### ET 438a Automatic Control Systems Technology Laboratory 5 Control of a Separately Excited DC Machine

**Objective:** Apply a proportional controller to an electromechanical system and observe the effects that feedback control has on system performance. Model an electromechanical system using differential and algebraic equations. Determine the steady-state and dynamic performance of a torque-controlled permanent magnetic dc motor with different controller parameters. Observe the effects of feedback control on motor-generator system that is subject to external disturbances. Apply a proportional-integral controller to the motor-generator system above and compare the performance to a proportional controller.

### **Theoretical Background**

Previous labs demonstrate that the response of a system is modified by negative feedback control. The response speed of the system is increased as the proportional gain on the system increases. This was demonstrated using an electrical analog for a process. This section of the circuit represented a first order lag process. This lab introduces a commonly used final control element and uses it in the proportional controller designed from the previous experiment.

The separately excited dc motor is widely used in process industries as an actuator. This machine has a linear model if no saturate is assumed. Motor speed, torque or power can be controlled to meet a number of industrial applications. This lab uses small permanent magnet dc machines in motor, speed sensing, and generator applications. This is equivalent to a larger separately excited machine that has a constant field current.

The schematic model of a separately excited voltage controlled dc machine is shown in Figure 1. For armature voltage control the speed of the motor is proportional to the armature voltage assuming that there is no saturation of the field. The electrical parameters of the circuit are the armature resistance  $R_a$ , the armature inductance,  $L_a$  and the counter emf,  $e_b$ . The value of  $e_b$  is proportional to the speed of the motor for linear operation. The proportionality constant is called the back emf constant,  $K_e$ . This can be found from test or in the manufacturer's data sheets.

Writing a KVL equation around the armature circuit gives the steady-state response for the armature.

$$\mathbf{e}_{\mathrm{b}} = \mathbf{e}_{\mathrm{a}} - \mathbf{I}_{\mathrm{a}} \cdot \mathbf{R}_{\mathrm{a}} \tag{1}$$

Where

$$e_{b} = K_{e} \cdot \omega_{m}$$

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Figure 1. Separately excited dc motor model.

The steady-state speed of the motor is found by combining Equations 1 and 2.

$$\omega_{\rm m} = \frac{\mathbf{e}_{\rm a} - \mathbf{I}_{\rm a} \cdot \mathbf{R}_{\rm a}}{\mathbf{K}_{\rm e}} \tag{3}$$

The steady-state developed motor torque depends on the strength of the magnetic field flux and the armature current. This is also a linear relationship if no saturation is assumed in the magnetic circuits. Since the motor magnetic flux is constant, the motor torque is linearly related to the armature current through the proportionality constant  $K_T$ .

$$\mathbf{T}_{\mathsf{d}} = \mathbf{K}_{\mathsf{T}} \cdot \mathbf{I}_{\mathsf{a}} \tag{4}$$

For motor speed to remain constant, the motor load torque,  $T_L$  must be equal to the motor developed torque plus any rotational loss torque.

$$T_{L} + T_{f} = T_{d}$$
 (5)

Since the developed torque of the motor is proportional to the armature current, equations 3 and 4 can be combined to give an equation that relates the armature current to the load torque,

$$\mathbf{T}_{\mathrm{L}} = \mathbf{K}_{\mathrm{T}} \cdot \mathbf{I}_{\mathrm{a}} - \mathbf{T}_{\mathrm{f}} \tag{6}$$

where  $T_f$  is the frictional torque of the machine. Since all practical machines have nonzero values of  $T_f$ , an unloaded motor must draw at least enough current to overcome its rotational losses. The motor's no-load speed will depend on the mechanical parameters.

The mechanical parameters of the dc machine are the viscous damping of the motor  $B_m$  and the rotational inertia of the motor armature,  $J_m$ . The value of  $J_m$  and the armature acceleration determine the inertial torque that the machine must overcome while the value of  $B_m$  relates the dynamic friction to the armature speed.

The dynamic equations of the machine are differential equations that relate the electrical inputs to the developed torque and speed of the motor. When arranged in the form shown below they are called state equations. State equation formulations are more flexible than transfer function models because that allow non-zero initial conditions and produce the time function solutions directly when solved by computer.

