

EET 438a
Automatic Control Systems Technology
Laboratory 5
Control of a Separately Excited DC Machine

Objective: Apply a proportional controller to an electromechanical system and observe the effects that feedback control has on system performance. Model an electromechanical system using differential and algebraic equations. Determine the steady-state and dynamic performance of a torque-controlled permanent magnetic dc motor with different controller parameters. Observe the effects of feedback control on motor-generator system that is subject to external disturbances. Apply a proportional-integral controller to the motor-generator system above and compare the performance to a proportional controller.

Theoretical Background

Previous labs demonstrate that the response of a system is modified by negative feedback control. The response speed of the system is increased as the proportional gain on the system increases. This was demonstrated using an electrical analog for a process. This section of the circuit represented a first order lag process. This lab introduces a commonly used final control element and uses it in the proportional controller designed from the previous experiment.

The separately excited dc motor is widely used in process industries as an actuator. This machine has a linear model ignoring saturation. Motor speed, torque or power can be controlled to meet a number of industrial applications. This lab uses small permanent magnet dc machines in motor, speed sensing, and generator applications. This is equivalent to a larger separately excited machine that has a constant field current.

The schematic model of a separately excited voltage controlled dc machine is shown in Figure 1. For armature voltage control the speed of the motor is proportional to the armature voltage assuming that there is no saturation of the field. The electrical parameters of the circuit are the armature resistance R_a , the armature inductance, L_a and the counter emf, e_b . The value of e_b is proportional to the speed of the motor for linear operation. The proportionality constant is called the back emf constant, K_e . This can be found from test or in the manufacturer's data sheets.

Writing a KVL equation around the armature circuit gives the steady-state response for the armature.

$$e_b = e_a - I_a \cdot R_a \quad (1)$$

Where

$$e_b = K_e \cdot \omega_m \quad (2)$$

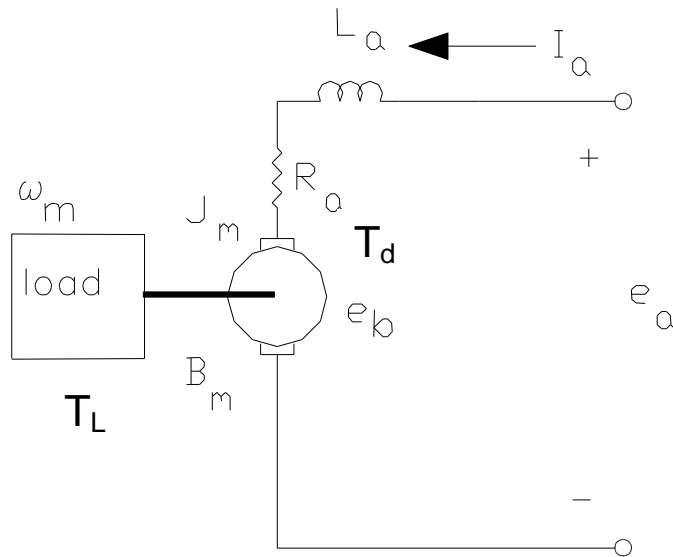


Figure 1. Separately excited dc motor model.

The steady-state speed of the motor is found by combining Equations 1 and 2.

$$\omega_m = \frac{e_a - I_a \cdot R_a}{K_e} \quad (3)$$

The steady-state developed motor torque depends on the strength of the magnetic field flux and the armature current. This is also a linear relationship if no saturation is assumed in the magnetic circuits. Since the motor magnetic flux is constant, the motor torque is linearly related to the armature current through the proportionality constant K_T .

$$T_d = K_T \cdot I_a \quad (4)$$

For motor speed to remain constant, the motor load torque, T_L must be equal to the motor developed torque plus any rotational loss torque.

$$T_L + T_f = T_d \quad (5)$$

Where

- T_d = motor developed torque
- T_L = mechanical load torque
- T_f = frictional torque due to rotational losses

Since the developed torque of the motor is proportional to the armature current, equations 3 and 4 can be combined to give an equation that relates the armature current to the load torque,

$$T_L = K_T \cdot I_a - T_f \quad (6)$$

Since all practical machines have non-zero values of T_f , an unloaded motor must draw at least enough current to overcome its rotational losses. The motor's no-load speed will depend on the mechanical parameters.

