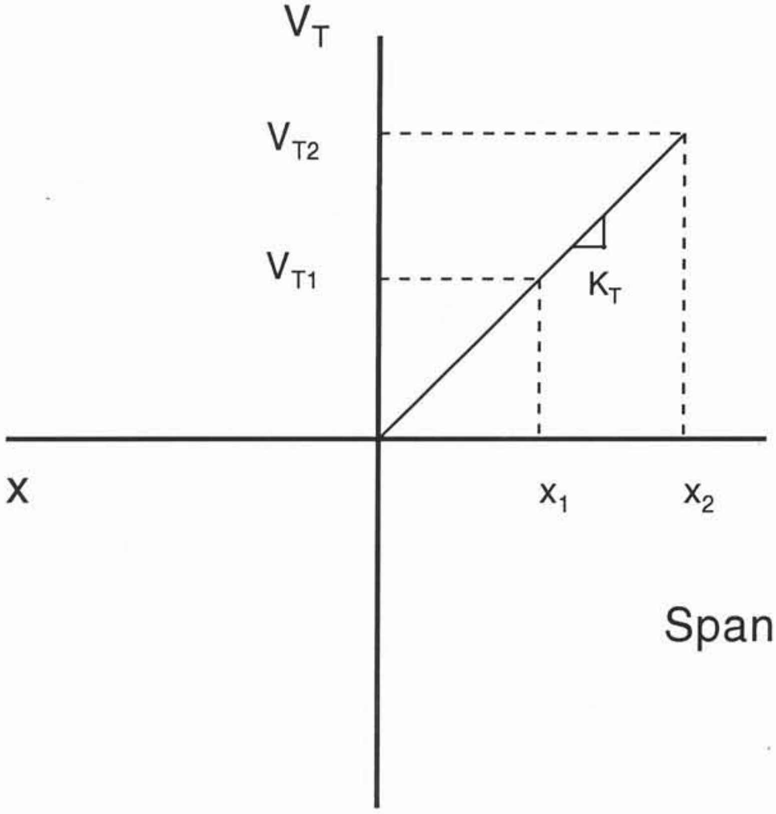
$$\text{Check} \quad 5\left(\frac{0.5 + 0.45 + 0.4}{3}\right) = \underline{\underline{2.25}}\ \text{checks}$$

NOTE: R_f is non-standard value

use closest standard value and series Potentiometer then calibrate.

Practical R_f

Scaling Linear Transducer Outputs



x= transducer input (measured value)

V_T = transducer output voltage

K_T = transducer gain (slope)

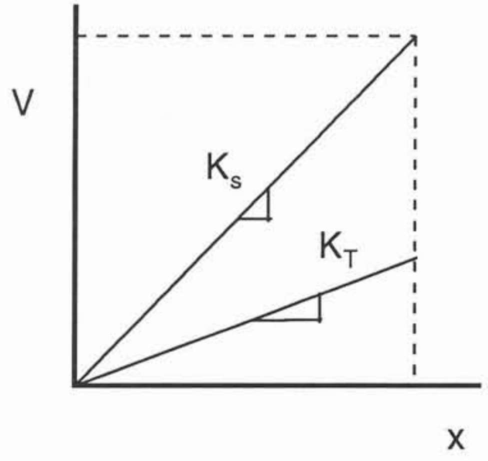
K_s = scaler gain

V_s = scaler output

Span = max. value - min. value

$$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i}$$

Case 1: no offset in transducer or scaler characteristics



Transducer gain formula: $V_T = K_T x$

Required scaler gain:

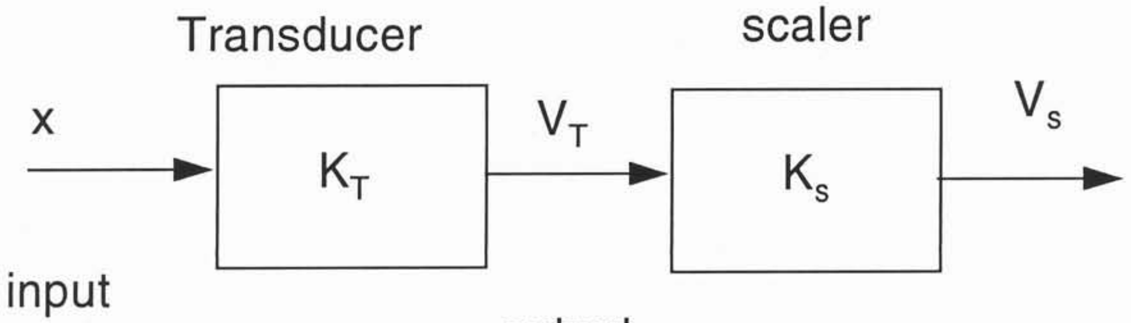
$$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

scaler output formula

$$V_s = K_s \cdot K_T \cdot x$$

$$V_s = K_s \cdot V_T$$

Block Diagram Case 1: multiply by constant



Transducer

scaler

x

V_T

V_s

K_T

K_s

input

output

$V_T = K_T x$

$V_s = K_s V_T$

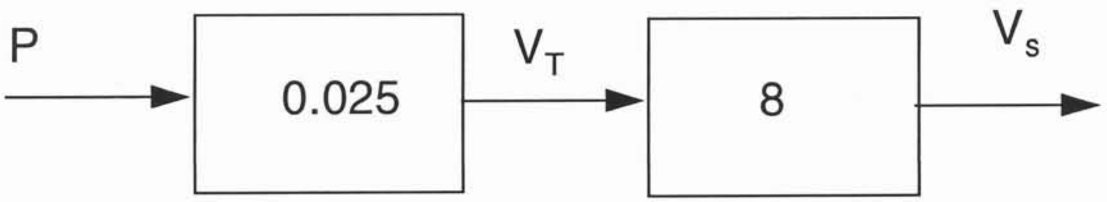
Example: Pressure transducer: P (x) range 0 - 50 pisg
V (V_T) range 0 - 1.25 Vdc

Desired output range: 0 - 10 Vdc
Find transducer gain, gain required to scale transducer to new range and draw the block diagram with calculated values shown.

Find K_T $$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i} \qquad K_T = \frac{1.25 - 0\ \text{V}}{50 - 0\ \text{psig}} = 0.025\ \text{V / psig}$$

Find scaler gain $$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

$$K_s = \frac{10 - 0\ \text{V}}{1.25 - 0\ \text{V}} = 8\ \text{V / V}$$



P

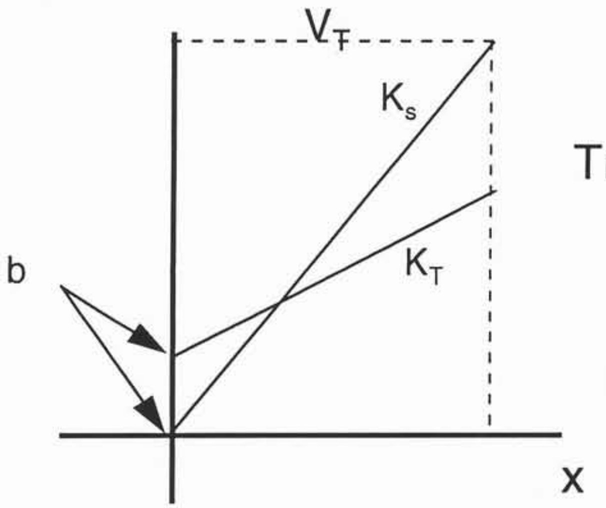
0.025

V_T

8

V_s

Case 2: Transducer with offset - Output with no offset



b = transducer offset

Transducer gain formula:

$$V_T = K_T x + b$$

Scaler gain formula
Must subtract offset

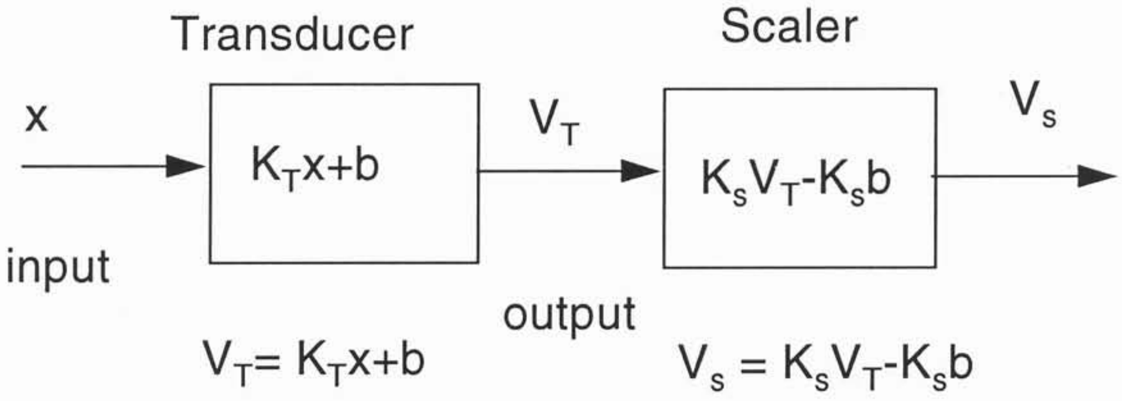
$$V_s = K_s(K_T x + b) - K_s b$$

$$V_s = K_s K_T x + K_s b - K_s b$$

correct form of scaler output ⟶ $V_s = K_s K_T x$

scaler equation ⟶ $V_s = K_s V_T - K_s b$

Block diagram



Transducer

Scaler

x

input

$K_T x + b$

V_T

output

$K_s V_T - K_s b$

V_s

$V_T = K_T x + b$

$V_s = K_s V_T - K_s b$

Example: offset transducer output transducer range

Temperature transducer Temp range 0 - 100 C (x)
output 1 - 5 Vdc (V_T)

Desired output: 0 - 10 Vdc
Find K_T and b for transducer. Also find K_s to get desired output. Also draw the block diagram and insert gains and formulas

Find K_T $K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i}$ $K_T = \frac{5-1\text{ V}}{100\ -\ 0\text{ C}} = 0.04\text{ V / C}$

Find b from point slope form of line $V_T - V_{T1} = K_T(x - x_1)$

$$V_T - 1 = 0.04(x - 0)$$

$$V_T = 0.04x + 1$$ so b = 1

Find scaler gain $K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$

$$\frac{S_d}{S_T} = \frac{10-0\text{ V}}{5-1\text{ V}} = \frac{10}{4} = 2.5\text{ V / V} = K_s$$

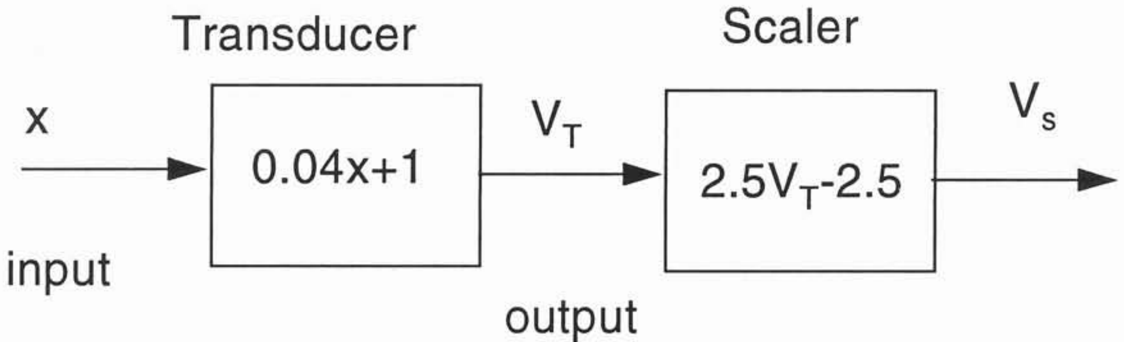
$$V_s = K_s V_T - K_s b$$

$$V_s = 2.5V_T - 2.5(1)$$

scaler formula $V_s = 2.5V_T - 2.5$

Example: Case 2 continued

Block Diagram



Transducer

Scaler

x

V_T

V_s

0.04x+1

$2.5V_T$-2.5

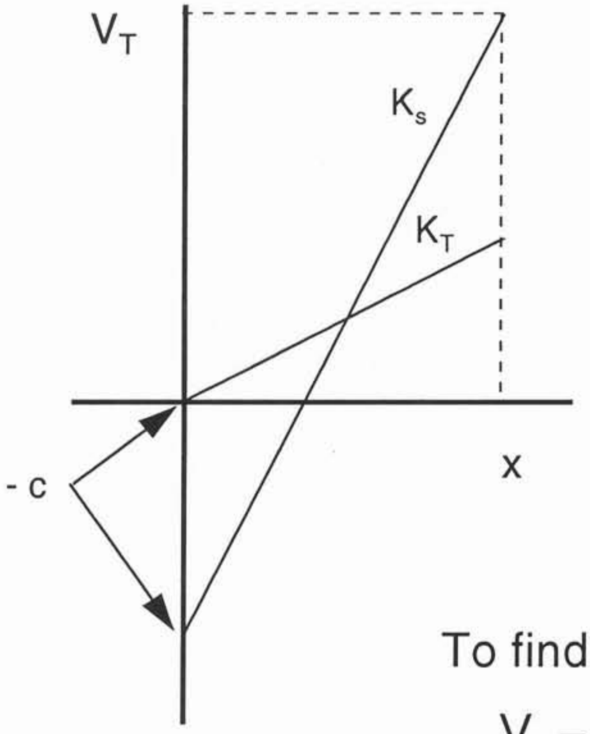
input

output

V_T= 0.04x+1

$V_s = 2.5V_T$-2.5

Case 3: Transducer with no offset, Output offset



c = scaler offset can be +- value

Transducer gain formula:

$$V_T = K_T x$$

Scaler formula must add a constant

$$V_s = K_s V_T + c$$

$$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

To find c, use point slope form

$$V_s - V_{s1} = K_s (V_T - V_{T1})$$

Case 3: Example: Pressure transducer
range 0 - 25 psig (x)
0 - 1 V (V_T)
Desired range (-5 V to 5 Vdc) Find transducer gain and scaler gain formula. Draw the block diagram of the system

