EET 438a
Automatic Control Systems Technology
Laboratory 2
Proportional Control Action

Objective: Construct a proportional controller using OP AMP circuits and measure its steady-state and transient response. View the response of a first order process to proportional control action.

Theoretical Background

Automatic control has two principle functions: to maintain control output as close as possible to the desired setpoint value while the system is subject to outside disturbances (regulator action), and to respond as quickly as possible to changes in the setpoint value.
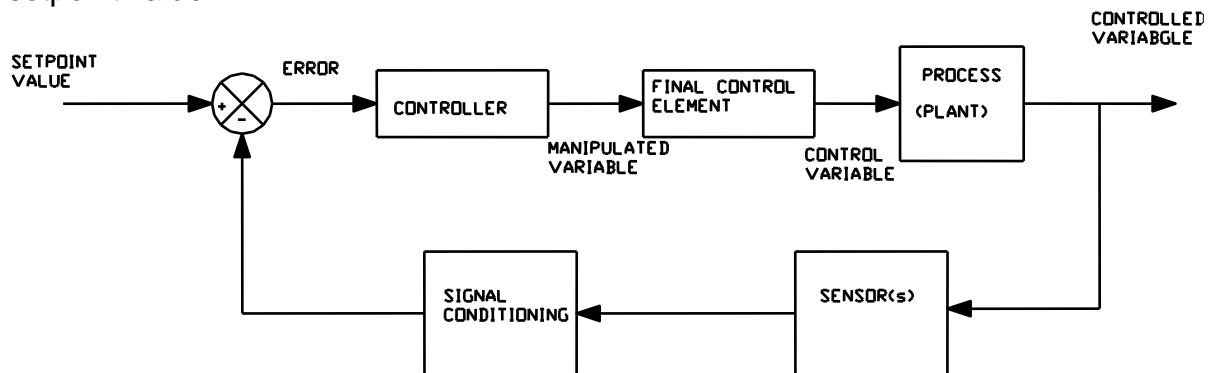

Figure 1. Feedback Control System.

Figure 1 shows block diagram of a typical control system. This is a negative feedback control system because the error signal is produced by subtracting the output from the setpoint value at the summing node. Another characteristic of this type of feedback is that the output is measured after the changes are made in the system under control, also called the plant. Sensors and signal conditioning may be used to transform a mechanical variable (e.g. flow, speed, pressure, temperature) into electrical signals. In some cases such as OP AMPs, the signals may already be electrical quantities such as voltage or current.

An error signal is produced by taking the difference between the desired value and the measured value. This is the control error signal. Ideally this signal should be zero after the setpoint has been changed or an external disturbance has occurred on the process. If the control was initially in a balanced state the system will come into a new balanced condition after the changes. The controller block takes the error signal and modifies it

to produce an output signal called the manipulated variable. This output signal is used to control some element in the final process. The final control element is the actual apparatus that modifies the process. An example of this is a dc motor speed controller where:

Control variable = motor speed
Sensor/signal conditioning = speed transducer
Set-point = desired speed
Final control element = variable dc power supply
Control variable = motor terminal voltage
Manipulated variable = power supply voltage

The controller can take several forms. The simplest mode of control is to amplify the error signal with a constant gain, K_p, and use this as the manipulated variable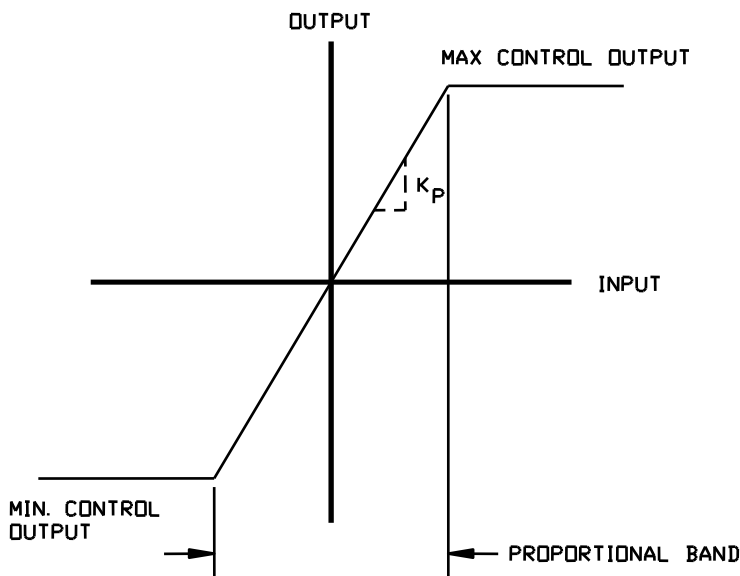. This is called proportional control. Figure 2 shows the input output response of a practical proportional controller. All practical devices have maximum and minimum limits on output. The final control elements, such as valves and heaters, have limits on flow and temperature. The proportional controller will also have limited output. The controllable range will be between these limits. This range is called the proportional band.