	$ \frac{di_{a}(t)}{dt} = -\frac{R_{a}}{L_{a}} \cdot i_{a}(t) - \frac{K_{e}}{L_{a}} \cdot \omega_{m}(t) + \frac{e_{a}(t)}{L_{a}}  (a) $ $ \frac{d\omega_{m}(t)}{dt} = \frac{K_{T}}{J_{mL}} \cdot i_{a}(t) - \frac{B_{mL}}{J_{mL}} \cdot \omega_{m}(t) - \frac{T_{L}}{J_{mL}}  (b) $
Where	$R_a$ = the motor armature resistance $L_a$ = the motor armature inductance $e_a$ = the motor armature voltage $\omega_m(t)$ = the motor speed (rad/sec)
	$B_{mL}$ = the total motor/load viscous friction coefficient (N-m-s/rad) $J_{mL}$ = the total motor/load rotational inertia (N-m-s <sup>2</sup> /rad)
	$K_e$ = back emf constant (V-s/rad)
	$K_T$ = torque constant (N-m/A)
	T <sub>L</sub> = motor load torque (N-m)

In this formulation of the motors dynamic response the variables  $i_a(t)$  and  $\omega_m(t)$  are called state variables. The inputs to the system are the armature voltage,  $e_a(t)$  and the motor load torque,  $T_L$ . Equation 7a describes the electrical dynamics of the motor and equation 7b the mechanical dynamics. Solving these equations using appropriate computer routines will give plots of the responses of the motor speed and armature current. These variables completely describe the response of the motor.

### **Torque-controlled Current-fed Dc Machines**

In the state equation model of a dc motor above, the motor speed is controlled by changing the armature voltage. For this experiment a current source will supply the machine. This will simplify the modeling equations. Figure 2 shows the schematic for a current source dc motor.



Figure 2. Current driven dc motor model.

In this schematic, an ideal current source delivers current to the motor armature. A KVL equation around the armature loop is not necessary to find the armature current since it is the same as the source current  $I_s$ . The only equation necessary to describe the motor dynamics is a torque balance at the armature.

$$J_{mL} \frac{d\omega_{m}(t)}{dt} + B_{mL} \cdot \omega_{m}(t) + T_{L} = K_{T} \cdot i_{a}(t)$$
(8)

The terms on the left-hand side of the equation represent the inertial torque, the viscous friction torque, and the motor load torque. These torques must equal the developed armature torque.

The armature current and the viscous friction coefficient determine the speed of the current-fed dc motor. At equilibrium there is no acceleration so the derivative in equation 8 is zero. This gives the relationship for speed as a function of the armature current.

$$\omega_{\rm m} = \frac{\mathbf{K}_{\rm T} \cdot \mathbf{I}_{\rm a} - \mathbf{T}_{\rm L}}{\mathbf{B}_{\rm m}} \tag{9}$$

Where

 $\omega_m$  = the motor armature speed. (rad/sec) B<sub>m</sub> = the motor viscous friction coefficient. T<sub>L</sub> = load torque

If the motor is unloaded then  $T_L$  is zero and the value of  $B_m$  limits the motor speed to the no-load value. The armature current must be set to a value that will produce at least the friction torque,  $T_f$  or there will not be enough torque developed to start the motor spinning. Increasing the value of  $I_a$  above the no-load current will cause the armature to accelerate until it reaches a new steady-state speed. Additional mechanical load can then be applied until  $T_L = K_T I_a$ . If more torque is required by the motor but the armature

current remains fixed then the motor will stall. The armature current must increase to meet the torque demand placed on the motor.

The block diagram show in Figure 3 shows how the armature current and load torque act as inputs to the mechanical system comprised of the motor armature and its load.



Figure 3. Block Diagram of a Current Driven Dc Motor with an External Load.

The motor load combination is a first order lag process with a time constant that is given by  $J_{mL}/B_{mL}$ . The total rotational inertia is  $J_{mL}$  and total viscous friction is  $B_{mL}$ .

