

Lesson 9: Proportional Control Action

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Learning Objectives

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After this presentation you will be able to:

- Identify the components of a proportional feedback control system.
- Write a mathematical model for a proportional controller.
- Compute the proportional bandwidth of a proportional controller.
- Explain the relationship between steady-state error and proportional gain.
- Perform lab experiment 2 more effectively.

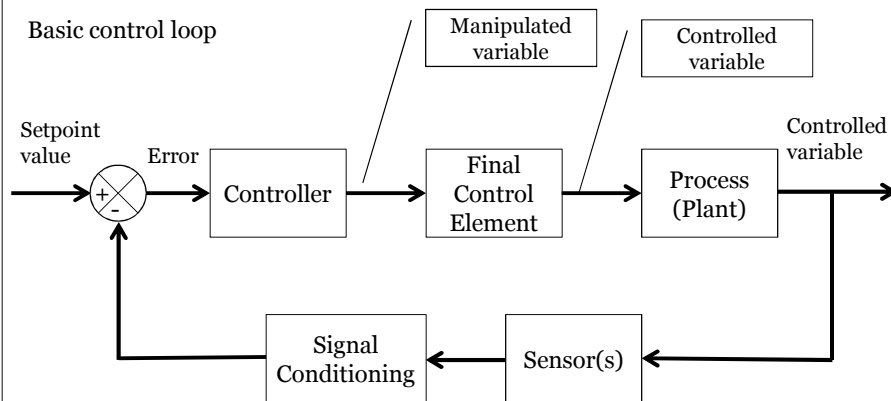
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Proportional Control Action

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Simplest form of controller - amplify error and apply signal to the process through final control element.

Basic control loop

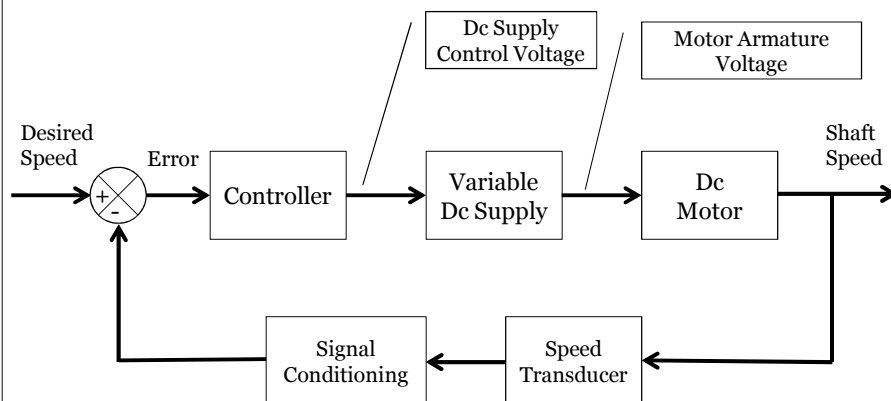


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Proportional Control Action

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Example: Dc Motor Speed Control



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Proportional Control Math Relationship

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Amplify error and send to final control element.

$$C_o = K_p e + C_b$$

Where C_o = the controller output
 C_b = the controller output with $e = 0$
 K_p = the proportional gain
 e = the control error

$$e = SP - \text{Measurement,}$$

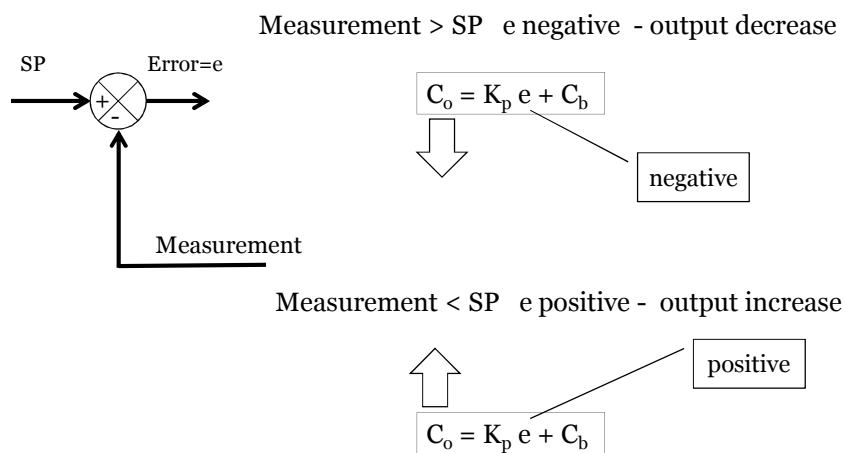
e = error signal
 SP = setpoint value
 Measurement = sensor measurement