Find K_T $$K_T = \frac{\text{output span}}{\text{input span}} = \frac{S_o}{S_i} \quad K_T = \frac{1-0\text{ V}}{25\ -\ 0\text{ psig}} = 0.04\text{ V / psig}$$

Find K_s $$K_s = \frac{\text{desired span}}{\text{transducer span}} = \frac{S_d}{S_T}$$

$$\frac{S_d}{S_T} = \frac{5-(-5)\text{ V}}{1-0\text{ V}} = \frac{10}{1} = 10\text{ V/V} = K_s$$

Use point-slope form of line to find the value of c

$$V_s - V_{s1} = K_s(V_T - V_{T1}) \quad (V_{T1}, V_{s1}) = (1,\ 5)$$

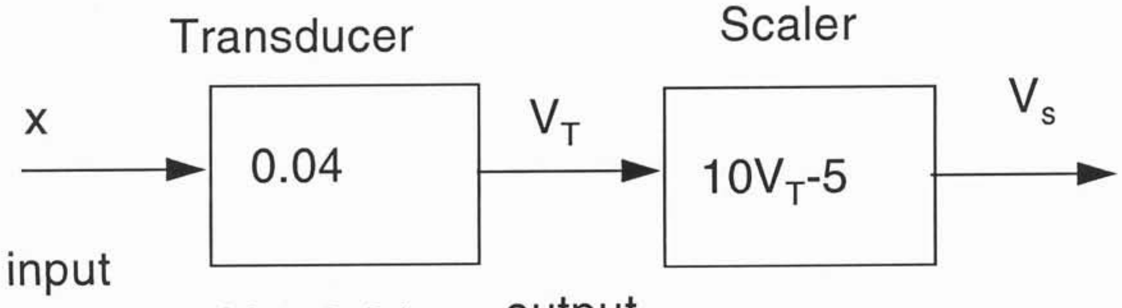
can use either point in pair defining range

$$V_s - 5 = 10(V_T - 1)$$

$$V_s = 10V_T - 5$$

scaler gain formula
c = -5 in this case

Block Diagram



Transducer

Scaler

x

0.04

V_T

$10V_T$-5

V_s

input

V_T= 0.04x

output

V_s = $10V_T$-5

Example 1

Range of linear temperature transducer is 32- 212 F with a transducer gain of 10 mV/F. The desired output of the transducer for the range of temperature is 0 - 10 Vdc. Find the gain formula.

$K_T = 0.01\,V/F$ FIND SPAN AT $T_1 = 32F$ $V_{T1} = 0.01V/F\,(32F) = 0.32\,V$

AT $T_2 = 212F$ $V_{T2} = 0.01\,V/F\,(212F) = 2.12\,V$

$S_T = V_{T2} - V_{T1} = 2.12 - 0.32V$

$S_T = 1.8\,V$

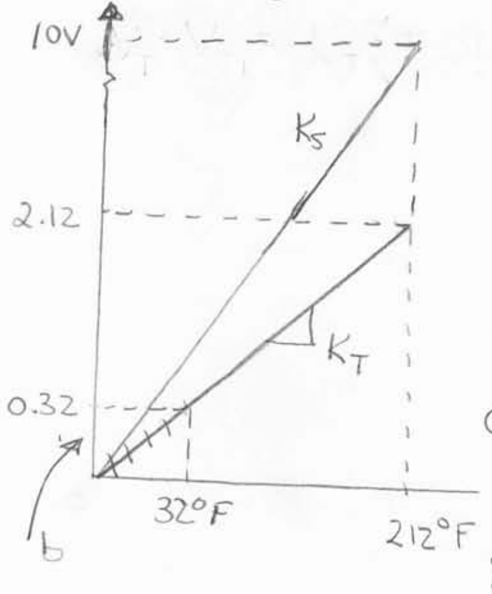
OUTPUT AT 32°F scaled to 0 V

S_d = desired span $S_d = 10 - 0V = 10V$ $V_T = K_T T + b$

$$K_S = \frac{S_d}{S_T} = \frac{10\,V}{1.8\,V} = 5.56$$

CASE 2 SCALING MUST REMOVE OFFSET

$V_s = K_S V_T - K_S b$ FIND b from Graph $b = 0.32\,V$



$V_s = 5.56\,V_T - 5.56(0.32)\ V/V$

$V_s = 5.56V_T - 1.779\ V/V$ check

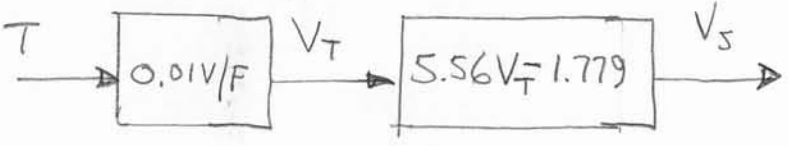
at 32°F $V_T = 0.32\,V$

$V_s = 5.56(0.32) - 1.779 = 0\,V$

at 212°F $V_T = 2.12\,V$ Checks

$V_s = 5.56(2.12) - 1.779 = 10.01\,V$

Block Diagram



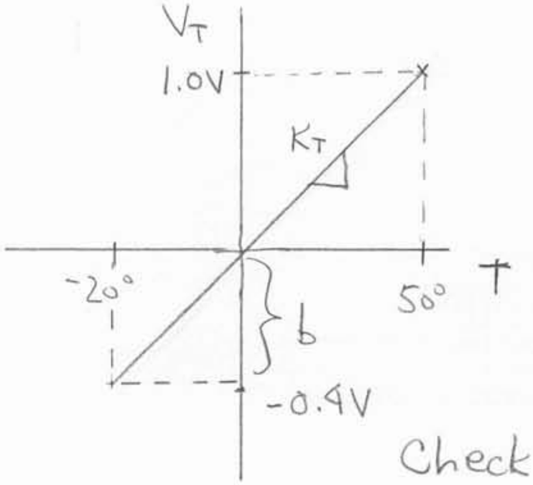
Example 2

Linear temperature transducer
Range: -20 C to 50 C $K_T = 20$ mV/C
Desired range: 0 - 5 Vdc.
Transducer output voltage is bipolar (+-)

V_T at -20 °C $\quad V_{T1} = 20\text{ mV/°C}(-20\text{°C}) = -0.4\text{V}$

V_T at 50°C $\quad V_{T2} = 20\text{ mV/°C}(50\text{°C}) = 1.0\text{ V}$

SCALER FORMULA $\quad V_S = K_S V_T - K_S b$ Find b Graphically



$$K_S = \frac{S_d}{S_T} = \frac{(5-0)V}{1-(-0.4)} = 3.571\text{ V/V}$$

b = -0.4V from plot

$$V_S = 3.571 V_T - 3.571(-0.4)$$

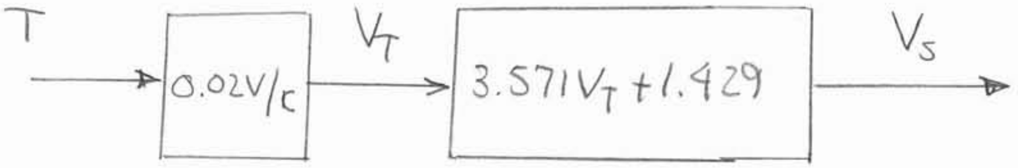
$$V_S = 3.571 V_T + 1.429\text{ V/V}$$

Check $V_T = -0.4\text{ V}$ at -20°C

$$V_0 = 3.571(-0.4v) + 1.429 \approx 0\text{V}$$

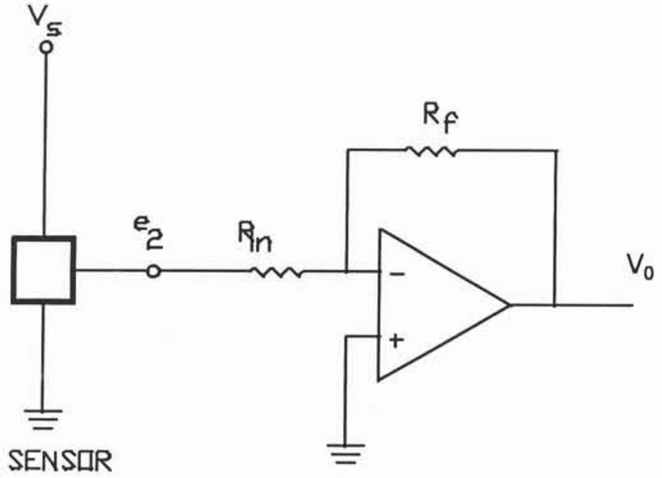
$V_T = 1\text{V}$ at 50°C

$$V_0 = 3.571(1) + 1.429 = 5.0\text{V}$$



Practical Realization of gain formula

Use OP AMP circuits



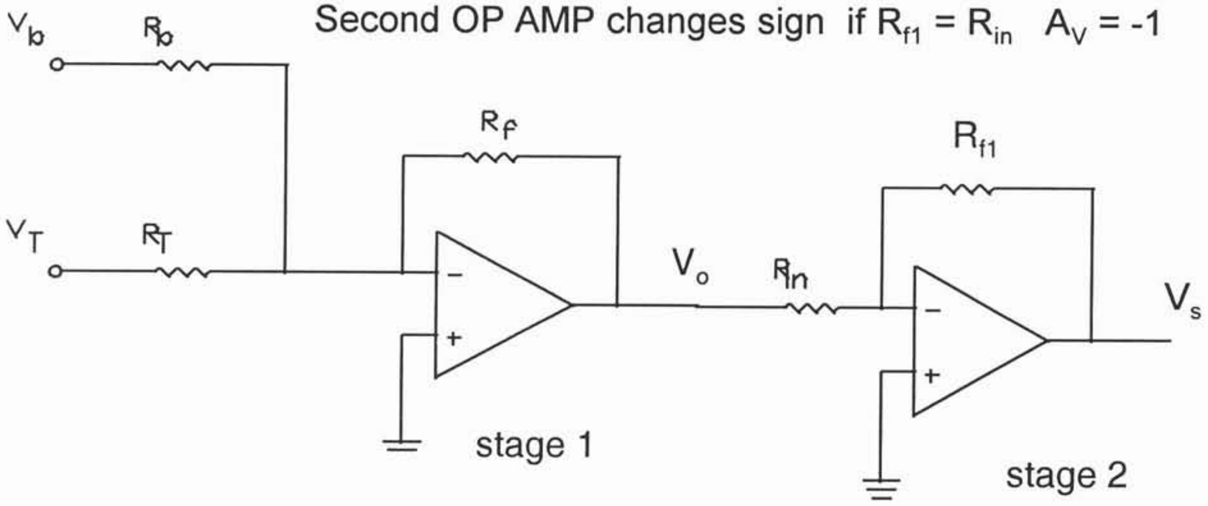
scaling without offset:
use inverting or non-inverting amps to implement K_s

For inverting

$$K_s = \frac{-R_f}{R_{in}}$$

For transducers with offset use inverting summation amps

Second OP AMP changes sign if $R_{f1} = R_{in}$ $A_V = -1$



stage 1

overall gain

$$V_o = \frac{-R_f}{R_T}V_T + \frac{-R_f}{R_b}V_b \qquad V_s = \left(\frac{-R_f}{R_T}V_T + \frac{-R_f}{R_b}V_b\right)(-1) = \frac{R_f}{R_T}V_T + \frac{R_f}{R_b}V_b$$