# Tachogenerator Model

The speed of a dc machine can be measured with a dc tachometer. A dc tachometer is a permanent magnet dc generator that is connected to the same shaft as the drive motor. The output voltage of the dc generator under constant load is proportional to the generator shaft speed. The time domain and Laplace equations for this device are:

$$v_{t}(t) = k_{t} \cdot \omega_{m}(t)$$

$$V_{t}(s) = k_{t} \cdot \Omega_{m}(s)$$
(10)

where

 $k_t$  = the tachogenerator constant (V-s/rad)  $v_t(t)$  = the tachogenerator output voltage (V)  $\omega_m(t)$ = motor armature speed (rad/sec)

It may be necessary to scale the output of a tachometer to reduce the span of its output. OP AMP scaling circuits, similar to those in Lab 1, can scale the tachometer's output to any practical range.

### **Dc Motor Current Drive Circuit**



Figure 4. Typical Dc motor Current Dive Circuit Using a unity Gain Follower.

Figure 4 shows a dc motor mechanically coupled to a dc tachometer and a permanent magnet generator. The motor is connected between a 38 Vdc supply and a current sinking transistor, Q2. The resistor Ra represents the dc resistance of the motor armature and is not an external component. The diode D1 protects Q2 from inductive voltage transients. The armature current of the motor is set by the base current of Q2 when the transistor is operated in the active region. The dc current gain of Q2 relates the base current to the armature current of the motor.

$$\mathbf{I}_{\mathsf{a}} = \mathbf{h}_{\mathsf{FE}} \cdot \mathbf{I}_{\mathsf{b}}$$

As long as the transistor is in the active region, this relationship is valid. If base current is increases until Q2 saturates, the collector-to-emitter voltage,  $V_{ce}$  will drop to approximately 0.2 V and  $I_a$  will become independent of the base current. For analog control of the motor the transistor must remain in the active region.

The base current of Q2 depends on the value of R2 and the voltage developed at the emitter of Q1. The value of R2 is set to limit the base current into Q2 when the emitter voltage of Q1 is at its maximum value. The value of R2 should still be low enough to produce collector currents that drive the motor to its rated loading.

The OP AMP U1 and Q1 form a voltage follower circuit that has greater current output capabilities than the OP AMP alone. This is done to make certain that the power transistor Q2 can be driven over a wide range of current outputs when  $h_{FE}$  is low. The resistor R1 limits the collector current of Q1 to safe values. This collector current is approximately equal to the emitter current which drives the base of Q2. The resistor R3 is also a current limiting resistor but in this case for the base of Q1. The potentiometer, R4 controls the emitter voltage at Q1 which controls  $I_a$  of Q2. The proportional controller output is connected here when the system is constructed.

The armature current as a function of the emitter voltage is given below.

$$I_{a} = \frac{V_{e} - V_{be}}{R_{2}} \cdot h_{FE}$$
(11)

where

 $V_e$  = the emitter voltage of Q1  $V_{be}$  = the base to emitter voltage of Q2 in active region (0.7 V)  $h_{FE}$  = the dc current gain of Q2 (from data sheets)

The value of R2 is set to allow a maximum value of  $I_a$  when  $V_e$  is maximum. When these maximums are specified, the a linear function with input  $V_e$  and output  $I_a$  results. Since the OP AMP and Q1 are connected as a voltage follower circuit, the input control voltage  $V_c$  will equal  $V_e$ .

# Separately Excited Dc Generator Model



Figure 5. Separately Excited Dc Generator Model.

Figure 5 shows the schematic of a separately excited dc generator. A permanent magnet dc motor acts as a separately excited dc generator if it is coupled to a source of mechanical power. The source of mechanical power is called the prime mover of the generator. When the generator is not connected to an electrical load the prime mover supplies enough power to overcome the rotational losses of the generator. If the

generator is driven at a constant speed then the rotational power losses can be expressed as a frictional torque just as in the motor case.

As electrical load is applied to the machine additional torque must be supplied by the prime mover or the speed of the system will decrease. The steady-state power balance at the armature of the generator is given as:

$$T_{d} \cdot n \cdot \left(\frac{\pi}{30}\right) = E_{a} \cdot I_{a}$$
 (12)

where  $T_d$  = the torque developed at the armature (N-m) n = armature speed (RPM)  $E_a$  = induced armature voltage (V)  $I_a$  = armature current (A)

The left-hand side of the equation is the mechanical power input and the right-hand side is the total electrical power output. The power transferred to the load is the armature power given in Equation 12 minus the power losses of the armature resistance.