Figure 2. Proportional Controller Input/Output characteristic.

An OP AMP circuit will be used to design the proportional controller for this lab. For OP AMPs, controller limits will be the saturation voltage of the IC used in circuit construction.

The proportional control band is inversely proportional to the gain of the proportional controller. The gain of the controller is defined as

$$K_p = \frac{\Delta \text{Output}}{\text{SP - Measurement}}$$ (1)

Where, K_p = proportional controller gain
               SP = setpoint value
               ΔOutput = change in controller output.

The difference between the setpoint and the measurement is the controller error signal, so the gain formula for the proportional controller simplifies to
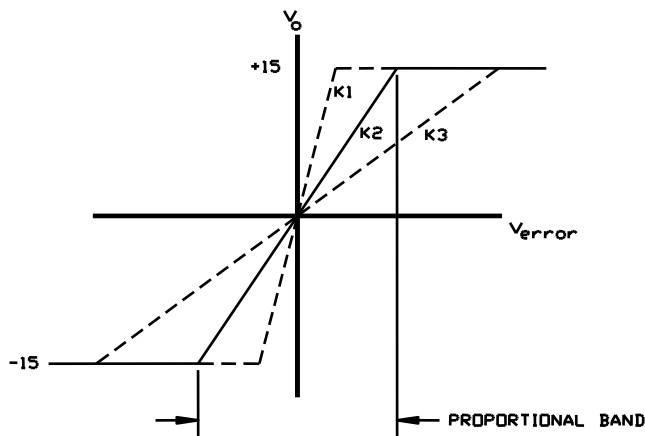
$$K_p = \frac{\Delta \text{Output}}{\Delta e}$$ (2)

Figure 3. Effects of Increased Proportional Gain.

Where, Δe = the change in error value.

The proportional band as a percentage of the controller output is given by,

$$\%PB = \frac{1}{K_p} \bullet 100\%$$ (3)

The effects of increasing the proportional gain on the proportional band are shown in Figure 3. In this figure, K1 > K2 >K3. The proportional band decreases as the gain increases which makes the overall control more sensitive. It also reduces the controllable range of the overall system since a small error signal may cause the controller to reach a limit.
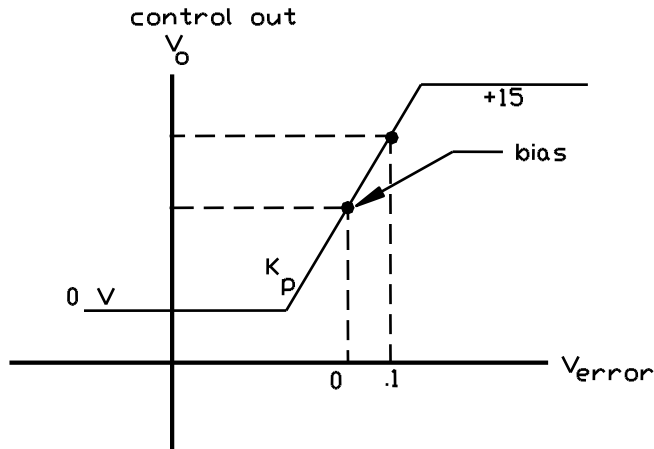
Figure 4. Effects of Imbalance on Proportional Controller Output.