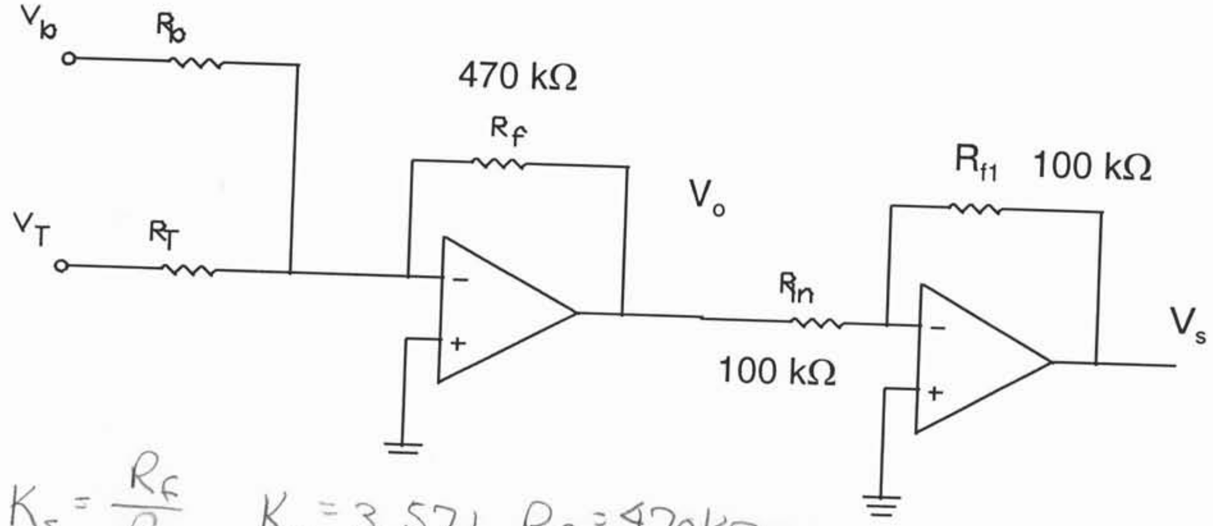
Gain formula with offset

$$V_s = \left(K_s V_T - K_s b\right) = \frac{R_f}{R_T}V_T + \frac{R_f}{R_b}V_b \quad \text{so} \quad K_s = \frac{R_f}{R_T} \quad K_s b = \frac{R_f}{R_b}V_b$$

Example: Design an OP AMP circuit that will implement the scaler equation from Example 2

Assume $R_{f1} = R_{in} = 100\ k\Omega$ $R_f = 470\ k\Omega$

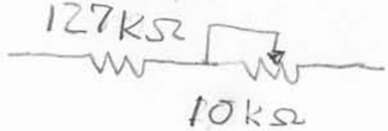
$V_s = 3.571V_T + 1.429$ scaler relationship for Example 2



$K_s = \frac{R_f}{R_T}$ $K_s = 3.571$ $R_f = 470k\Omega$

SOLVE FOR R_T $R_T = \frac{R_f}{K_s}$ $R_T = \frac{470k\Omega}{3.571} = \underline{131.61k\Omega}$

for R_T use standard value & Potentiometer



Calibrate with $V_T = -0.4$ and $1.0V$

$K_s b = 1.429$ $R_f > 0$ AND $R_b > 0$

$K_s b = \frac{Rf}{R_b} V_b$ Assume $V_b = 2.0V$ must be > 0

$1.429 = \frac{Rf}{R_b}(2) \Rightarrow \frac{1.429}{2} = \frac{Rf}{R_b}$ $R_f = 470k\Omega$ so

$0.7145 R_b = 470k\Omega \rightarrow R_b = \frac{470k\Omega}{0.7145} = \underline{657.80k\Omega}$

Simple case make $\underline{\frac{Rf}{R_b} = -1}$ so $\underline{V_b = K_s b}$

Models of Components

Types of components in controls systems

electrical
mechanical
liquid flow
gas flow
thermal

Behavior of systems defined by characteristics
Example: electrical

resistance
capacitance
inductance
delay

voltage
current
charge

Resistance - amount of potential required to produce a unit current.

Capacitance - amount of charge required to make a unit change in potential.

Inductance - amount of potential required to make a unit change in rate of flow (current).

Delay (Dead-time) - time interval between signal appearing on input and response appearing on output.

Defining Equations

Resistance

Static resistance (linear)

$$R = \frac{e}{i}$$

Dynamic Resistance (non-linear) Depends on the values of e and i.

$$R = \frac{\Delta e}{\Delta i} = \frac{de}{di}$$

Can estimate dynamic R with slope of tangent line at operating point.

$$R = \frac{\Delta e}{\Delta i} = \frac{e_2 - e_1}{i_2 - i_1}$$

Capacitance

$$C = \frac{\Delta q}{\Delta e}$$

I is rate of change of flow
Columb/sec = Amp

$$C \cdot \Delta e = \Delta q \qquad C \cdot \frac{\Delta e}{\Delta t} = \frac{\Delta q}{\Delta t} = i$$

$$i = C \cdot \frac{de}{dt}$$

Definition of capacitance

Inductance

$$e = L \cdot \frac{\Delta i}{\Delta t} = L \cdot \frac{di}{dt}$$

Potential required to make change in flow

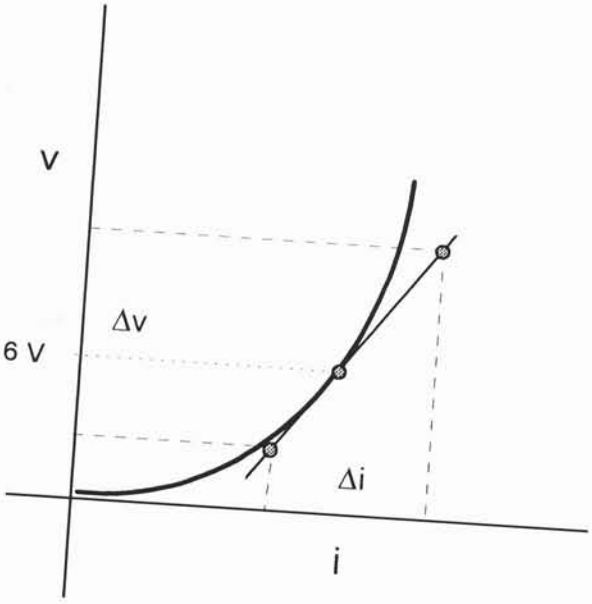
Dead-time Delay - use in high frequency transmission lines and satellite communications

$$t_d = \frac{D}{v_p}$$

Where: D = distance (m)

V_p = velocity of propagation (m/s)

Example: Non-linear resistance Volt-amp characteristic. Dynamic resistance.



Find R at 6 V operation

$$R = \frac{\Delta v}{\Delta i}$$

$i_1 := 0.5$ $i_2 := 0.504$

$v_1 := 5.95$ $v_2 := 6.05$

$\Delta v := v_2 - v_1$ $\Delta i := i_2 - i_1$

$\Delta v = 0.1$ $\Delta i = 0.004$

$$R := \frac{\Delta v}{\Delta i}$$

$R = 25$ ohm

Example: Sine voltage across a capacitor. What is the current?

$$e(t) = V_{max} \cdot \sin(\omega \cdot t)$$

$$i(t) = C \cdot \frac{dv}{dt} = C \cdot \frac{d}{dt} V_{max} \cdot \sin(\omega \cdot t) = C \cdot V_{max} \cdot \omega \cdot \cos(\omega \cdot t)$$

90 degree lead between current and voltage

Example: Current pulse of 0.1 sec and amplitude of 0.1 mA applied to a capacitor produces a rise in voltage from 0 to 25 V. What is the capacitance?

$i = C \cdot \frac{\Delta e}{\Delta t}$ solve for C $i \cdot \frac{\Delta t}{\Delta e} = C$

$i := 0.1 \cdot 10^{-3} \cdot amp$ $e_1 := 0 \cdot volt$ $e_2 := 25 \cdot volt$

$t_1 := 0 \cdot sec$ $t_2 := .1 \cdot sec$

$\Delta e := e_2 - e_1$ $\Delta t := t_2 - t_1$

$\Delta e = 25 \cdot volt$ $\Delta t = 0.1 \cdot sec$

$C := i \cdot \frac{\Delta t}{\Delta e}$ $C = 4 \cdot 10^{-7} \cdot farad$

$C = 4.0\ \mu F$ **Ans**

Example: voltage pulse of amplitude 5 with a duration of 0.02 sec is applied across an inductor. This causes a current increase from 1 amp to 2.1 amp. Find L.

$e = L \cdot \frac{\Delta i}{\Delta t}$ Solve for inductance $L = e \cdot \frac{\Delta t}{\Delta i}$

$t_1 := 0 \cdot sec$ $t_2 := 0.02 \cdot sec$ $e := 5 \cdot volt$

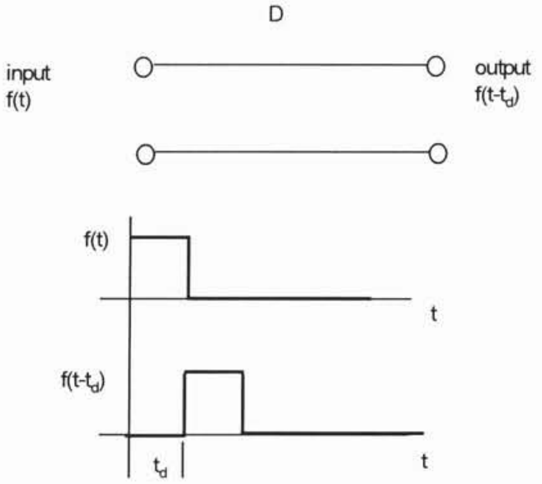
$i_1 := 1 \cdot amp$ $i_2 := 2.1 \cdot amp$

$\Delta t := t_2 - t_1$ $\Delta i := i_2 - i_1$

$\Delta t = 0.02 \cdot sec$ $\Delta i = 1.1 \cdot amp$

$L := e \cdot \frac{\Delta t}{\Delta i}$ $L = 0.091 \cdot henry$ (V*s/A)

Example: Electrical delays Transmission lines



$$t_d = \frac{D}{v_p}$$

v_p = velocity of propagation typical values between 2-3 x10^8 m/s a.) Find the delay of a 600 m transmission line with $v_p = 2.3\text{x}10^8$ m/s

b.) Find the delay of a satellite transmission with a path length of 2000 km and a velocity of propagation of $3\text{x}10^8$ m/s.

a.)

$$D := 600 \cdot m \qquad v_p := 2.3 \cdot 10^8 \cdot \frac{m}{sec}$$

$$t_d := \frac{D}{v_p} \qquad t_d = 2.609 \cdot 10^{-6} \cdot sec$$

t_d =2.609 μS **<u>Ans</u>**

b.) Convert km to m

$$2000 \cdot km \cdot 1000 \cdot \frac{m}{km} = 2 \cdot 10^6 \cdot m$$

$$t_d := \frac{2 \cdot 10^6 \cdot m}{3 \cdot 10^8 \cdot sec} \qquad t_d = 6.667 \cdot 10^{-3} \cdot \frac{m}{sec}$$

t_d = 6.667 mS **<u>Ans</u>**

Liquid Flow Characteristics

Flow = volume of liquid/unit time
(e.g. gallons/minute)
Potential = pressure

Liquid flow resistance - depends on the type of flow

Types of liquid flow

Laminar - low velocity flows. Stream lines are parallel. Liquid flows in layers. Linear flow resistance

turbulent - relatively high velocity flow. non-linear flow resistance

Flow type determined by the Reynold's Number

$$R = \frac{\rho \cdot v \cdot d}{\mu}$$

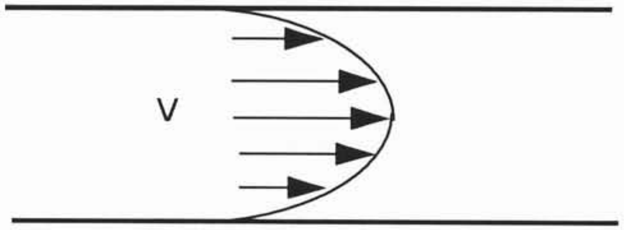
Where ρ = density of the fluid (kg/m^3)
v = average velocity of the fluid (m/s)
d = diameter of pipe (m)
μ = absolute viscosity of fluid (Pa-s)

Note: Reynold's number is dimensionless

Laminar flow : $R<2000$
Turbulent flow: $R>4000$
Transition flow: $2000< R< 4000$

Average velocity of liquid

Laminar Flow



Velocity distribution

$$v = \frac{Q}{A}$$

Where:

A = area of pipe (m^2)
Q = flow (m^3/s)

Since pipe diameter is usually given

$$v = \frac{4 \cdot Q}{\pi \cdot d^2}$$

Where: Q = flow (m^3/s)
d = pipe diameter (m)

Laminar Flow Equations for Round Pipes

$$p = R_L \cdot Q \quad \text{(Pa)}$$

$$R_L = \frac{128 \cdot \mu \cdot l}{\pi \cdot d^4} \quad (\text{Pa-s/m}^3)$$

Where: p = pressure drop (Pascals)
R_L = laminar flow resistance
Q = flow (m^3/s)
l = length of pipe (m)
μ = absolute viscosity (Pa-s)
d = pipe diameter (m)

Turbulent flow Equations for Round Pipes

$$p = K_t \cdot Q^2 \qquad \text{Pa}$$

$$R_t = 2 \cdot K_t \cdot Q \qquad \text{Pa-s/m}^3$$

$$K_t = \frac{8 \cdot \rho \cdot f \cdot l}{\pi^2 \cdot d^5}$$

Where: f= friction factor (see table 3.3 p. 80 text)
l = length (m)
d = pipe diameter (m)
ρ = density of liquid (kg/m^3)
R_t = turbulent flow resistance ($Pa\text{-}s/m^3$)
p = pressure (Pa)
Q = flow (m^3/s)