If  $I_a$  and  $V_t$  are measured and the value of armature resistance,  $R_a$  is known, the developed torque is given by Equation 13.

$$T_{d} = \left(\frac{30}{\pi}\right) \cdot \left(\frac{V_{T} \cdot I_{a} + I_{a}^{2} \cdot R_{a}}{n}\right)$$
(13)

This relationship shows that if speed is held constant then the torque developed in the prime mover must increase. Increases in load occur when more resistors are connected in parallel across the generator terminals. This causes an increase in  $I_a$  which also increases the losses due to armature resistance.

The induced voltage is proportional to the armature speed for a separately excited generator. The emf constant of the generator gives the relationship between the prime mover speed and the induced voltage.

$$E_a = K_e \cdot n$$

The dynamic equations for the separately excited generator are shown in state variable form. These equations assume that the prime mover is a separately excited dc motor.

$$\begin{aligned} \frac{d\omega_{g}(t)}{dt} &= \frac{K_{T}}{J_{g}} \cdot i_{am}(t) - \frac{B_{g}}{J_{g}} \cdot \omega_{g}(t) - \frac{K_{Tg}}{J_{g}} \cdot i_{a}(t) \qquad (a) \\ \frac{di_{a}(t)}{dt} &= -\frac{R_{a} + R_{L}}{L_{a}} \cdot i_{a}(t) + \frac{\omega_{g}(t) \cdot K_{e}}{L_{a}} \qquad (b) \\ v_{T}(t) &= R_{L} \cdot i_{a}(t) \qquad (c) \end{aligned}$$

where 
$$\omega_g(t) = \text{generator armature speed (rad/sec)}$$
  
 $i_{am}(t) = \text{motor armature current (A)}$   
 $i_a(t) = \text{generator armature current (A)}$   
 $R_a = \text{generator armature resistance (\Omega)}$   
 $L_a = \text{generator armature inductance (H)}$   
 $R_L = \text{load resistance (\Omega)}$   
 $K_e = \text{generator emf constant (V-s/rad)}$   
 $K_T = \text{motor torque constant (N-m/A)}$   
 $J_g = \text{generator rotational inertia (N-m-s^2/rad)}$   
 $B_g = \text{generator viscous friction coefficient (N-m-s/rad)}$   
 $v_T(t) = \text{load terminal voltage}$ 

Equation 14a describes the mechanical dynamics of the generator. The prime mover armature current is considered an input to the system. This set of equations can be combined with the model for the current driven dc motor to give an overall response of the system. Equation 14b represents the electrical dynamics of the generator armature circuit. The load is considered to be a fixed resistance. Equation 14c is the output equation that relates the state variables  $\omega_q(t)$  and  $i_a(t)$  to the output.

### Design Project I – Proportional Control of a Dc Motor-Generator System

- Construct the voltage-to-current converter shown in Figure 4 using the values shown in the schematic. The 2N3055 (TIP41C) power transistor should be mounted on a heat sink. Use two power supplies to derive the voltage sources for the circuit. One supply will provide the 38 Vdc for the motor only. The other supply will provide the voltages for the controller and current converter circuits. Set the OP AMP supply for ±15 Vdc.
- 2.) Test the circuit by connecting it to the motor-generator test setup provided. It consists of 3 identical permanent magnet motors with coupled shafts. The machine parameters are:

 $\begin{array}{l} {\sf K}_{e} = 0.06398 \; V\text{-sec/rad} \\ {\sf K}_{T} = 0.06426 \; N\text{-m/A} \\ {\sf R}_{a} = 22.6 \; ohms \\ {\sf L}_{a} = 0.0144 \; H \\ {\sf B}_{m} = 10.745 \times 10^{-6} \; N\text{-m-s/rad} \\ {\sf J}_{m} = 3.39 \times 10^{-6} \; N\text{-m-s}^{2}\text{/rad} \end{array}$ 

Assume that all three machines are identical so the above parameters describe all machines mounted on the test setup.

