LESSON 15: BODE PLOTS OF TRANSFER FUNCTIONS

ET 438a Automatic Control Systems Technology

Learning Objectives

After this presentation you will be able to:

- Compute the magnitude of a transfer function for a given radian frequency.
- Compute the phase shift of a transfer function for a given radian frequency.
- Construct a Bode plot that shows both magnitude and phase shift as functions of transfer function input frequency
- Use MatLAB instructions it produce Bode plots of transfer functions.

Construction of Bode Plots

Bode plots consist of two individual graphs: a) a semilog plot of gain vs frequency b) a semilog plot of phase shift vs frequency. Frequency is the logarithmic axis on both plots.

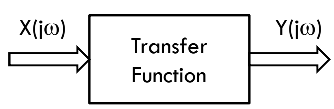
Bode plots of transfer functions give the frequency response of a control system

To compute the points for a Bode Plot:

1) Replace Laplace variable, s, in transfer function with $j\omega$
2) Select frequencies of interest in rad/sec ($\omega=2\pi f$)
3) Compute magnitude and phase angle of the resulting complex expression.

Construction of Bode Plots

Bode plot calculations- magnitude/phase



Gain: $G(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$

$dB = 20 \cdot \log(|G(j\omega)|)$

Where for a given frequency

$\mathbf{Y} = y\angle\theta_o$

$\mathbf{X} = x\angle\theta_i$

Phase: $\phi = \theta_o - \theta_i$

To find the magnitude and phase shift of a complex number in rectangular form given: $\mathbf{z} = a + jb$

$$|\mathbf{z}| = \sqrt{a^2 + b^2} \qquad \phi = \tan^{-1}\left[\frac{b}{a}\right]$$

Computing Transfer Function Values

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Example 15-1: A self-regulating tank has a transfer function of the form shown below.

$$\frac{H(s)}{Q(s)} = \frac{G}{1+\tau \cdot s}$$

The tank has a time constant, τ=1590 seconds and a gain, G=2000 s/m^2. Determine the amplitude and phase shift of the system to a sinusoidal flow input of 0.0001592 Hz

Solution: Substitute values of G, τ, and jω into transfer function and compute the gain magnitude and phase shift.

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Example 15-1 Solution (1)

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$$G = 2000\ \mathrm{s/m^2} \qquad \tau = 1590\ \mathrm{s} \qquad f = 0.0001592\ \mathrm{Hz}$$

$$\frac{H(j\omega)}{Q(j\omega)} = \frac{2000}{1+1590\cdot j\omega}$$

$$\omega = 2\pi \cdot f \qquad \omega = 2\pi \cdot (0.0001592\ \mathrm{Hz}) = 0.001\ \mathrm{rad/s}$$

Place radian frequency into transfer function and compute complex number

$$\frac{H(j0.001)}{Q(j0.001)} = \frac{2000}{1+1590\cdot j0.001} = \frac{2000}{1+j1.590} = 566.9 - j901$$

Convert the result to polar form to find magnitude and phase shift

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Example 15-1 Solution (2)

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Complex gain

$$G(j0.001)=\frac{2000}{1+1590\cdot j0.001}=\frac{2000}{1+j1.590}=566.9-j901$$

$$|G(j\omega)|=\sqrt{a^2+b^2} \qquad \phi=\tan^{-1}\left[\frac{b}{a}\right]$$

Magnitude Calculation

$a=566.9 \quad b=-901$

$$|G(j0.001)|=\sqrt{566.9^2+(-901)^2}$$

$$|G(j0.001)|=1065$$

$$dB=20\cdot\log(1065)=60.5\ dB$$ Ans

Phase shift

$$\phi=\tan^{-1}\left[\frac{b}{a}\right]=\tan^{-1}\left[\frac{-901}{566.9}\right]=-58^{\circ}$$ Ans

At 0.001 rad/sec, the system has a gain of 60.5 dB and the output changes in height lag the flow changes by 58 degrees

Constructing Bode Plots Using MatLAB

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MatLAB has control system toolbox functions for defining Linear Time-invariant systems (LTI) and constructing the Bode plots.