Liquid Flow Capacitance - increase in volume of liquid required to make unit increase in pressure

$$C_L = \frac{\Delta V}{\Delta p}$$

Where: C_L = capacitance (m^3/Pa)
ΔV = volume change (m^3)
Δp = pressure change (Pa)

Pressure relationship

$$\Delta p = \rho \cdot g \cdot \Delta H$$

Where: ρ = density of fluid
g = acceleration due to gravity
H = height of liquid in tank

Derive relationship

$$\Delta H = \frac{\Delta V}{A} \qquad \Delta p = \rho \cdot g \cdot \frac{\Delta V}{A}$$

$$C_L = \frac{\Delta V}{\Delta p} = \frac{\Delta V}{\rho \cdot g \cdot \frac{\Delta V}{A}} = \frac{A}{\rho \cdot g} \quad (m^3/Pa)$$

Liquid Flow Inertance (Inductance)

Amount of pressure drop required to increase flow rate by 1 unit/second

$$I_L = \frac{p}{\left(\frac{\Delta Q}{\Delta t}\right)}$$

Where:

I_L = inertance (Pa/(m^3/s^2)

p = pressure drop (Pa)

$\Delta Q/\Delta t$ = change in flow

Inertance defined using physical parameter

$$I_L = \frac{\rho \cdot l}{A} \quad (\text{Pa}/(\text{m}^3/\text{s}^2)$$

Where A = area of pipe

ρ = density of liquid

l = length of pipe

Dead-time Delay of Liquid - time required to transport liquid from one point to another

$$t_d = \frac{D}{v}$$

v=average velocity of fluid (m/s)

D = distance traveled (m)

Example 1: Oil at a temperature of 15 C flows in a horizontal, 1 cm diameter tube with a flow rate of 9.42 L/min. Tube length is 10 m. Find:

Reynold's Number
Flow resistance
Pressure drop in tube

From Appendix A in text

$$\rho := 880 \cdot \frac{kg}{m^3} \qquad \mu := 0.160 \cdot Pa \cdot sec$$

CONVERT ALL UNITS TO SI UNITS $\quad d = 1\,cm = 0.01\,m$

$$Q = (9.42\ L/min)(1.6667 \times 10^{-5}\ (m^3/s)/L/min)$$

Conversion in appendix

$$Q = 1.57 \times 10^{-4}\ m^3/s$$

AVE. VELOCITY $\quad V = \frac{4Q}{\pi d^2} = \frac{4(1.57 \times 10^{-4}\ m^3/s)}{\pi (0.01m)^2} = 2.00\ m/s$

Reynold's # $\quad R = \frac{\rho v d}{\mu} = \frac{(880\ kg/m^3)(2\ m/s)(0.01m)}{(0.160\ Pa \cdot s)} = \boxed{109.95}$ ANS

$R < 2000$ LAMINAR FLOW LINEAR RELATIONSHIP BETWEEN FLOW AND PRESSURE

$$R_L = \frac{128 \mu L}{\pi d^4} = \frac{128(0.160\ Pa\text{-}s)(10\ m)}{\pi (0.01m)^4} = \boxed{6.519 \times 10^9\ Pa \cdot s/m^3}$$ ANS

Pressure Drop

$$p = R_L Q = (6.519 \times 10^9\ Pa\text{-}s/m^3)(1.57 \times 10^{-4}\ m^3/s)$$

$$\boxed{p = 1.0235 \times 10^6\ Pa}$$ ANS

Convert to PSI

$$p = (1.0235 \times 10^6\ Pa)(1.45 \times 10^{-4}\ PSI/Pa) = \boxed{148.41\ psi}$$ ANS

Example 2: Water at 15 C flows through a commercial steel pipe with a diameter of 0.4 inch with a flow rate of 6 gal/min. The line is 50 ft long. Find:

Reynold's Number
Resistance to flow
Pressure drop in pipe

FROM APPENDIX A LIQUID DENSITY AND VISCOSITY

$\rho = 1000\ \mathrm{Kg/m^3} \quad \mu = 0.001\ \mathrm{Pa\text{-}s}$

Convert English units to SI units

$d = (0.4\ \mathrm{in})(0.0254\ \mathrm{m/in}) = 0.01016\ \mathrm{m}$

$Q = (6\ \mathrm{gal/min})(6.3088 \times 10^{-5}\ \mathrm{m^3/s/gpm}) = 3.7853 \times 10^{-4}\ \mathrm{m^3/s}$

$l = (50\ \mathrm{ft})(0.3048\ \mathrm{m/ft}) = 15.240\ \mathrm{m}$

Find Average Velocity $V = \dfrac{4Q}{\pi d^2} = \dfrac{4(3.7853 \times 10^{-4}\ \mathrm{m^3/s})}{\pi (0.01016\mathrm{m})^2}$

$V = 4.669\ \mathrm{m/s}$

$R = \dfrac{\rho v d}{\mu} = \dfrac{(1000\ \mathrm{Kg/m^3})(4.669\ \mathrm{m/s})(0.01016\mathrm{m})}{0.001\ \mathrm{Pa \cdot s}}$

$R = 47{,}440 \quad R > 4000$

FLOW IS TURBULENT. MUST USE FRICTION FACTOR TABLE TO FIND RESISTANCE AND PRESSURE DROP

$P = K_t Q^2 \quad R_t = 2 K_t Q \quad K_t = \dfrac{8 \rho f l}{\pi^2 d^5}$

Find f from Table 3.3 p.80

Commerical Steel pipe diameter 1-2 cm

R between $R_a > 10{,}000 \quad R_b < 100{,}000$

f values @ R_a $f_a = 0.035 \quad f_b = 0.028$

Example 2 (cont.)

MUST USE LINEAR INTERPOLATION FORMULA TO FIND f for R = 47,440

$$f = f_a + (f_b - f_a)\left[\frac{R - R_a}{R_b - R_a}\right]$$

f must be between f_a and f_b

$$f = 0.035 + (0.028 - 0.035)\left[\frac{47440 - 10,000}{100,000 - 10,000}\right]$$

$$f = 0.035 + (-0.007)\left(\frac{3.744}{9}\right)$$

$$f = 0.03209$$

Now find K_t

$$K_t = \frac{8(1000\ kg/m^3)(0.03209)(15.24m)}{\pi^2\ (0.01016m)^5}$$

$$K_t = 3.6614 \times 10^{12}\ Pa\text{-}s/m^6$$

Find R_t

$$R_t = 2K_tQ = 2(3.6614 \times 10^{12}\ Pa\text{-}s/m^6)(3.7853 \times 10^{-4}\ m^3/s)$$

$$\boxed{R_t = 2.7719 \times 10^9\ Pa\text{-}s/m^3}$$ ANS

CONVERT TO ENGLISH UNITS

$$(2.7719 \times 10^9\ Pa\text{-}s/m^3)(9.148 \times 10^{-9}) = \boxed{25.357\ psi/gpm}$$ ANS

↗ from appendix

$$P = K_tQ^2$$

$$P = (3.6614 \times 10^{12}\ Pa\text{-}s/m^6)(3.7853 \times 10^{-4}\ m^3/s)^2$$

$$P = 5.246 \times 10^5\ Pa$$

CONVERT TO ENGLISH UNITS

$$P = (5.246 \times 10^5\ Pa)(1.45 \times 10^{-4}\ psi/Pa) = \boxed{76.07\ psi}$$

ANS

Example 4: A tank has a diameter of 1.83 meters and a height of 10 ft. Determine the capacitance of the tank when it holds: a.) water b.) oil c.) kerosene d.) gasoline

a.) $C_L = \frac{A}{\rho g}$ $\quad A = \frac{\pi d^2}{4} = \frac{\pi (1.83\ m)^2}{4} = 2.63\ m^2$

$\rho = 1000\ kg/m^3 \quad C_L = \frac{2.63\ m^2}{1000\ kg/m^2\ 9.81\ m/s^2} = \boxed{2.68 \times 10^{-4}\ m^3/Pa}$

b.) $\rho = 880\ kg/m^3 \quad C_L = \frac{2.63\ m^2}{(880\ kg/m^3)(9.81\ m/s^2)} = \boxed{3.05 \times 10^{-4}\ m^3/Pa}$

c.) $\rho = 800\ kg/m^3 \quad C_L = \frac{2.63\ m^2}{800\ kg/m^3 (9.81\ m/s^2)} = \boxed{3.35 \times 10^{-4}\ m^3/Pa}$

d.) $\rho = 740\ kg/m^3 \quad C_L = \frac{2.63\ m^2}{740\ kg/m^3 (9.81\ m/s^2)} = \boxed{3.62 \times 10^{-4}\ m^3/Pa}$

Example 5: Determine the inertance of water in a pipe with a diameter of 2.1 cm and a length of 65 meters.

$I_L = \frac{\rho l}{A} \quad \rho = 1000\ kg/m^3 \quad A = \frac{\pi d^2}{4}$

$l = 65\ m$

$A = \frac{\pi (0.021 m)^2}{4}$

$A = 3.46 \times 10^{-4} m^2$

$I_L = \frac{(1000\ kg/m^3)(65 m)}{3.46 \times 10^{-4} m^2}$

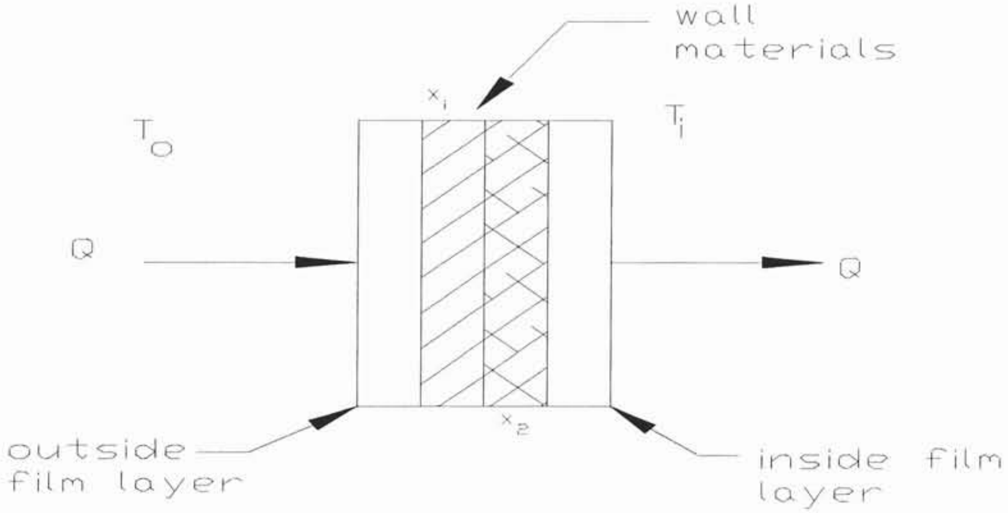
$\boxed{I_L = 1.88 \times 10^8\ Pa/(m^3/s^2)}$

$\frac{ANS}{5}$

Thermal Elements
Thermal Conduction and Convection

Thermal Resistance - opposition to heat flow. Heat flows from high temperature to low temperature

$$Q = \frac{T_o - T_i}{R_T} \quad \text{Watts}$$



R_u = unit thermal resistance

$$R_T = \frac{R_u}{A} \quad \text{K/W}$$

A = area of surface

$$Q = \frac{(T_o - T_i) \cdot A}{R_u} \quad \text{Watts}$$

Resistance of film layers depends on type of fluid, velocity of fluid. High velocity flow makes thin film. Convection cooling. "Wind Chill".

Film Conductance $h = \dfrac{1}{R_{film}}$ Called film coefficient

Film coefficients: Natural Convection In Air

a). Horizontal surface facing up

$$h = 2.5 \cdot T_d^{.25}$$

T_d = temperature difference between wall and fluid

a). Horizontal surface facing down

$$h = 1.32 \cdot T_d^{.25}$$

a). vertical surfaces

$$h = 1.78 \cdot T_d^{.25}$$

Natural convection in still water

$$h = 2.26 \cdot \left(T_w + 34.3\right) \cdot T_d^{.5}$$

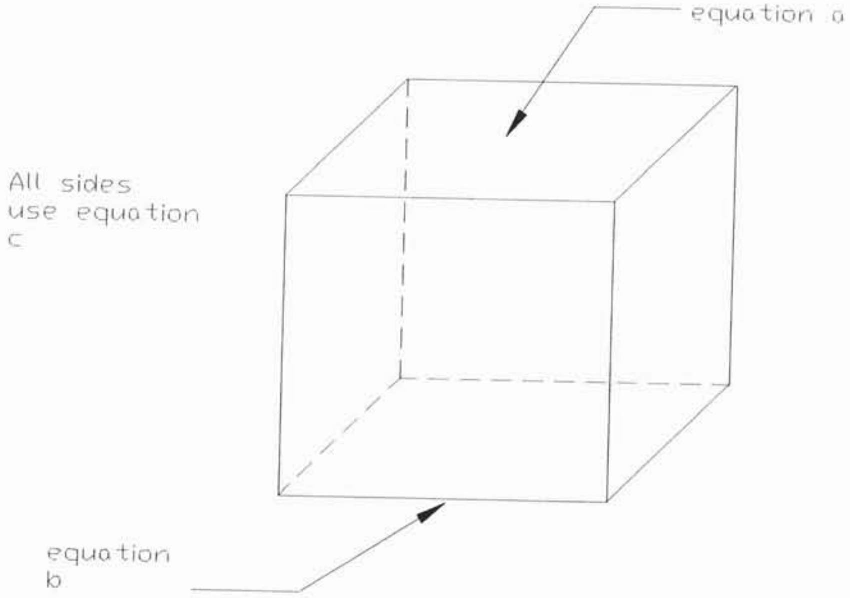
T_w = water temperature

Forced convection-air against smooth surfaces and inside pipes

$$v_{air} \le 4.6 \cdot \frac{m}{s} \qquad h = 4.54 + 4.1 \cdot v_{air}$$

$$v_{air} > 4.6 \cdot \frac{m}{s} \qquad h = 7.75 \cdot v_{air}^{0.75}$$

Using the natural convection in air formulas



Thermal resistance of inner solid layers given by

$$R_{ui} = \frac{x_i}{k_i} \qquad \text{K-m}^2\text{/W}$$

Where: x_i = thickness of i-th material

k_i = thermal conuctivity of material (see appendix in text)

Unit resistance of a composite wall is the sum of inside and outside film resistances and the resistances comprising the wall.