e can be positive or negative. Error used to take corrective action

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Proportional Control Action

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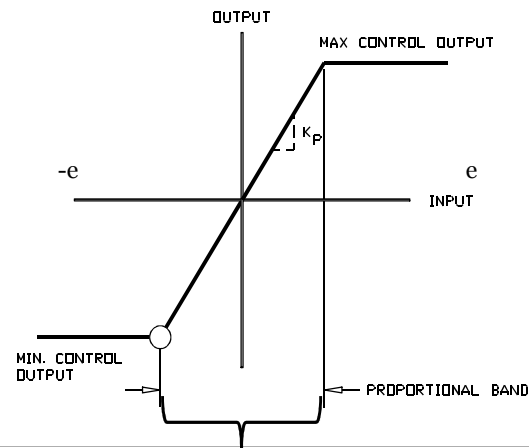


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Controller Output Limits

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Practical controller output devices have limits on C_o and final control element. Examples: flow valve position, motor power supply



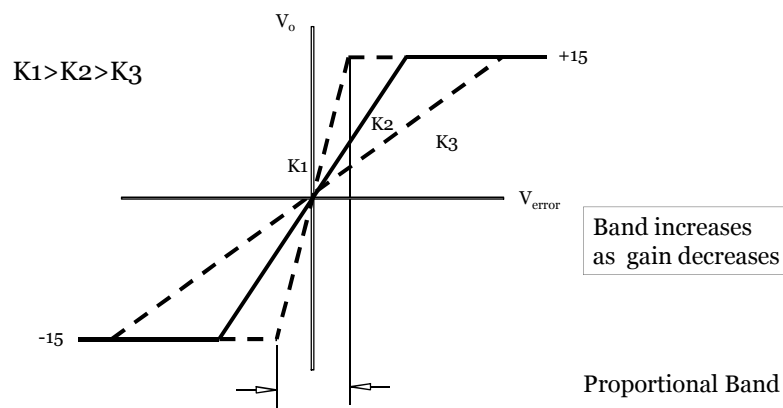
Proportional band sets limits of control output. Determined by value of K_P .

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Controller Output Limits

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At higher K_p , e produces more correction but reduces the range between controller limits



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Computing Proportional Band and K_p

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Proportional gain - change in output / change in error

$$K_p = \frac{\Delta C_o}{\Delta e} = \frac{\Delta \text{Output}}{\Delta(\text{SP} - \text{Measurement})}$$

Increasing proportional gain decreases proportional band (PB). In terms of percent:

$$\%PB = \left(\frac{1}{K_p} \right) \cdot 100\%$$

%PB inversely proportional to K_p

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Proportional Control Action

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Example 9-1: A proportional controller uses an OP AMP with ± 15 Vdc output limits when the error input is ± 3 Vdc. Saturation of the OP AMP sets these limits. Find the proportional gain and the percent proportional bandwidth from this information. Determine the proportional band and max output if $K_p = 2$.

Find K_p by using $K_p = \frac{\Delta C_o}{\Delta e}$

Find the %PB

$$K_p = \frac{C_{\max} - C_{\min}}{e_{\max} - e_{\min}}$$

$$K_p = \frac{15 - (-15) \text{ Vdc}}{3 - (-3) \text{ Vdc}}$$

$$K_p = \frac{30}{6} = 5 = \frac{\Delta \text{output}}{\Delta e}$$

$$\%PB = \left[\frac{1}{K_p} \right] 100\%$$

$$\%PB = \frac{1}{5} (100\%) = 20\%$$

20% change in error gives maximum change in output

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Example 9-1 Solution

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Now set the value of proportional gain to 2 and compute the %PB and output

$$\%PB = \frac{1}{K_p}(100\%)$$

$$\%PB = \left(\frac{1}{2}\right)(100\%) = 50\%$$

$$K_p = \frac{\Delta \text{output}}{\Delta e} \Rightarrow \Delta \text{output} = \Delta e K_p$$