Proportional control has two components, the bias and the amplified error. The bias is also called the control offset. The bias is the control output when the error signal is zero. This is usually set at 50% of the controller output. This is the point where the setpoint and sensor signals are equal.

When the setpoint is moved from this value an error signal is produced and the controller must increase or decrease its output to maintain the system in balance. Figure 4 shows this action graphically.

The proportional controller output can be described mathematically as an equation for a line.

$$C_o = K_p\, e + C_b \qquad\qquad (4)$$

Where
- C_o = the controller output
- C_b = the controller bias value
- e = the control input error
- K_p = the controller proportional gain

The error signal is the difference between the desired setpoint value and the measured value of the control output. This relationship can be represented by

$$e = r - b \qquad\qquad (5)$$

Where,
- r = the setpoint value
- b = the measured indication of the control output.

Given the desired ranges for input error and controller output the values of K_p and C_b in

Equation 4 can be found using the point-slope formula for a straight line. This relationship is shown below.

$$(C_o - C_{min}) = \frac{C_{max} - C_{min}}{e_{max} - e_{min}} (e - e_{min})$$ (6)

Figure 5 shows a block diagram of a proportional controller. The error signal,e is multiplied by the proportional gain, K_p. The result is then added to the bias signal C_b to form the output C_o.
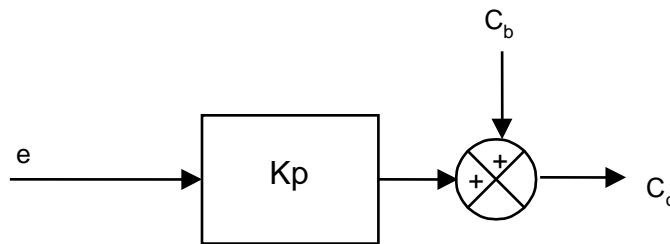

Figure 5. Block Diagram Equivalent of a Proportional Controller.

An inverting summing amplifier can implement this relationship. Figure 6 shows a schematic for an OP AMP implementation of a proportional controller. The performance of this circuit was investigated in Lab 1. The input/output relationships for the inverting summing amplifier are given by:

$$V_c = -\left(\frac{R_f}{R_1} \bullet V_1 + \frac{R_f}{R_2} \bullet V_b \right)$$

$$C_o = K_p \bullet e + C_b$$

$$\frac{R_f}{R_1} = K_p$$

$$\frac{R_f}{R_2} \bullet V_b = C_b$$

$$V_1 = e$$

In this formulation, the bias value is determined by a constant input voltage and a gain set by the values of the resistors R_f and R_2. The proportional gain is set by the ratio of R_f and R_1. Selecting values for R_f and V_b gives the control designer the ability to compute the values of R_1 and R_2 for the desired values of K_p and C_b.
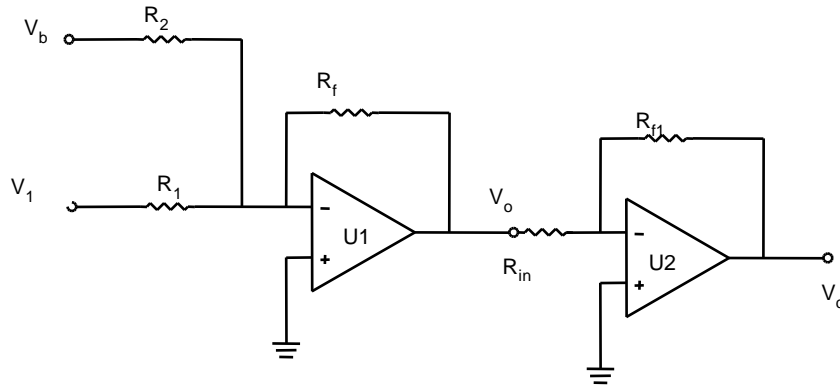
Figure 6. Proportional Controller Implementation Using OP AMP Circuits.