Series resistance analogy
Temperature analogous to voltages

$$R_u = \frac{1}{h_o} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \ldots + \frac{x_n}{k_n} + \frac{1}{h_i} \qquad \text{K-m}^2\text{/W}$$

$$R_T = \frac{1}{A}\cdot\left(\frac{1}{h_o} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \ldots + \frac{x_n}{k_n} + \frac{1}{h_i}\right) \qquad \text{K/W}$$

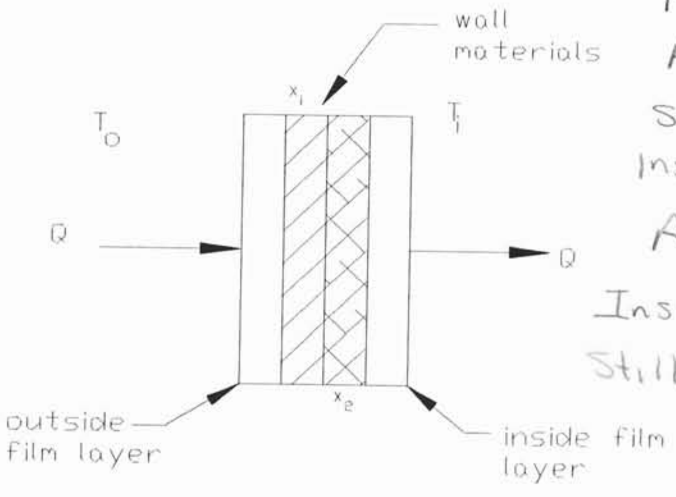
Thermal Capacitance - increase in heat required to make a unit change in temperature (SI unit J/K)

Heat Capacity (specific heat) - heat required to raise temperature of 1 kg of material by 1 K.

$$C_T = m\cdot S_T$$

Where: C_T = thermal capacitance
m = mass (kg)
S_T = heat capacity (J/kg)

Example 1: A wall section shown below has 2 layers: steel 1 cm thick, and insulation 2 cm thick. Still Water at 45 C is inside the wall. The temperature difference between the water and the inner wall is estimated to be 10 C. The outer wall is surrounded by air at a temperature of 85 C that has a velocity of 6 m/s. The wall dimensions are 2 m x 3 m. Determine the unit thermal resistance, the total thermal resistance, the total heat flow, the direction of heat flow.



Find K_1 and K_2 from Appendix A

steel $K_1 = 45 W/m \cdot K$

Insulation $K_2 = 0.036 \ W/m\text{-}K$

$A = 2m \times 3m = 6 m^2$

Inside film coefficient

Still water

$h_i = 2.26(T_w + 34.3) T_d^{0.5}$

$T_w = 45 C \quad T_d = 10 C$

$h_i = 2.26(45 + 34.3) = \underline{566.7 \ W/m^2\text{-}K}$

Outside film coefficient

air forced convection

$V_{air} = 6 m/s > 4.6 m/s$

use $h_o = 7.75 V_{air}^{0.75} = 7.75(6 m/s)^{0.75} = 29.71 w/m^2\text{-}K$

$$R_u = \frac{1}{h_o} + \frac{x_1}{k_1} + \frac{x_2}{k_2} + \frac{1}{h_i} = \frac{1}{566.7 \ w/m^2\text{-}k} + \frac{0.01m}{45w/m\text{-}k} + \frac{0.02m}{0.036 w/m\text{-}K} + \frac{1}{29.71 w/m^2\text{-}k}$$

$x_1 = 0.01 m$ $\quad R_u = 0.001765 + 0.000222 + 0.5556 + 0.033656$

$x_2 = 0.02 m$ $\quad \boxed{R_u = 0.5912 \ K\text{-}m^2/W}$ ANS

$$R_T = \frac{R_u}{A} = \frac{0.5912 \ K\text{-}m^2/W}{6 m^2} = \boxed{0.09854 \ K/W}$$ ANS

Into wall

$$\text{Heat Flow } Q = \frac{T_o - T_i}{R_T} = \frac{(85 - 45)C}{0.09854 K/W} = \boxed{406 \ W}$$

NOTE: ΔT UNITS C OR K WORK

Example 2: Determine the thermal resistance for each of the following film conditions: a.) Natural convection in still air of surface T_d = 20 C. b.) Natural convection in still water where T_d = 30 C and T_w is 20 C c.) Forced convection in air with velocity of 4 m/s.

a.) Natural Convection air on vertical surface

$T_d = 20C \quad h = 1.78 T_d^{0.25} = 1.78(20)^{0.25}$

$h = 3.764\ W/m^2\text{-}K$

$R_u = \frac{1}{h} = \boxed{0.266\ K\text{-}m^2/W}$ ANS

b.) Natural Convection in Still Water

$T_d = 30 \quad T_w = 20C \quad h = 2.26(T_w + 34.3)T_d^{0.5}$

$h = 2.26(20 + 34.3)(30)^{0.5}$

$h = 2.26(54.3)(30)^{0.5}$

$h = 672.2\ W/m^2 \cdot K$

$\boxed{R_u = \frac{1}{h} = 0.00149\ K\text{-}m^2/W}$ ANS

c.) Forced Convection Air $V_{air} \leq 4.6\ m/s$

$V_{air} = 4\ m/s \quad h = 4.54 + 4.1 V_{air}$

$h = 4.54 + 4.1(4) = 20.94\ W/m^2\text{-}K$

$R_u = \frac{1}{h} = \boxed{0.0478\ K\text{-}m^2/W}$ ANS

Mechanical Elements

Mechanical resistance = friction
Opposition to motion. Force required to increase velocity.

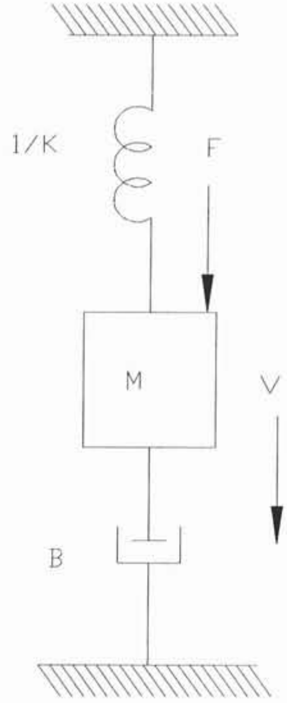
Viscous friction - friction of motion. Related to velocity
Coulomb friction - force independent of velocity

Example of viscous friction - shock absorber, dash pot

linear viscous friction $R_m = B = \frac{F}{v}$ N-s/m

Non-linear viscous friction $B = \frac{\Delta F}{\Delta v}$ $B = \frac{dF}{dt}$

At operating point use tangent line to find approx. B



Mechanical Capacitance - change in length of spring required to make unit increase in force. Inverse spring constant

$C_m = \frac{1}{K}$ N/m

Mechanical inertia (mass) - force required to make unit increase in acceleration

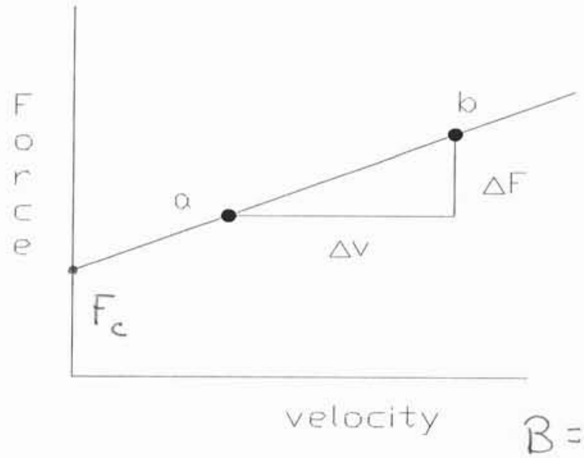
$F_{ave} = m \cdot \frac{\Delta v}{\Delta t}$ $F = m \cdot \frac{dv}{dt}$

Mechanical Dead-time - time required to transport material from one place to another

$$t_d = \frac{D}{v}$$

Where D = distance
v = velocity of material

Example 1: A mechanical system consists of a sliding load and a shock absorber. The force versus velocity curve is shown below



Test data

Run	F (N)	v (m/s)
a	7.1	10.5
b	9.6	15.75

Determine R_m and coulomb friction

$B = R_m = \frac{\Delta F}{\Delta v}$ use point slope form of Line

USE $F = F_c + B v$

$F - F_1 = \frac{\Delta F}{\Delta v}(v - v_1) \Rightarrow F - 7.1 = \frac{9.6 - 7.1}{15.75 - 10.5}(v - 10.5)$

$F - 7.1 = \frac{2.5}{5.25}(v - 10.5)$

$F - 7.1 = 0.4762v - 5$

$F = 0.4762v + 2.1$ N

$$\boxed{\begin{array}{l} B = 0.4762 \text{ N-s/m} \\ F_c = 2.1 \text{ N} \end{array}}$$

$\underline{\text{ANS}}$

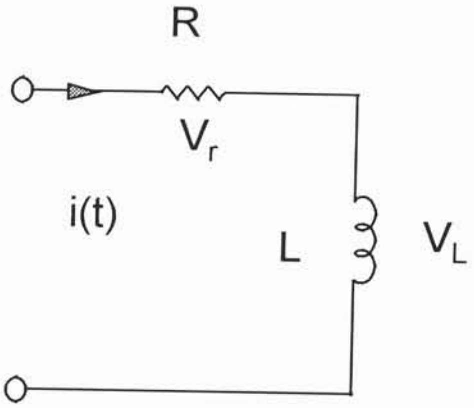
Modeling Physical Systems

Linear systems - modeled with linear differential equations

Derivatives capture how system variables change with time

Definition
Linear Differential Equation - a linear combination of derivatives of an unknown function and the unknown function

Example Series RL circuit. What does current do over time



$$L \cdot \frac{d}{dt} i(t) + R \cdot i(t) = 0$$

To solve - find how i varies with time-i(t)

Can do analytically or numerically

More Complex Differential Equations

Can have higher order derivatives... 2, 3, 4...etc.

$$\frac{d^2}{dt^2} i(t) + 2\cdot\frac{d}{dt} i(t) - 7\cdot i(t) = 0$$

2nd order- linear ODE (ordinary differential equation)

$$\frac{d^2}{dt^2} i - 3\cdot\frac{d}{dt} i + i^2 = 0 \qquad i = i(t)$$

2nd order non-linear

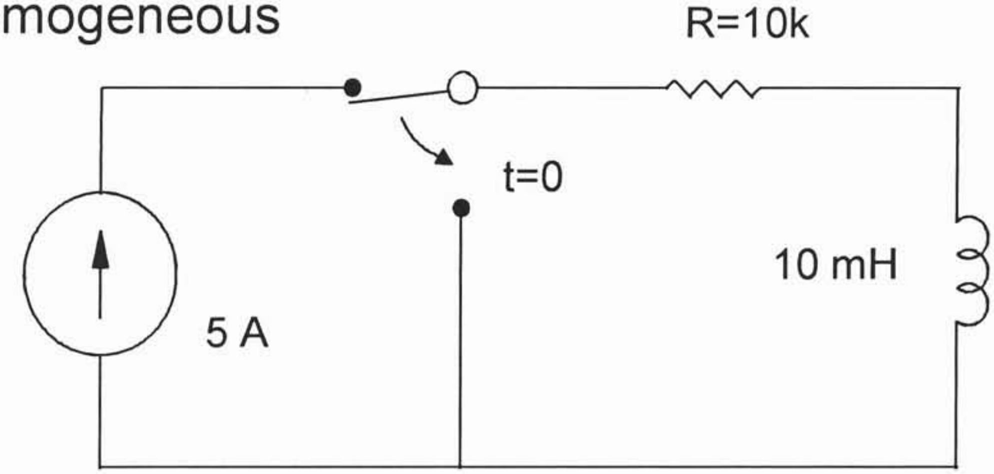
$$\left(\frac{d^2}{dt^2} v\right)\cdot\sin(v) + v = 0$$

2nd order non-linear

When right-hand side (RHS) is 0, equation called homogeneous. Implies no outside stimulation.