$$\Delta e = 3 - (-3) = 6 \quad \Delta \text{output} = 6(2) = \pm 12V_{dc}$$

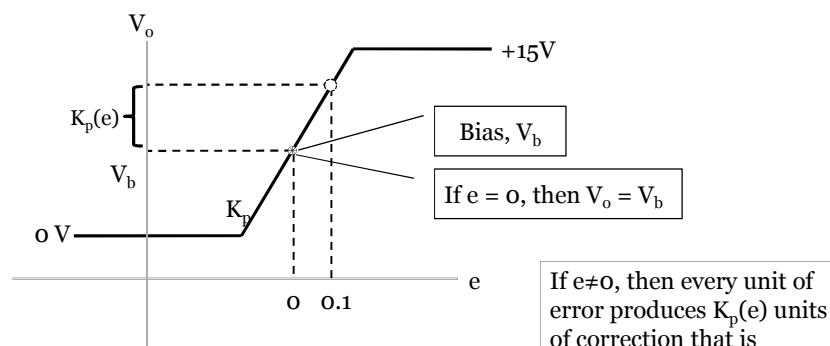
Lower gain allows the OP AMP to handle more error before saturation.

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Characteristics of Proportional Control

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Control formula $V_o = K_p \cdot e + V_b$



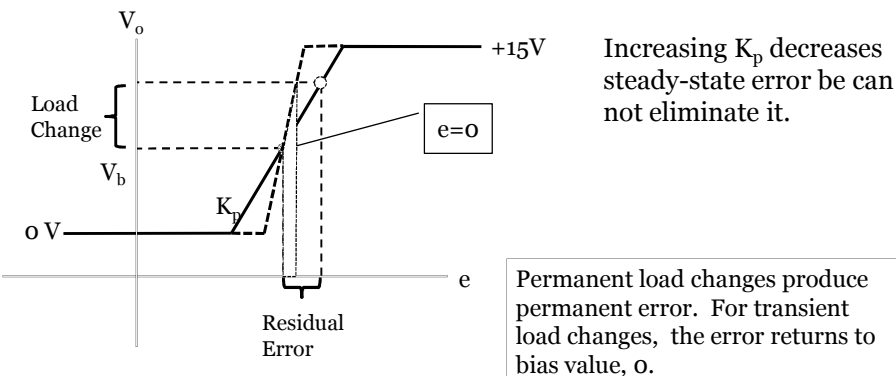
There is a band of steady-state error about 0 with magnitude of PB% where the output is not saturated.

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Control Offset and Residual Error

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Proportional control always produces a steady-state or residual error when a change in the process load occurs. (When $e \neq 0$)



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Increased Gain & Residual Error

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High gain reduces steady-state error but increases chances of instability

Example 9-2: A proportional controller with a gain of 2 has an output range of 0-15 Vdc for an error input range of 0 to ± 1 Vdc. The output has a balance value of 7.5 Vdc. Determine: a.) controller output when error is zero, b.) residual error when a process load change produces an error voltage of -0.25 Vdc, c.) residual error when control gain increases to 5.

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Example 9-2 Solution (1)

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For $K_p=2$, $V_o=C_o=0-15$ Vdc, and $V_e=e=0$ to ± 1 Vdc. Also $V_b=C_b=7.5$ Vdc.

$$C_o = K_p C_e + C_b \quad V_o = K_p V_e + V_b \quad C_o = 2V_e + 7.5$$

At $V_e=0$ $V_o = 2(0) + 7.5$ V
 $V_o = 7.5$ V Ans

No residual error
(Steady-state error)

Process load change produces... Find residual error....

$$V_e = -0.25 \text{ V}$$

$$V_o = 2(-0.25) + 7.5$$

$$V_o = 7.0$$

$$K_p = \frac{C_o}{e}$$

$$e = K_p C_o$$

$$e = \frac{C_o}{K_p}$$

$$V_e = \frac{V_o}{K_p}$$

$$V_e = \frac{7.0}{2} = 3.5$$
 Ans

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Example 9-2 Solution (2)