One of the machines will be driven by the voltage-to-current converter circuit while another machine will function as a dc tachometer to measure the speed of the motor. The last machine will be connected as a dc generator. Initially the generator will operate with no-load. The functional block diagram of the desired system is shown in Figure 6.



Figure 6. Block Diagram of the Motor-Generator Control System.

Test the combination of the voltage-current driver and motor-generator-tachometer system by applying different values of input voltage, V<sub>c</sub>, to the OP AMP U1 using the potentiometer. Record the values of motor armature current I<sub>a</sub>, tachometer output voltage V<sub>t</sub>, Q2 collector-to-emitter voltage, V<sub>CE2</sub> and the generator output voltage. Make the measurements for the following values of V<sub>c</sub> shown in Table 1-A. Determine the value of V<sub>c</sub> that just causes the armature to turn (between 160 and 180 mA of armature current) when there is no-load connected to the generator. Record this data in Table 1-B.

- 3.) Modify the difference amplifier and proportional controller from the Experiment 2 so that a bias voltage produces a V<sub>c</sub> that just causes the motor armature to spin. The feedback signal to the difference amplifier and the set-point voltage signals should be disconnected and both inputs to the difference amplifier grounded when testing this circuit modification. Grounding the inputs is equivalent of having zero error.
- 4.) Unground the set-point input to the difference amplifier and connect it to the wiper arm of a potentiometer that gives 0-5 Vdc output. With the motor armature current set to the value found in step 3 by the proportional controller bias, Adjust the output of the set-point potentiometer and measure the voltage output of the tachometer. Adjust the set-point value until the tachometer output reads 12.06 Vdc. Using a voltage divider circuit and an OP AMP voltage follower circuit, scale the tachometer output voltage to be equal to 2.5 Vdc.
- 5.) De-energize the whole system and set the value of Kp to 2. Disconnect the inputs of the difference amplifier from ground and connect them to the set-point value and the feedback signal supplied from the scaling circuit. Set the set-point voltage to

2.5 Vdc and energize the whole system. Measure the motor armature current  $I_a$ , the controller output  $V_c$ , the feedback signal voltage,  $V_m$ , the error voltage  $V_e$ , and the tachometer output voltage  $V_t$ . For the set-point voltages shown in Table 2-A Estimate the motor speed,  $n_m$ , by dividing  $V_t$  by the tachometer constant,  $K_e$ =0.0067 V/RPM, and also record this in Table 2-A.

- 6.) Set the set-point value to 2.5 V. Connect a 1000 ohm 1/2 watt resistor across the generator terminals with the proportional controller in operation. Measure I<sub>a</sub>, V<sub>c</sub>, V<sub>m</sub>, V<sub>e</sub> and V<sub>t</sub> again. Record this data in Table 2-B. Parallel 1000 ohm resistors to get the remaining loads in Table 2-B and record the results for each value of R<sub>L</sub>.
- 7.) Repeat steps 5 and 6 with values of Kp set to10 and 50. Enter the data for Kp=10 into Tables 3-A and 3-B. For Kp=50 enter the data into Tables 4-A and 4-B.

Include all the data collected from the tests on the system in the report. Also derive a closed loop transfer function for the system with the set-point voltage as the input and the generator terminal as the output using Kp=2 and  $R_L$ =1000 ohms. The block diagrams for this system and the derived equations are in Appendix A of this handout.

Using the data in Tables 2-A, 3-A, 4-A, produce three plots of the set-point voltage,  $V_{sp}$ , (x) vs. the error voltage,  $V_{e}$ , (y). In the laboratory report, comment on how increasing the proportional gain affects the speed error. Also, produce three plots using the data in Tables 2-B, 3-B, 4-B of the generator load resistance,  $R_L$ , (x) vs. the motor speed,  $n_m$ , (y). Discuss how the motors speed is affected by the load change with the controller in operation. Compare the performance with an uncontrolled system.

### Design Project II – Proportional-Integral Control of a Dc Motor-Generator System

Proportional control gives fast response but requires high gains to achieve small values of steady-state error. Applications that need more precise control can employ a proportional-integral (PI) controller. Figure 7 shows an OP AMP implementation of a PI controller.