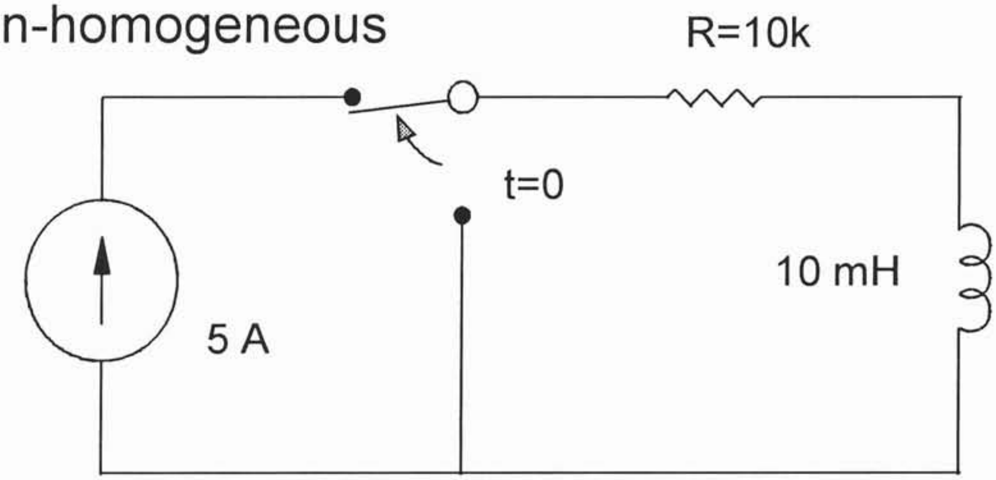
Examples

homogeneous



No current source after the switch is thrown.

non-homogeneous



Energize relay with the current source
Current source drives the system.

Differential equations for control

Equations have constant coefficients and are linear.
Single input stimulation r(t)
Single output variable x(t)

$$a_n \cdot \frac{d^n}{dt^n} x(t) + \dots\dots + a_2 \cdot \frac{d^2}{dt^2} x(t) + a_1 \cdot \frac{d}{dt} x(t) + a_0 \cdot x(t) = b_0 \cdot r(t)$$

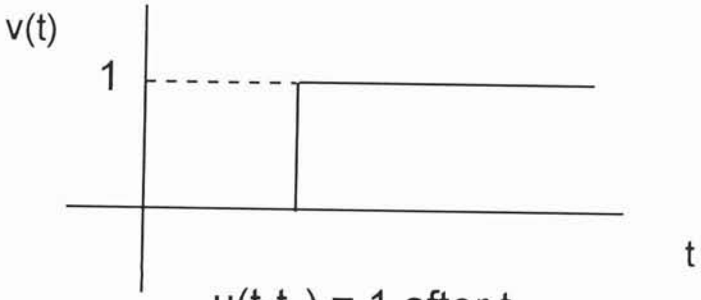
$a_n \dots\dots a_2$, a_1 a_0 and b_0 are constants

What can r(t) be?

constant	-5, 3, 10
ramp	3t, -10t, 0.2t-3
sin/cos	$100\sin(\omega t)$, $-5\cos(\omega t+\theta)$

w and q are constants

Unit-step (square wave)



$u(t-t_d)$ = 1 after t_d
0 before

Examples

Identify which of the following equations are linear and which are non-homogeneous

$$-3\cdot\frac{d}{dt}x(t)+x(t)^2=0$$

$$\left(\frac{d}{dt}v\right)^2+\sin(v)=0$$

$$6\cdot\frac{d}{dt}v+2\cdot v=V_m\cdot\sin(\omega t)$$

$$4\cdot\frac{d^2}{dt^2}i-2\cdot\frac{d}{dt}i-7\cdot i-I_o\cdot e^{-\frac{t}{\tau}}=0$$

$$L\cdot\frac{d}{dt}i(t)+R\cdot i(t)=0$$

Characteristics of Linear Systems

1.) Multiplying by constant is reflected though system.

If input r(t) gives output x(t) then,
K (r(t)) gives K (x(t))
I/O proportional

2.)Superposition from circuits holds

If input r1(t) gives y1(t) and
input r2(t) gives y2(t) then
total output to inputs is

r1(t)+r2(t) = y1(t)+y2(t)

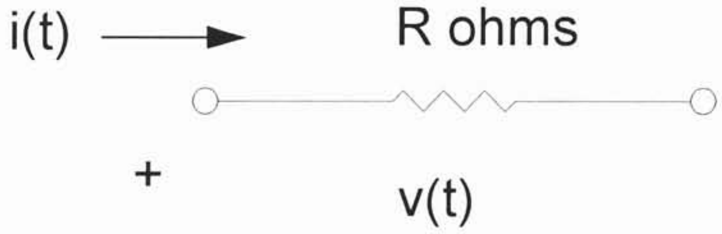
Total output is the sum of the individual input responses.

From circuits - transients and sine steady-state

Deriving Dynamic Equations
Input/output relationships

Dynamics represented by integrals and derivatives

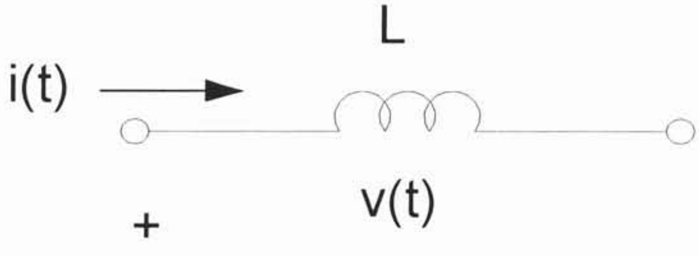
Electrical elements: Resistance



Defining equations $v(t) = R \cdot i(t)$

$$i(t) = G \cdot i(t) \qquad \frac{1}{R} = G$$

Inductance (henrys)



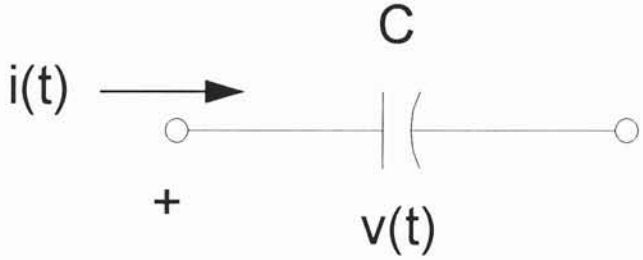
Defining equations

initial current at start of time

$$v(t) = L \cdot \frac{d}{dt} i(t) \qquad i(t) = \frac{1}{L} \cdot \int_{0}^{t} v(t)\,dt + \underset{\nwarrow}{i(0)}$$

Lumped Circuit Elements: capacitance

Capacitance (farads)



initial voltage at start time

Defining equations

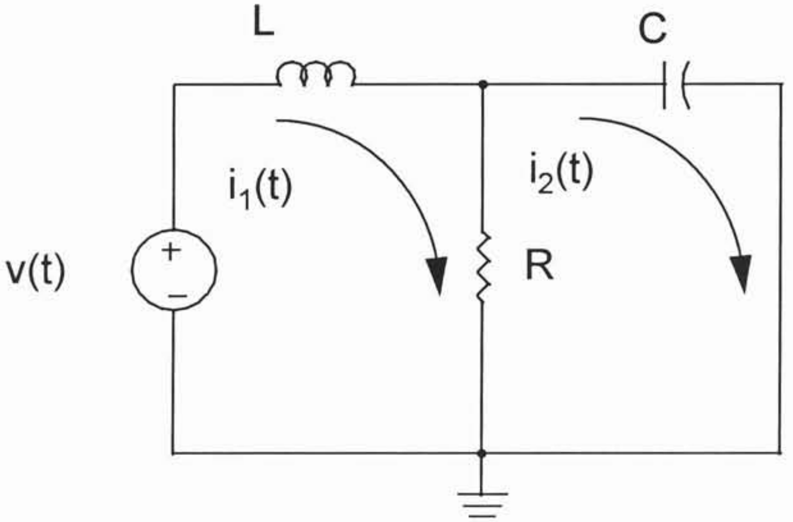
$$i(t) = C \cdot \frac{d}{dt} v(t) \qquad v(t) = \frac{1}{C} \cdot \int_0^t i(t)\,dt + v(0)$$

All law from circuit theory hold for the analysis of lumped circuits. KCL, KVL, mesh analysis, nodal analysis can all be performed. Substitute the appropriate integral or derivative into the mesh or nodal formulation.

When the current or voltage in a circuit element involves the two currents or voltages, the derivative or integral take the difference of the voltages or currents

Example

Write mesh equations for the circuit below using the lumped circuit element representations



KVL around mesh 1

$$-v(t) + L\cdot\frac{d}{dt}i_1(t) + R_1\left(i_1(t) - i_2(t)\right) = 0$$

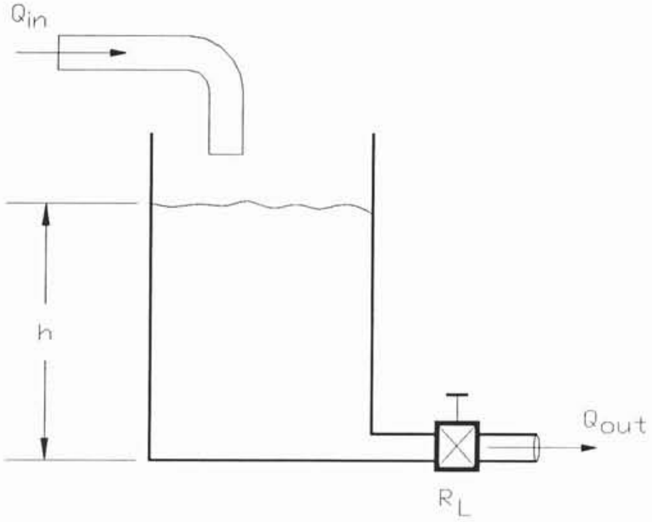
$$L\cdot\frac{d}{dt}i_1(t) + R_1\left(i_1(t) - i_2(t)\right) = v(t)$$

KVL around mesh 2

$$R_1\left(i_2(t) - i_1(t)\right) + \frac{1}{C}\cdot\int_0^t i(t)\,dt + v(0) + R_2\cdot i_2(t) = 0$$

system of integral-differential equations in unknown functions $i_1(t)$ and $i_2(t)$

Self-regulating tank system



Need relationship for how level changes with time. Derive differential equation

$$\Delta V = (Q_{in} - Q_{out}) \cdot \Delta t$$

level change

$$\Delta h = \frac{\Delta V}{A}$$

$$\Delta h = \frac{\Delta V}{A} = \frac{(Q_{in} - Q_{out}) \cdot \Delta t}{A}$$

average level change over time interval Δt

$$\frac{\Delta h}{\Delta t} = \frac{(Q_{in} - Q_{out})}{A} \quad \text{take limit } \Delta t \rightarrow 0 \quad \frac{dh}{dt} = \frac{(Q_{in} - Q_{out})}{A}$$

Assume laminar flow for simplicity. Q_{out} determined by p of tank.

$$P = R_L \cdot Q_{out} \qquad P = \rho \cdot g \cdot h$$

Combine these two equations and solve for Q_{out}

$$\frac{P}{R_L} = Q_{out} \qquad \frac{\rho \cdot g \cdot h}{R_L} = Q_{out}$$

Self-regulating tank system (cont.)

$$\frac{dh}{dt} = \frac{\left(Q_{in} - \frac{\rho \cdot g \cdot h}{R_L}\right)}{A} \longrightarrow \frac{dh}{dt} = \left(\frac{Q_{in}}{A} - \frac{\rho \cdot g \cdot h}{R_L \cdot A}\right)$$

bring all terms with
h or derivative of h
to one side

$$\frac{dh}{dt} + \left(\frac{\rho \cdot g}{R_L \cdot A}\right) \cdot h = \frac{Q_{in}}{A}$$

multiply through by
coefficient on h
Remember finding h(t)

$$\left(\frac{R_L \cdot A}{\rho \cdot g}\right) \cdot \frac{dh}{dt} + h = \frac{R_L}{\rho \cdot g} \cdot Q_{in}$$

Q_{in} is independent of the liquid height. consider it along with density, area and flow resistance to be constant

Let $\tau = \frac{R_L \cdot A}{\rho \cdot g}$ $G = \frac{R_L}{\rho \cdot g}$

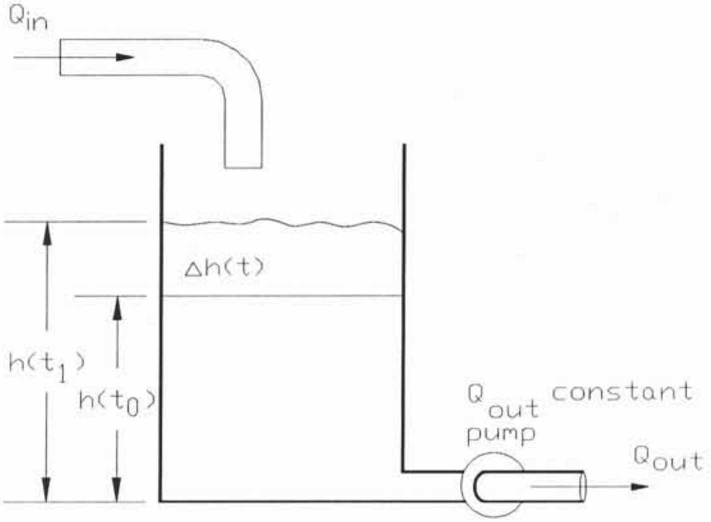
$$\tau \cdot \frac{dh}{dt} + h = G \cdot Q_{in}$$

non-homogeneous equation that determines how height of liquid in tank varies with time

Shut off value to find natural response of tank for some inital height

Non-regulating Tank

Output flow is fixed by pump flow rate. It is independent of the liquid height in tank.



$$\Delta h = \frac{\Delta V}{A} = \frac{\left(Q_{in} - Q_{out}\right)\cdot\Delta t}{A}$$

Define

$$\Delta t = t_1 - t_0$$

$$\Delta h(t) = h(t_1) - h(t_0)$$

$$\frac{\Delta h}{\Delta t} = \frac{\left(Q_{in} - Q_{out}\right)}{A}$$

take limit $\Delta t \rightarrow 0$

$$\frac{dh}{dt} = \frac{\left(Q_{in} - Q_{out}\right)}{A}$$

Right hand side is independent of h. To solve integrate both sides with respect to t

$$\int_{t_0}^{t_1} \frac{dh}{dt}\, dt = \int_{t_0}^{t_1} \frac{1}{A}\cdot\left(Q_{in} - Q_{out}\right) dt$$

Non-regulating Tank (cont.)