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For residual (stead-state) error to reach 0, K_p must increase to infinity.
 Set $K_p=5$ and compute new residual error

$$\frac{V_o}{K_p} = \frac{7.0}{5} = 1.4 \text{ Vdc}$$

Residual or steady-state error decreases inversely as the proportional gain increases

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Error As a Function of Proportional Gain

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Computing residual error

$$\%_error = \frac{SS_d - SS_a}{SS_d}$$

Where: SS_d = desired steady-state output (1 for unit step)

SS_a = actual output at steady-state

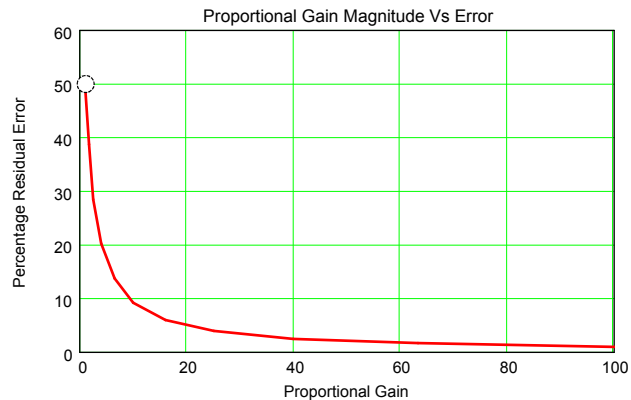
$\%_error$ = percentage of residual error based on desired value.

Plot the change in residual error as the proportional gain increases using a typical control system

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Error As a Function of Proportional Gain

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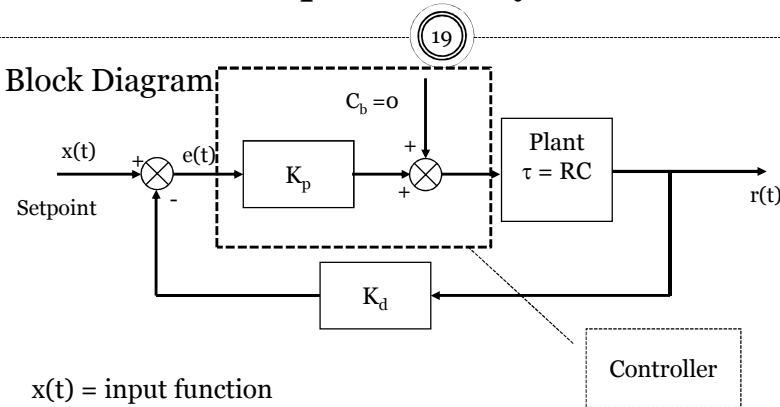


Low values of K_p produce high residual error

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Model of Proportional System (Bias = 0)

Block Diagram:



$x(t)$ = input function
 K_p = Proportional controller gain
 Plant is modeled using RC circuit
 $r(t)$ = output response of the control system
 K_d = feedback gain (voltage divider in lab)

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Proportional System Time Response

Method of solution: Signal flow algebra and Laplace transforms
 Final solution to step change input:

$$r(t) = \left[\frac{K_p}{1 + K_p \cdot K_d} \right] \left(1 - e^{-\frac{t(1 + K_p \cdot K_d)}{R \cdot C}} \right)$$

K_p affects response speed

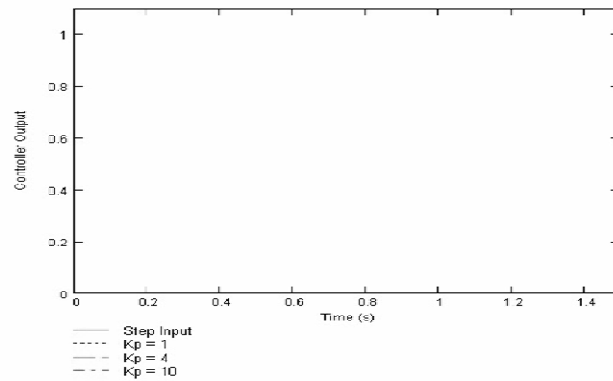
K_p affects final value

Plot the response of this system to a step input (0-1) for several values of proportional gain and compare them based on response speed and residual error.

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Proportional System Time Response

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Comparison of response time and residual errors

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End Lesson 9: Proportional Control Action

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