The mechanical parameters of the dc machine are the viscous friction of the motor B_m and the rotational inertia of the motor armature, J_m . The value of J_m and the armature acceleration determine the inertial torque that the machine must overcome while the value of B_m relates the dynamic friction to the armature speed.

The dynamic equations of the machine are differential equations that relate the electrical inputs to the developed torque and speed of the motor. When arranged in the form shown below they are called state equations. State equation formulations are more flexible than transfer function models because that allow non-zero initial conditions and produce the time function solutions directly when solved by computer.

$$\begin{aligned} \frac{di_a(t)}{dt} &= -\frac{R_a}{L_a} \cdot i_a(t) - \frac{K_e}{L_a} \cdot \omega_m(t) + \frac{e_a(t)}{L_a} \quad (a) \\ \frac{d\omega_m(t)}{dt} &= \frac{K_T}{J_{mL}} \cdot i_a(t) - \frac{B_{mL}}{J_{mL}} \cdot \omega_m(t) - \frac{T_L}{J_{mL}} \quad (b) \end{aligned} \quad (7)$$

Where

- R_a = the motor armature resistance
- L_a = the motor armature inductance
- e_a = the motor armature voltage
- $\omega_m(t)$ = the motor speed (rad/sec)
- B_{mL} = the total motor/load viscous friction coefficient (N-m-s/rad)
- J_{mL} = the total motor/load rotational inertia (N-m-s²/rad)
- K_e = back emf constant (V-s/rad)
- K_T = torque constant (N-m/A)
- T_L = motor load torque (N-m)

In this formulation of the motors dynamic response the variables $i_a(t)$ and $\omega_m(t)$ are called state variables. The inputs to the system are the armature voltage, $e_a(t)$ and the motor load torque, T_L . Equation 7a describes the electrical dynamics of the motor and equation 7b the mechanical dynamics. Solving these equations using appropriate computer routines will give plots of the responses of the motor speed and armature current. These variables completely describe the response of the motor.

Armature-Controlled Dc Machines

In the state equation model of a dc motor above, the motor speed is controlled by changing the armature voltage. Figure 2 shows the schematic for an armature-controlled dc motor. This method of control requires a variable dc voltage source and assumes a constant field flux. Permanent magnet dc motors have constant field flux. This lab activity will use this type of motor.

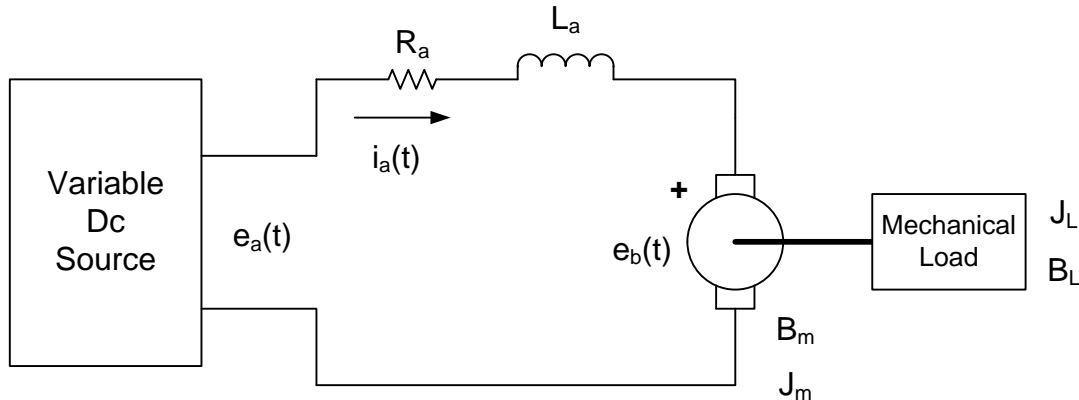


Figure 2. Armature-Controlled Dc Motor Model.