From Calculus

$$h(t_1) - h(t_0) = \int_{t_0}^{t_1} \frac{1}{A} \cdot (Q_{in} - Q_{out})\, dt$$

$$\Delta h(t) = \int_{t_0}^{t_1} \frac{1}{A} \cdot (Q_{in} - Q_{out})\, dt$$

Note: Right hand side is not a function of t. It is all a constant and can be taken out of the integral

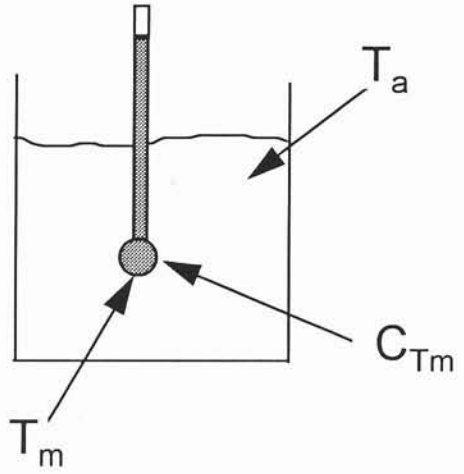
Change in height depends on the difference in flows and the time that the difference is applied.

If Q_{in} is not a constant but changes with time, the formula below models the response of the tank system

$$\Delta h(t) = \int_{t_0}^{t_1} \frac{1}{A} \cdot (Q_{in}(t) - Q_{out})\, dt$$

Solution techniques: convert all equations into derivatives only. Approximate derivatives using mathematical methods calculate approximate derivative values for some small increment in time. Results are a list of computed points that approximate variable over a time interval. Graph these points to see system response

Liquid Filled Themometer



T_a = fluid temperature
T_m = measured temperature
C_{Tm} = thermal capacitance of themometer

How does measured temperature change with time. Heat transfered to themometer depends on ΔT, R_T and time interval

$$\Delta Q = \frac{(T_a - T_m)\cdot\Delta t}{R_T}$$

definition of thermal capacitance $\frac{\Delta Q}{\Delta T_m} = C_{Tm}$

$$\Delta T_m = \frac{\Delta Q}{C_{Tm}} \qquad \frac{\Delta Q}{C_{Tm}} = \frac{(T_a - T_m)\cdot\Delta t}{R_T\cdot C_{Tm}}$$

$$\Delta T_m = \frac{(T_a - T_m)\cdot\Delta t}{R_T\cdot C_{Tm}}$$

Liquid Filled Themometer (cont.)

Determine the average change in temperature

$$\frac{\Delta T_m}{\Delta t} = \frac{\left(T_a - T_m\right)}{R_T \cdot C_{Tm}}$$

take limit $\Delta t \rightarrow 0$

$$\frac{dT_m}{dt} = \frac{\left(T_a - T_m\right)}{R_T \cdot C_{Tm}}$$

Get all T_m and its derivatives on one side of the equation

$$\frac{dT_m}{dt} = \frac{T_a}{R_T \cdot C_{Tm}} - \frac{T_m}{R_T \cdot C_{Tm}}$$

$$\frac{dT_m}{dt} + \frac{T_m}{R_T \cdot C_{Tm}} = \frac{T_a}{R_T \cdot C_{Tm}}$$

$$R_T \cdot C_{Tm} \cdot \frac{dT_m}{dt} + T_m = T_a$$

This is similar in form to the self-regulating tank equation
This is a non-homogeneous differential equation that describes how the measured temperature changes with time

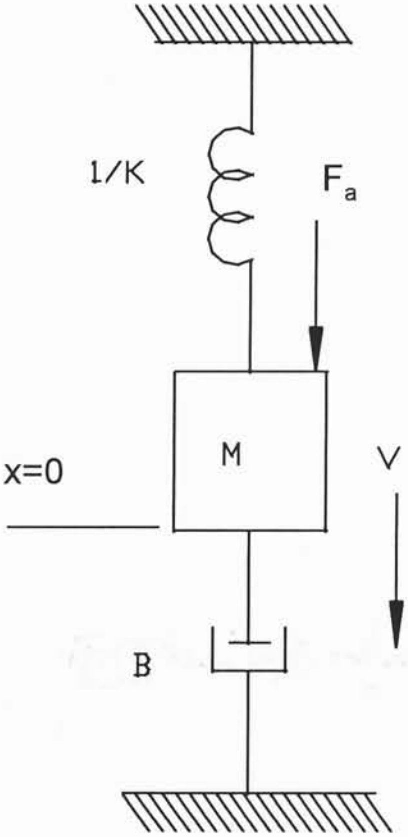
Mechanical Systems

Pneumatic Control Value Position

Determine how the position of a air actuated control value changes with time after pressure is applied

Schematic of value
(See text p.116 for details)

$F_a = P_a(A)$ = input air force



All forces must balance at each instance in time so:

$$F_a = F_I(t) + F_R(t) + F_C(t)$$

Where $F_I(t)$ = inertial force
$F_R(t)$ = viscous friction force
$F_C(t)$ = spring force

Need equation that relates position, x to time

$$F_I = m \cdot a \qquad a = \frac{d}{dt} v = \frac{d^2}{dt^2} x$$

$$v = \frac{d}{dt} x$$

Inertial force

$$F_I = m \cdot \frac{d^2}{dt^2} x$$

Pneumatic Control Value Position, (cont.)

Inertial force

$$F_I = m \cdot \frac{d^2}{dt^2} x$$

Acceleration is second derivative of position, x

Friction force

$$F_R = B \cdot v = B \cdot \left(\frac{d}{dt} x \right)$$

Viscous friction for laminar flow is porportional to the velocity. Velocity is the rate of change of position

Spring force

$$x = C_m \cdot F_C \qquad F_C = \frac{x}{C_m} \qquad \frac{1}{C_m} = k$$

$F_c(t) = K\,x(t) = K \int x(t)\, dt$

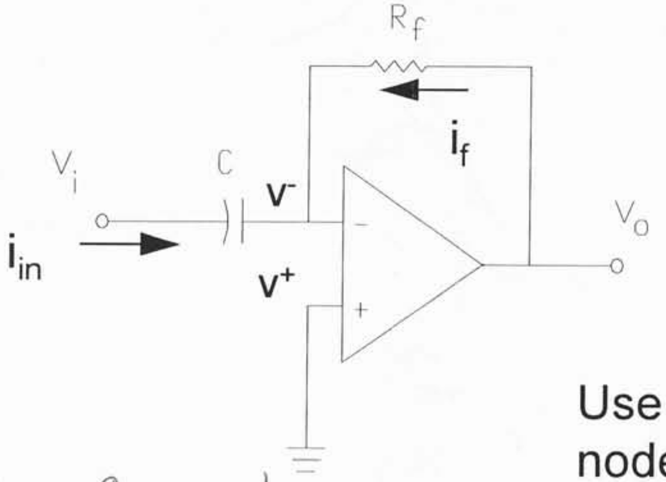
Force from spring is porportional to its length, x. C_m is spring capacitance, k = spring constant.

$$F_a = m \cdot \frac{d^2}{dt^2} x + B \cdot \left(\frac{d}{dt} x \right) + \frac{1}{C_m} \cdot x$$

Fa is constant. Equation describes how position changes with time. Second order equation.

Simulating differentiation with OP AMPs

Use rules of circuit analysis and ideal OP AMPs to find the input/output relationship for the circuit below.



Rules of OP AMPS

no current flows into OP AMP

$V^- = V^+$

Use nodal analysis at inverting node of OP AMP

Sum Currents

$i_{in}(t) + i_f(t) = 0 \rightarrow i_{in}(t) = -i_f(t)$

$i_{in}(t) = i_c(t)$ Current in C

define currents in terms of voltage

$i_{in}(t) = C\frac{d}{dt}\left(V_{in}(t) - V^-(t)\right)$

$i_f(t) = \frac{V_o(t) - V^-(t)}{R_f}$

$C\frac{d}{dt}\left(V_{in}(t) - V^-(t)\right) = \frac{-V_o(t) - V^-(t)}{R_f}$

$i_c(t) = C\frac{d}{dt}V_c(t)$

$V_c(t) = V_{in}(t) - V^-(t)$

$V^-(t) = V^+(t) = 0$ Pos. terminal Grounded

$\rightarrow C\frac{d}{dt}V_{in}(t) = \frac{-V_o(t)}{R_f}$

$-R_f C\frac{d}{dt}V_{in}(t) = V_o(t)$

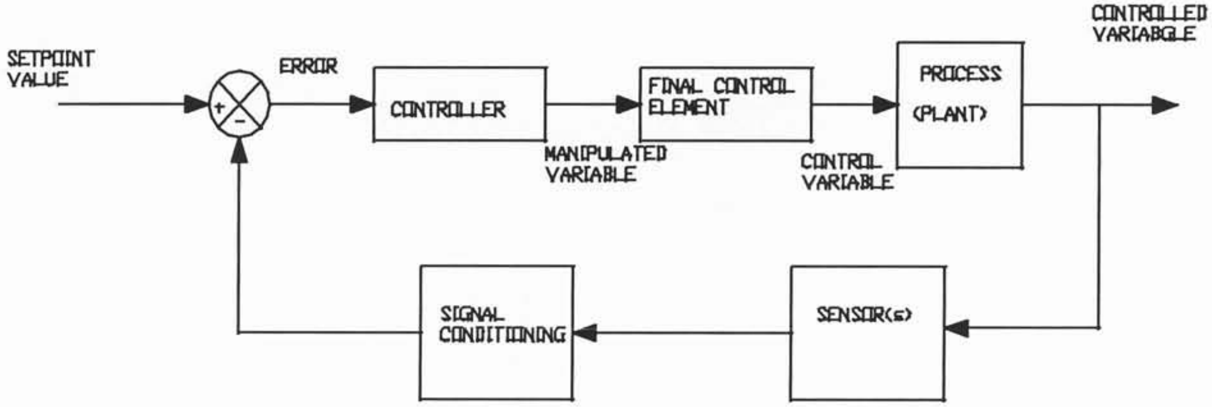
write as input/output

$V_o(t) = -R_f C\frac{d}{dt}V_{in}(t)$

output input

Proportional Control Action

Basic control loop



Simplest form of controller - amplify error and apply signal to the process through final control element.