When the inverting summing amplifier is used to implement proportional control, the output will be the negative of the desired control response. To produce the correct sign on the control output, an inverting OP AMP circuit should be added to the controller design

To simplify the design process, let $R_f=R_2$. This gives the gain value on V_b a value of -1. This will make the bias voltage value, V_b, equal to the value of C_b in the proportional control equation and simplify the calibration of the circuit. The gain of U2 should be set to -1 by making $R_{f1}=R_{in}$. A voltage divider circuit or a potentiometer makes an excellent source for the bias voltage.

Process Modeling

For this lab, the process will be represented by a first order RC circuit. Figure 7 shows this schematic for the simulated process or plant response. The input to this circuit will be the voltage of a proportional controller and the output is the control variable. The RC circuit responds to a change in voltage input with an exponential output response. The speed at which the circuit responds is determined by the RC time constant.
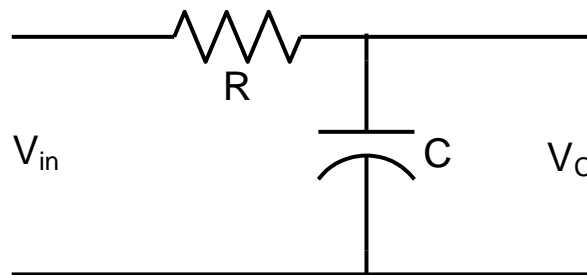

Figure 7. Process Plant for Lab 2. First Order Low Pass Filter.

The mathematical model of this circuit is shown below.

$$V_c = V\ (1 - e^{-t/\tau})\quad a$$
$$V_c = V\ e^{-t/\tau}\quad b$$

(7)

Equation 7a is the response to an instantaneous increase to V_{in} and equation 7b is the response to an instantaneous decrease.

Where $\tau = RC$
$t = $ time

This response is shown graphically in Figure 8.   When a rapid change in the input to a control system is made the control output will respond in a manner that is determined by the time response of the plant under control and the controller. For proportional control, the response of the OP AMPs is many times faster than the processes under control, so the response to rapid changes, also know as step-response, is determined by the time constant of the plant.
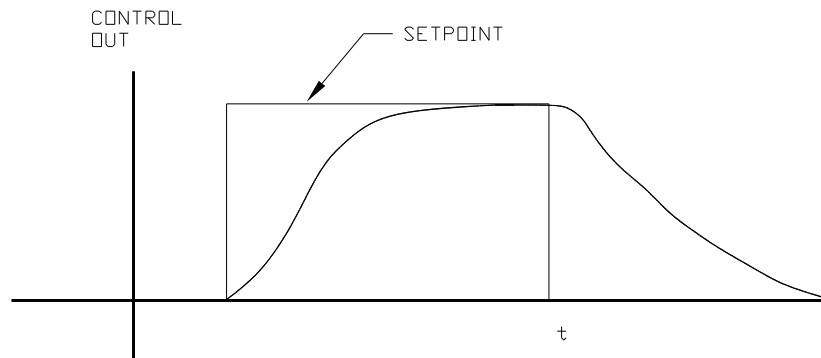
Figure 8 .   Control Response to Step Changes in the Setpoint Value.

Control System Performance Measures

The following terms are used to describe the operation and performance of control systems.

Process Lag -The time it takes for a process to reach a new value after the input changes. This is related to the time constants of the controller and the plant under control.

Process Load - The amount of control output needed to keep the process in balance.

Steady-state error - The difference between the setpoint value and the actual control output after the controller output has stabilized.

In a proportional only controller, the steady-state error of the controller is inversely related to the controller gain, K_p. As the value of K_p is increase, the value of steady-state error decreases. The value of steady-state error can never reach zero in proportional only control, because this would require and infinite gain. This is a limitation of proportional control.