In Figure 2 schematic, a variable dc voltage source supplies a voltage to the motor armature. Equation 7a describes the electrical response of the motor to a change in the variable input voltage, $e_a(t)$. This equation writes $e_b(t)$ in terms of the motor speed and the back EMF constant. Equation 8 below restates equation 7b as a torque balance at the armature.

$$J_{mL} \frac{d\omega_m(t)}{dt} + B_{mL} \cdot \omega_m(t) + T_L = K_T \cdot i_a(t) \quad (8)$$

The terms on the left-hand side of the equation represent the inertial torque, the viscous friction torque, and the motor load torque. These torques must equal the developed armature torque to maintain constant motor speed.

The armature voltage, the motor constants and the viscous friction coefficient determine the speed of the armature-controlled dc motor. At equilibrium there is no acceleration and not change in current so the derivatives in equations 8 and 7a are zero. Combining these equations gives the relationship for speed as a function of the armature voltage. Note that the in steady-state $T_f = B_{mL} \cdot \Omega_{mL}$.

$$\omega_m = \frac{K_T \cdot E_a - R_a (T_L + T_f)}{K_T \cdot K_e} \quad (9)$$

Where

- ω_m = motor armature speed. (rad/sec)
- K_T = motor torque constant (N-m/A).
- K_e = motor EMF constant (V-sec/rad)
- T_L = load torque (N-m)
- T_f = frictional torque

If the motor is unloaded then T_L is zero and the value of T_f limits the motor speed to the no-load value. The motor armature voltage, E_a , must be set to a value that will produce at least the friction torque or the motor will not start spinning. Increasing the value of E_a above the no-load voltage will cause the armature to accelerate until it reaches a new steady-state speed. Additional mechanical load can then be applied until $T_f + T_L = K_T I_a$. In steady-state, armature voltage relates to armature current through the following expression:

$$\frac{E_a - K_e \cdot \Omega_m}{R_a} = I_a \quad (10)$$

Where: Ω_m = motor steady-state speed

If more torque is required by the motor but the armature voltage remains fixed then the armature current cannot increase limiting the torque and the motor will stall. The armature current must increase to meet the torque demand placed on the motor. This means that the armature voltage must increase as well to produce more current.

The block diagram show in Figure 3 shows how the armature current and load torque act as inputs to the mechanical system comprised of the motor armature and its load.

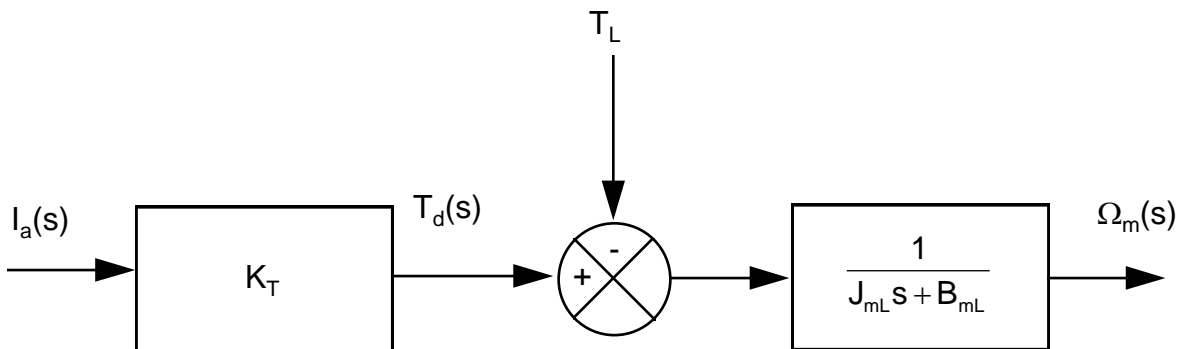


Figure 3. Block Diagram of a Current Driven Dc Motor with an External Load.

The direct-coupled motor/load combination is a first order lag process with a time constant that is given by J_{mL}/B_{mL} . The total rotational inertia is J_{mL} and total viscous friction is B_{mL} .

Tachogenerator Model

The speed of a dc machine can be measured with a dc tachometer. A dc tachometer is a permanent magnet dc generator that is connected to the same shaft as the drive motor. The output voltage of the dc generator under constant load is proportional to the generator shaft speed. The time domain and Laplace equations for this device are:

$$\begin{aligned} v_t(t) &= k_t \cdot \omega_m(t) \\ V_t(s) &= k_t \cdot \Omega_m(s) \end{aligned} \quad (11)$$

where

k_t = the tachogenerator constant (V-s/rad)
 $v_t(t)$ = the tachogenerator output voltage (V)
 $\omega_m(t)$ = motor armature speed (rad/sec)

It may be necessary to scale the output of a tachometer to reduce the span of its output. OP AMP scaling circuits, similar to those in Lab 1, can scale the tachometer's output to any practical range.