Example: Dc motor speed control

control variable = motor speed
sensor/signal conditioning = speed transducer
setpoint = desired speed
final control element = variable dc voltage supply
control variable = motor terminal voltage
manipulated variable = dc voltage control voltage

The control relationship for proportional control

Amplify error and send to final control element.

$$C_o = K_p e + C_b$$

Where C_o = the controller output
C_b = the controller output with $e = 0$
K_p = the proportional gain
e = the control error

$$e = SP - \text{Measurement},$$

e = error signal
SP = setpoint value
Measurement = sensor measurement

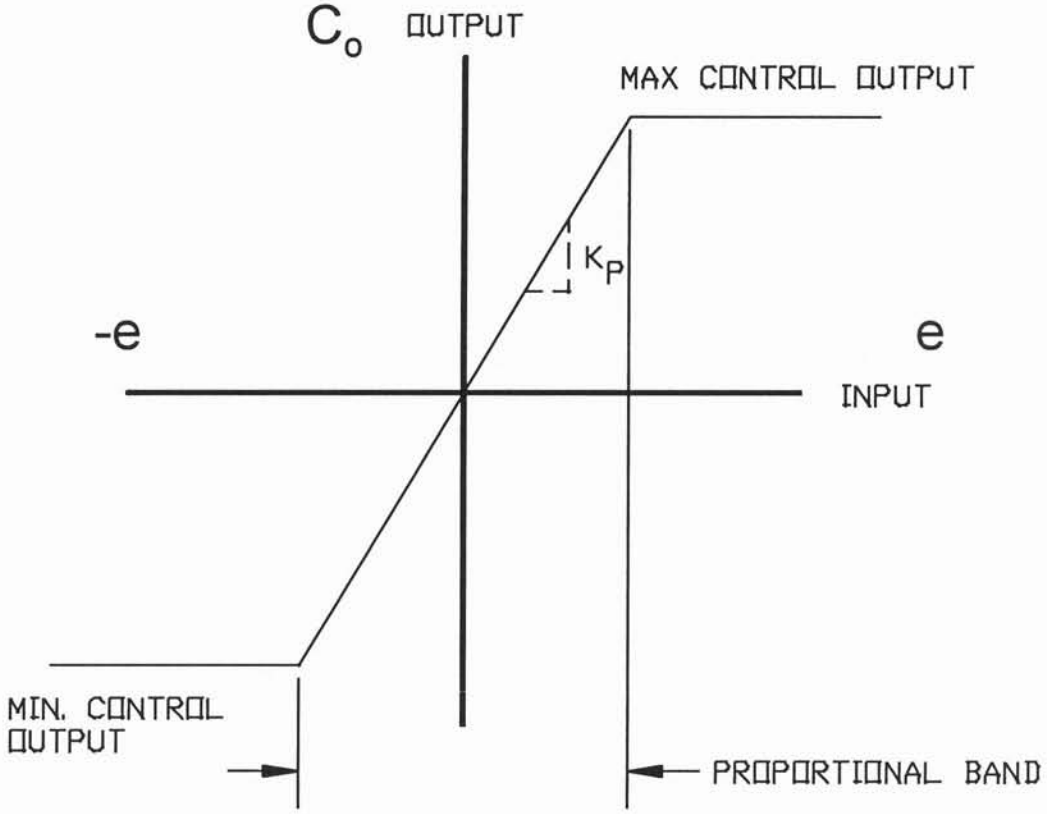
e can be positive or negative
e used to take corrective action

Measurement > SP   e negative - output decreases
Measurement < SP   e positive - output increases

Practical controller output devices have limits on C_o and final control element.

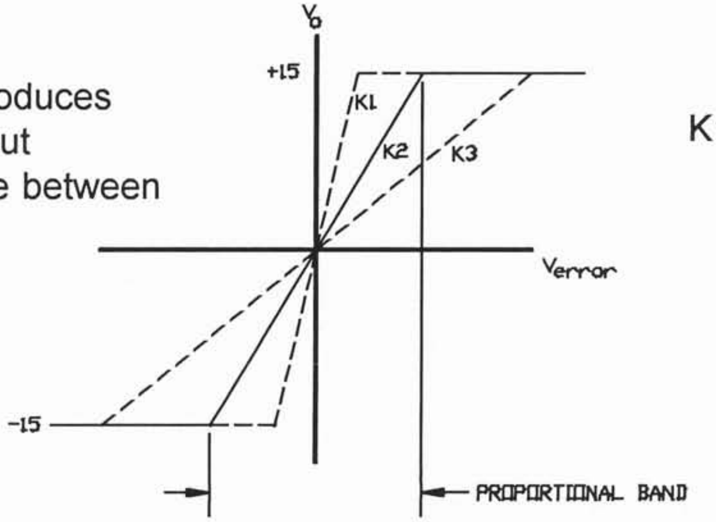
Examples: flow value position
motor power supply

Controller Gain Plots



Proportional band limits of control output. Determined by value of K_p.

At higher K_p, e produces more correction but reduces the range between controller limits



K1>K2>K3

Finding K_p and the Proportional Band

$$K_p = \frac{\Delta \text{Output}}{\text{SP} - \text{Measurement}} \qquad K_P = \frac{\Delta \text{Output}}{\Delta e}$$

Proportional gain - change in output /change in error

Increasing proportional gain decreases proportional band (PB). In terms of percent.

$$\%PB = \frac{1}{K_p} \cdot 100\%$$

%PB inversely proportional to K_p

Example: Δoutput ±15Vdc OP AMP Limits Fixed by saturation

$\Delta e = \pm 3\text{Vdc}$

Find K_p and % PB

$$K_p = \frac{C_{max} - C_{min}}{e_{max} - e_{min}} = \frac{15-(-15)}{3-(-3)} = \frac{30}{6} = 5 = \frac{\Delta \text{Output}}{\Delta e}$$

$$\%PB = \frac{1}{K_p} 100\% = \frac{1}{5}(100\%) = 20\%$$

Based on C_o, 20% change in Δe Gives Full Δ output

$$\Delta \text{output}(0.20) = \Delta e \Rightarrow 30(0.2) = 6$$

Set $K_p = 2$ find %PB and max Δoutput $\%PB = \frac{1}{2}(100\%) = 50\%$

$$K_p = \frac{\Delta \text{output}}{\Delta e} \Rightarrow \Delta e K_p = \Delta \text{output} \Rightarrow 6(2) = \pm 12\text{V}$$ output range

Greater error can be handled before Saturation

Proportional Gain and Proportional Band

Example. Continued

Decreasing %PB (increasing K_p) causes a smaller error signal to cause the controller to reach its limits

ORIGINAL %PB = 20% ±6Vdc Causes C_o to vary ±15Vdc

Decrease % PB to 10%

A VARIATION OF 1/2 OF ORIGINAL Δe Causes C_o to REACH LIMITS

$$\%PB = \frac{1}{K_p} 100\% \Rightarrow \frac{10\%}{100\%} = 1/K_p \Rightarrow K_p = 10$$

Find Δe input that Produces max Δ output

$\Delta output = 30$ MAX

$$\frac{\Delta C_o}{\Delta e} = K_p = \frac{\Delta output}{\Delta e} = \Delta e K_p = \Delta output$$

Solve for Δe

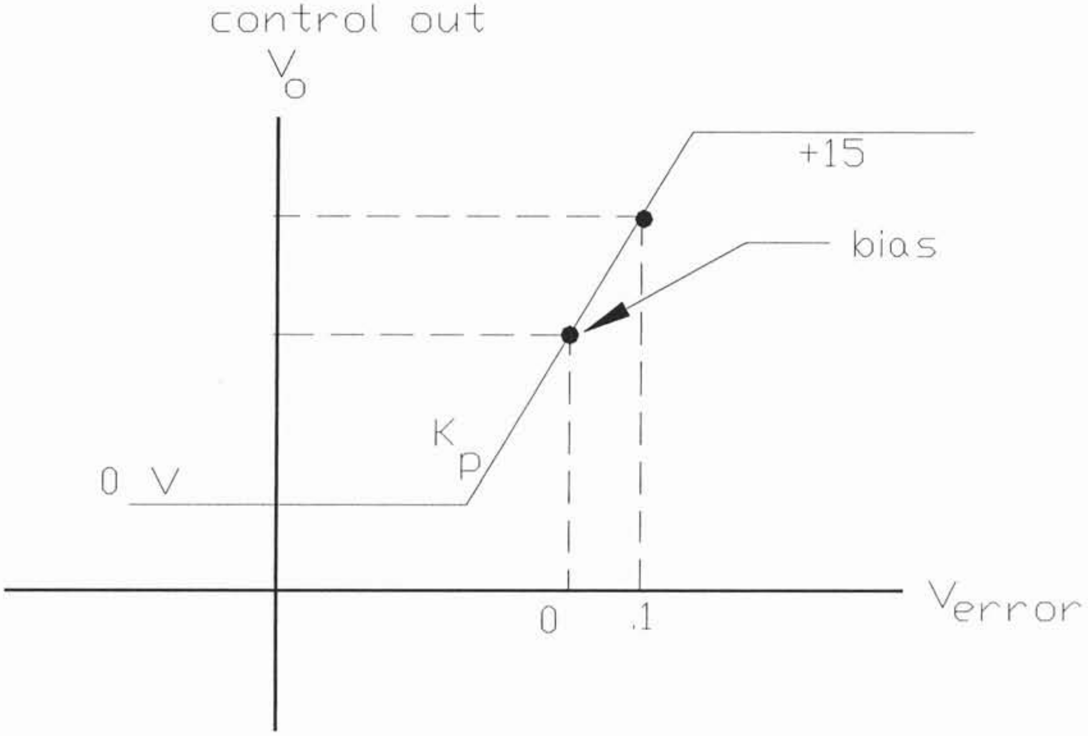
$$\Delta e = \frac{\Delta output}{K_p}$$

$$\Delta e = \frac{30V}{10} = 3V \ (\pm 1.5)$$

Proportion changes

Characteristics of Proportional Control

$C_o = K_p e + C_b$ Control formula



If e = 0 then $C_o = C_b$

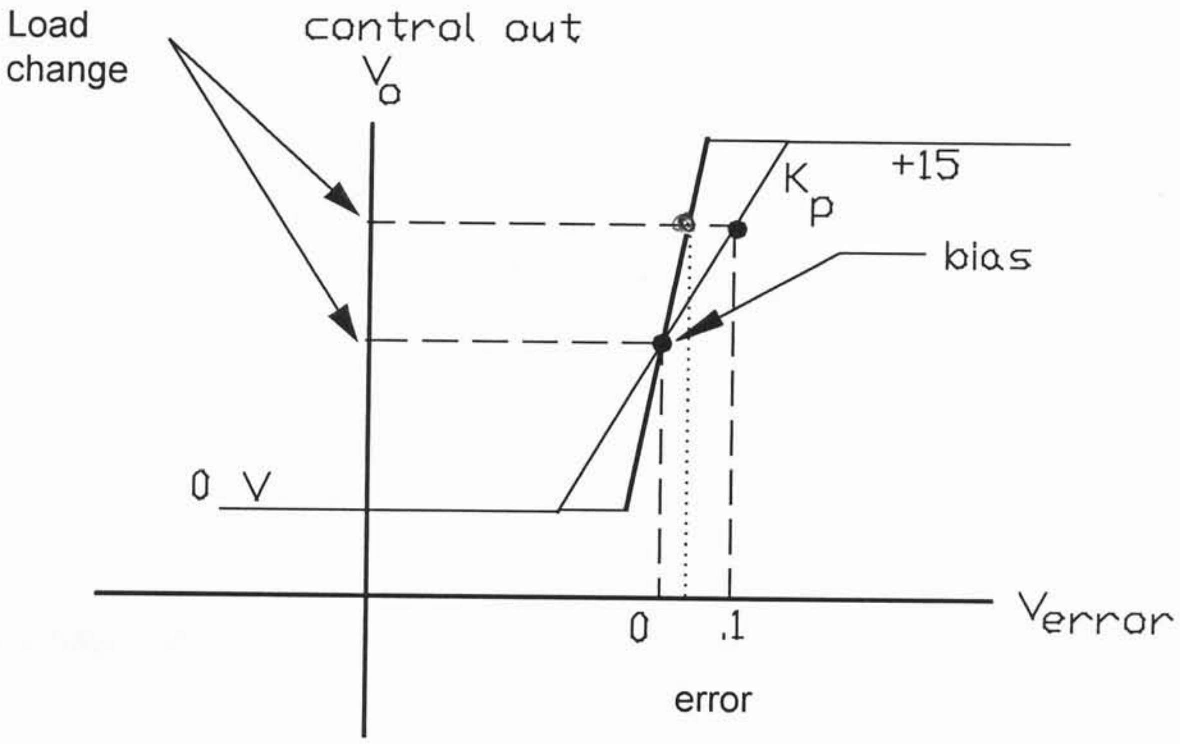
If e not = 0, then for every unit of error $K_p(e)$ units of correction are added/subtracted to control output.

There is a band of steady-state error about 0 that has a magnitude of PB% where the output is not saturated.

Offset and residual error

residual error - also known as steady-state error.

Proportional control always produces a steady-state or residual error when a change in the process occurs



If the load change is permanent, then the error is permanent. Increasing K_p decreases steady-state error be can not eliminate it.

If load change is transient, then error returns to bias value, 0.

Effects of Increased Gain on error and stability.

High gain reduces steady-state error but increases chances of instability

Example

FOR $K_p = 2$ $V_o = 0-15\ Vdc = C_o$

$V_e = 0 \pm 1\ vdc = e$

$V_{bias} = 7.5\ Vdc = C_b$

$V_o = K_p V_e + V_{bias} \Rightarrow C_o = K_p e + C_b \Rightarrow V_o = 2V_e + 7.5V$

$V_e = 0$

$V_o = 2(0) + 7.5 = 7.5$ with no steady-state error

Process Changes $V_e = -0.25$

$V_o = 2(-0.25) + 7.5 = 7.0\ V$

Find Residual error Remember $K_p = \frac{C_o}{e} \Rightarrow e = \frac{C_o}{K_p}$

error is inversely Proportional to K_p

$\frac{V_o}{K_p} = V_e = \frac{7.0}{2} = 3.5V$

For Control error to reach 0 $K_p \rightarrow \infty$ Set $K_p = 5$

Calculate

$\frac{V_o}{K_p} = \frac{7.0}{5} = 1.4\ V$