Steady-State Error and the Mathematical Model of the System

If the control process has come to a steady-state, then algebraic equations define the operation of the system. The theoretical values of system variables can be calculated using these equations. Figure 9 shows a typical proportional control system with the following variables

V_{sp} = setpoint input voltage
V_e = control error voltage
V_b = control bias voltage
V_c = controller output voltage
V_o = process output voltage
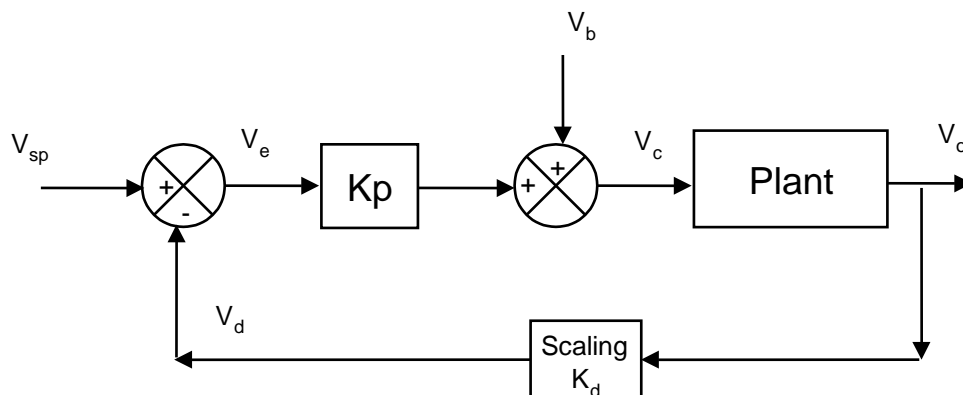V_d = scaled measurement signal

Figure 9. Proportional Control with Variables Identified.

The error voltage, V_e is defined as the difference between the scaled measurement of the plant output and the desired set point value.

$$V_e = V_{sp} - V_d \qquad (8)$$

The error signal from the summing point goes to the proportional controller where it is multiplied by the gain K_p and then summed to the bias voltage to produce the controller output

$$V_c = K_p V_e + V_b \qquad (9)$$

After the capacitor in the RC circuit charges, all transient responses in the system have ended and the input/output relationship of the circuit is $V_c = V_0$.

A measurement and scaling circuit senses the actual plant output and shifts the voltage levels to the same range as the setpoint input. For a linear sensing and scaling circuit, the output is proportional to the input and is given by the following equation.

$$V_d = K_d V_o \qquad (10)$$

In equation (10), V_d is the output of the scaling and measurement circuit. Equations (8), (9), and (10) can be combined to give:

$$V_c = K_p \left(V_{sp} - K_d V_c \right) + V_b$$

This equation can now be solved for the theoretical value of controller output when the other system parameters are known. If K_p, K_d and V_b are know, the controller output can be computed for different levels of setpoint input. Equation (11) is the result of the simplification.
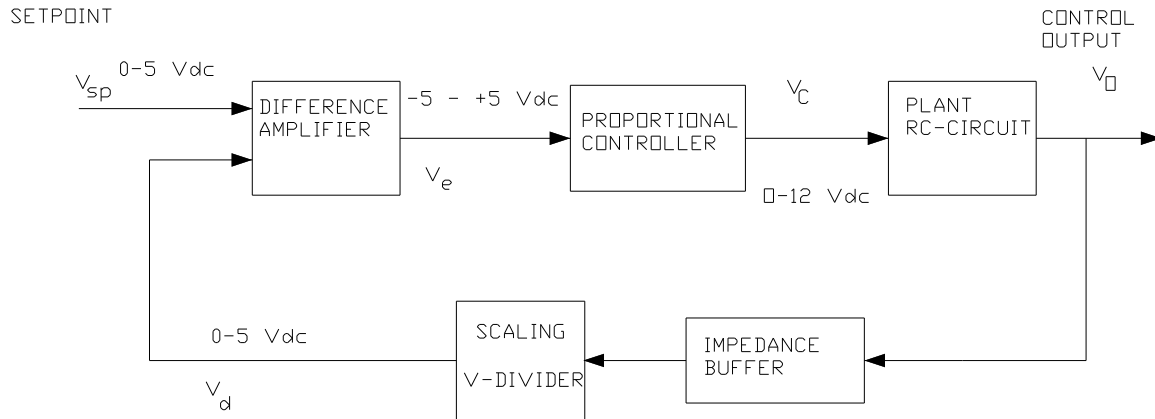
$$V_c = \frac{K_p V_{sp}}{1 + K_p K_d} + \frac{V_b}{1 + K_p K_d} \qquad (11)$$

The theoretical error voltage can now be found from:

$$V_e = V_{sp} - K_d V_c \qquad (12)$$

Design Project- Proportional Control

1.)   Design the appropriate circuits to implement the proportional controller shown in the block diagram below.