Figure 7. Proportional-Integral Controller Using a Single OP AMP

The transfer function of this controller is:

$$\frac{V_{c}(s)}{V_{e}(s)} = \left[\frac{R_{2}}{R_{1}} + \frac{1}{R_{2}C_{1}s}\right]$$
(15)

Equation (15) assumes that  $R_3=R_4$ . The ratio of  $R_2$  and  $R_1$  sets the proportional gain and the capacitor and  $R_2$  set the integral action rate. The reciprical of the integral action rate is the time required for the integral mode to match the change in output produced by the proportional mode. The proportional gain is defined in terms of the OP AMP parameters as

$$K_{p} = \frac{R_{2}}{R_{1}}$$
 (16)

$$K_{1} = \frac{1}{R_{2}C_{1}}$$
(17)

Disconnect the proportional controller from the feedback loop and substitute a PI controller that has the following parameters: Kp=2 and K<sub>1</sub>=213 sec<sup>-1</sup>. Let C<sub>1</sub>=0.01 $\mu$ F for this calculation and find values for R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, and R<sub>4</sub>. Figure 8 shows a block diagram of the motor-generator system with the PI controller added.



Figure 8. Motor Control Using Proportional Integral Controller.

- 1.) With the controller design from above installed in the control loop, vary the setpoint voltage over the range shown in Table 5-A, and make all the necessary measurements to fill the table.
- 2.) Adjust the set-point to 2.5 V dc and add the generator load resistors listed in Table 5-B. Make all necessary measurements to fill the table.
- 3.) Set Kp=10 and maintain  $K_1=213$  sec<sup>-1</sup>. Vary the set-point with no load resistor connected and record the data in Table 6-A.
- 4.) Adjust the set-point to 2.5 V dc and add the generator load resistors listed in Table 6-B. Make all necessary measurements to fill the table.

Using the data in Tables 5-A and 6-A, produce two plots of the set-point voltage,  $V_{sp}$ , (x) vs. the error voltage,  $V_e$ , (y). In the laboratory report, comment on how increasing the proportional gain affects the speed error and how the integral action changes the system performance. Also, produce two plots using the data in Tables 6-B, 6-B, of the generator load resistance,  $R_L$ , (x) vs. the motor speed,  $n_m$ , (y). Discuss how the motors speed is affected by the load change with the PI controller in operation. Compare the performance with an uncontrolled system and the proportional only controller.

### Appendix A Block Diagrams for Lab 5

The block diagram below describes the signal flows of torque-driven dc generator used in the lab.



Figure 9. Block diagram of Generator.

The parameters have the following definitions:

 $\begin{array}{l} T_d(s) = \mbox{mechanical torque developed by the prime mover} \\ T_g(s) = \mbox{counter torque developed by the generator} \\ J_{mL} = \mbox{the rotational inertia of the motor and generator} \\ B_{mL} = \mbox{the viscous friction of the motor and generator} \\ K_e = \mbox{generator emf constant} \\ L_a = \mbox{generator armature inductance} \\ R_a = \mbox{generator armature resistance} \\ R_L = \mbox{load resistance} \\ K_{Tg} = \mbox{generator torque constant} \\ \Omega_{mg}(s) = \mbox{shaft speed of motor-generator} \\ I_L(s) = \mbox{generator load current} \end{array}$ 

 $V_L(s)$ = generator terminal voltage

When the speed feedback control is added the following block diagram results:



Figure 10. Overall system block diagram.

The parameters and signals are defined below.

 $V_{sp}(s)$  = system control setpoint value  $V_c(s)$  = controller output voltage  $I_m(s)$  = motor armature current  $K_t$  = tachometer voltage constant  $K_s$  = voltage scaling circuit constant  $K_{VI}$  = voltage-to-current converter gain ( $h_{FE}/R_2$ )  $K_{Tm}$  = motor torque constant  $K_P$  = controller proportional gain

The overall transfer function is found using the formula below:

$$\frac{V_{L}(s)}{V_{sp}(s)} = \frac{G_{T}(s)}{1 + G_{1}(s)G_{2}(s)H_{1}(s) - G_{2}(s)G_{3}(s)H_{2}(s)}$$
(18)