Use tf and bode functions to create LTI and plot. Introducing zpk function

sys = zpk(z,p,k) Turns arrays of zeros, poles and gains into LTI called sys

Where z = array of transfer function zeros
p = array of transfer function poles
k = array of transfer function gains

Constructing Bode Plots Using MatLAB

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<u>Example 15-2:</u> Construct the Bode plot for the given transfer function shown in factored form using MatLAB control toolbox functions.

$$\frac{V_o(s)}{V_i(s)}=\frac{0.005\cdot s}{(0.001\cdot s+1)(0.001\cdot s+1)}$$

Solution: Transfer function has one zero at s=0 and two poles at s=-1/0.001=-1000

Dividing the transfer function denominator and numerator by 0.001 places it into standard form

$$\frac{V_o(s)}{V_i(s)}=\frac{0.005\cdot s\cdot\left[\frac{1}{0.001}\right]}{(0.001\cdot s+1)(0.001\cdot s+1)\cdot\left[\frac{1}{0.001}\right]}=\frac{s\cdot\left[\frac{0.005}{0.001}\right]}{\left(\frac{0.001}{0.001}\cdot s+\frac{1}{0.001}\right)\left(\frac{0.001}{0.001}\cdot s+\frac{1}{0.001}\right)}=\boxed{\frac{5\cdot s}{(s+1000)(s+1000)}}$$

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Constructing Bode Plots Using MatLAB

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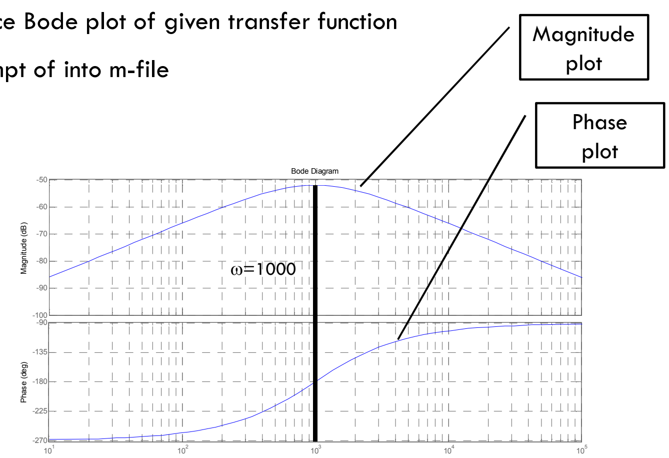
MatLAB code to produce Bode plot of given transfer function

Enter at command prompt of into m-file

```
k= [5]
p=[1000 1000]
z=[0]
sys=zpk(z,p,k)
bode(sys)
```



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Constructing Bode Plots Using MatLAB

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The bode(sys) function can plot more than one transfer function on the same figure axis. The figure produced by the bode(sys) function can be copied and pasted into wordprocessors and other programs. To plot more than one transfer function use the following syntax: bode(sys1,sys2,…).

Example 15-3: Compare the Bode plots of the transfer function given in Example 15-2 to the function given below. Use MatLAB to generate the Bode plots on a single set of axis. Note: the only difference is the gain is increased by a factor of 100.

$$\frac{V_o(s)}{V_i(s)} = \frac{500\cdot s}{(s+1000)(s+1000)}$$

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Constructing Bode Plots Using MatLAB

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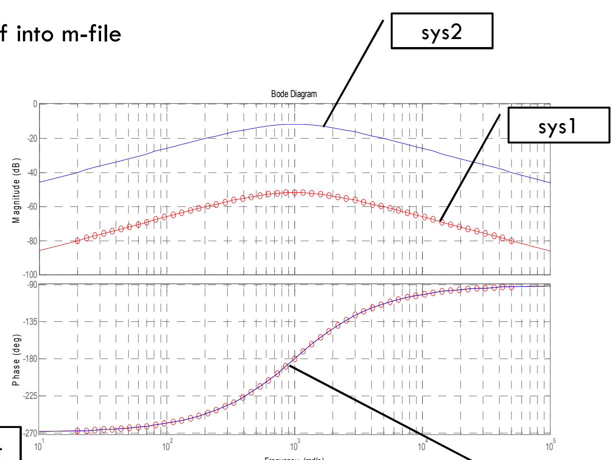
Enter at command prompt of into m-file

```
k= [5]
p=[1000 1000]
z=[0]
sys1=zpk(z,p,k)
k1=[500]
sys2=zpk(z,p,k1)
bode(sys1,'ro-
',sys2,'b-')
```



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End Lesson 15: Bode Plots of Transfer Functions

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