The time constant of the plant should be initially set for 2 seconds. Use a 100 kΩ resistor and an appropriately sized capacitor to set this value. Set the bias value of the control to 6 Vdc by grounding the other controller input. Set the initial proportional gain such that the controller has a proportional band of 100%. With the feedback signal from the scaling voltage divider disconnected, the following values should be found for a correctly operating system.

V_{sp} (Vdc)	V_d (Vdc)[1]	V_e (Vdc)	V_c (Vdc)	V_o (Vdc)
5.0	5.0	0.0	6.0	.6.0
0.0	2.5	-2.5	3.5	.3.5
2.5	0.0	2.5	8.5	.8.5
5.0	0.0	5.0	11.0[3]	.11.0[2,3]

1.This is the input to the inverting input of the difference amp with the feedback disconnected. Use a 5 volt supply to simulate this.
2.Note: if an electrolytic capacitor is used to make the RC circuit, a negative voltage could cause it to fail dramatically.(It will explode!) Connect a 1k resistor in series with a diode (1N4001) to the output of the proportional controller to prevent this. See sketch below
3. these values will be zero if the circuit above is used on the controller output.
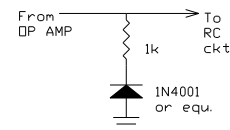
Close the control loop by connecting the scaling circuit output to the difference amplifier inverting input. Test the controller by changing the setpoint and measuring the V_{sp}, V_e, V_c, V_d, and V_o for each value of the setpoint. The output should track the feedback signal, V_d, with a difference between the two values. With $K_p = 1$ record the voltage values in Table 1 and save the data for the report. Compute the theoretical value of voltages V_e and V_d for each value of V_{sp} in the table using the equations from the previous section. Compare the difference between the theoretical error voltage and the measured error voltage. Repeat this procedure for $K_p = 100$. Enter the results in Table 2 for future use. Make two graphs that plot the setpoint voltage on the x-axis and the error voltage on the y-axis. Each graph should have two plots: the theoretical and measured error voltage. Include these in the report. Comment on the performance of the controller compared to the theoretical performance.

2.)   Fix the value of the setpoint to 5 volts and change the proportional gain of the controller to these values: $K_p = 1, 2, 5, 10, 20, 50, 100, 200$. Measure the values of V_e for each value of gain. Enter these values into Table 3. Plot these values on a semilog graph with the values of K_p on the log x-axis and the error on the y-axis. Comment on the effects of changing the gain of steady-state error in the report. Use the equations from the previous section to compute the theoretical values of error voltage at each value of K_p and include them in the report. Construct a graph the plots the theoretical values and the measured values on the same semilog graph.

3.) Change the time constant of the plant to 0.002 seconds by changing the size of the capacitor.   Set the proportional gain to 5 and use a TTL pulse set to 50 Hz instead of the setpoint input. View the input and V_o with a duel channel scope and note the response. Measure the time constant of the plant and the steady-state error using the scope. Include this information in the lab report.

Table 1 - Proportional Controller Voltage Readings

$K_p = 1$

V_{sp} (Vdc)	V_d (Vdc)	V_e (Vdc)	V_c (Vdc)	V_o (Vdc)
5.0 V				
4.5 V				
4.0 V				
3.5 V				
3.0 V				
2.5 V				
2.0 V				
1.5 V				
1.0 V				
0.5 V				
0.0 V				

Table 2 - Proportional Controller Voltage Readings

$K_p = 100$

V_{sp} (Vdc)	V_d (Vdc)	V_e (Vdc)	V_c (Vdc)	V_o (Vdc)
5.0 V				
4.5 V				
4.0 V				
3.5 V				
3.0 V				
2.5 V				
2.0 V				
1.5 V				
1.0 V				
0.5 V				
0.0 V				

Table 3 - Error Voltages for Different Gains

K_p	V_e (Measured)	V_e (Theoretical)
1		
2		
5		
10		
20		
50		
100		
200		