With:

$$G_{1}(s) = K_{p}K_{VI}K_{Tm} \qquad G_{2}(s) = \frac{1}{J_{mL}s + B_{mL}} \qquad G_{3}(s) = \frac{1}{L_{a}s + (R_{a} + R_{L})}$$
$$H_{1}(s) = K_{s}K_{t} \qquad H_{2}(s) = K_{Tg}$$

$$G_{T}(s) = \frac{K_{P}K_{VI}K_{Tm}K_{e}R_{L}}{(L_{a}s + (R_{a} + R_{L}))(J_{mL}s + B_{mL})}$$

	Table T-A								
V <sub>c</sub> (Vdc)	V <sub>CE2</sub> (Vdc)	V <sub>t</sub> (Vdc)	l <sub>a</sub> (mA)						
0.0									
1.0									
1.5									
2.0									
2.5									
3.0									
3.5									
4.0									
5.0									
6.0									

Table 1-A

Table 1-B Starting Current Values

I <sub>a</sub> (mA)	V <sub>CE2</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>c</sub> (Vdc)

$K_p = 2$ , $R_L = infinity$								
V <sub>sp</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>e</sub> (Vdc)	V <sub>C</sub> (Vdc)	n <sub>m</sub> (RPM)			
0.5								
1.0								
1.5								
2.0								
2.5								
3.0								
3.5								
4.0								
4.5								
5.0								

Table 2-A K₀ =2, R₁=infinit

Table 2-B Kp=2, V<sub>sp</sub>=2.5 Vdc

$R_{L}(\Omega)$	l <sub>a</sub> (mA)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>c</sub> (Vdc)	V <sub>L</sub> (Vdc)	n <sub>m</sub> (RPM)				
1000										
500										
250										
125										

V <sub>sp</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>e</sub> (Vdc)	V <sub>C</sub> (Vdc)	n <sub>m</sub> (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 3-A  $K_p = 10, R_l = infinity$ 

Table 3-B Kp=10, Vsp=2.5 Vdc

	Np=10, v <sub>sp</sub> =2.5 vuc							
$R_{L}(\Omega)$	I <sub>a</sub> (mA)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>c</sub> (Vdc)	V <sub>L</sub> (Vdc)	n <sub>m</sub> (RPM)		
1000								
500								
250								
125								

Table 4-A K<sub>a</sub> = 50  $B_1$  = infinity

V <sub>sp</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>e</sub> (Vdc)	V <sub>C</sub> (Vdc)	n <sub>m</sub> (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 4-B Kp=50, V<sub>sp</sub>=2.5 Vdc

R <sub>L</sub> (Ω)	I <sub>a</sub> (mA)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>c</sub> (Vdc)	V <sub>L</sub> (Vdc)	n <sub>m</sub> (RPM)
1000						
500						
250						
125						

# PI Controller Design Data

$K_p = 2$ , $K_l = 213$ , $R_L = infinity$								
V <sub>sp</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>e</sub> (Vdc)	V <sub>C</sub> (Vdc)	n <sub>m</sub> (RPM)			
0.5								
1.0								
1.5								
2.0								
2.5								
3.0								
3.5								
4.0								
4.5								
5.0								

Table 5-A

Table 5-B Kp=2 Ki=213 Var=2.5 Vdc

$Rp-2$ , $R_{l}-2$ 13, $V_{sp}-2$ .5 VUC								
$R_{L}(\Omega)$	I <sub>a</sub> (mA)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>c</sub> (Vdc)	V <sub>L</sub> (Vdc)	n <sub>m</sub> (RPM)		
1000								
500								
250								
125								

Table 6-A  $K_p = 10, K_l = 213, R_l = infinity$ 

		τφ το, τη Ξ			
V <sub>sp</sub> (Vdc)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>e</sub> (Vdc)	V <sub>C</sub> (Vdc)	n <sub>m</sub> (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 6-B Kp=10, K<sub>I</sub>=213, V<sub>sp</sub>=2.5 Vdc

$R_{L}(\Omega)$	l <sub>a</sub> (mA)	V <sub>t</sub> (Vdc)	V <sub>m</sub> (Vdc)	V <sub>c</sub> (Vdc)	V <sub>L</sub> (Vdc)	n <sub>m</sub> (RPM)
1000						
500						
250						
125						