Variable Dc Using Pulse-Width Modulation

Pulse-Width Modulation PWM can produce a controllable dc output that has high voltage and current. . PWM produces a variable dc voltage by switching a dc source on/off for a fraction of a pulse duty cycle. An ideal PWM circuit has a linear relationship between duty cycle and output voltage. This modulator compares a reference level to a triangle wave and produces an output square wave with variable duty cycle. This output signal can control a higher voltage and current source producing an adjustable dc level.

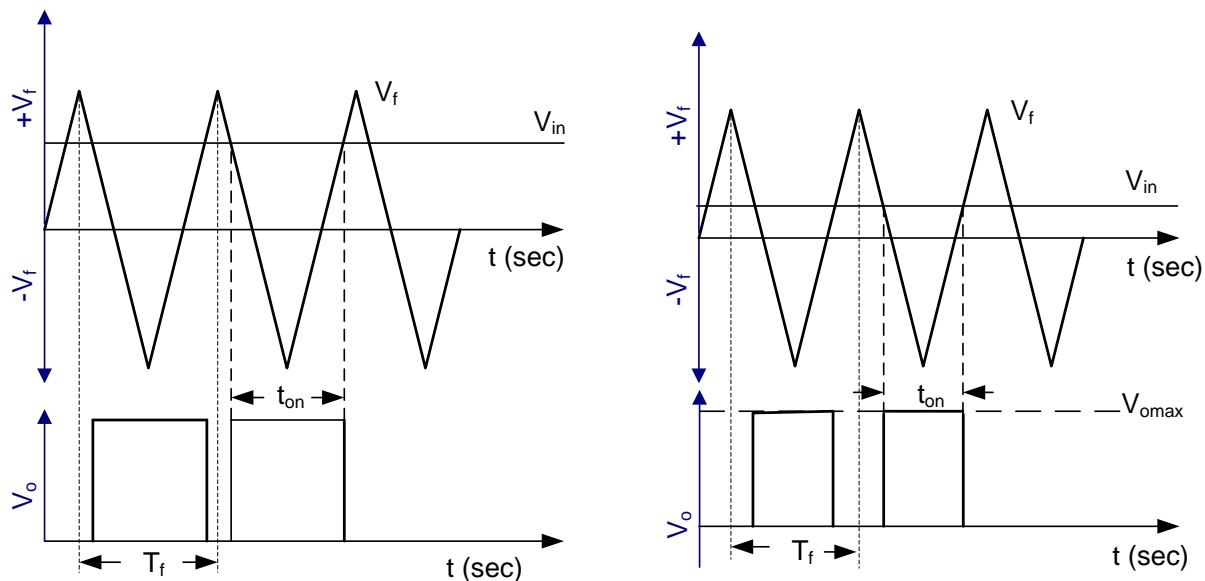


Figure 4. Pulse-Width Modulation Output for Varying Input Voltage.

Figure 4 shows the waveforms for a typical PWM output. In this realization, the output voltage pulse width increases as the control voltage, V_c , decreases. Increasing the pulse width increases the average dc output voltage linearly with the on time, t_{on} . Maximum output occurs when $t_{on}=T_f$, where T_f is the period of the triangle wave used to modulate the output. The duty cycle can vary from 10%-90% in practical realizations of PWM. The following equation describes the relationship between duty cycle and output voltage.

$$V_o = \frac{t_{on} \cdot V_{o\max}}{T_f} \quad (12)$$

Where: V_o =modulator output voltage
 $V_{o\max}$ = maximum supply voltage
 t_{on} = pulse on time (sec)
 T_f = triangle wave period (sec)

The slope of the triangle wave affects the sensitivity of the output voltage to the modulator input voltage, V_{in} . Using the concept of similar triangles on Figure 4 gives the following relationship between the control voltage V_{in} and the pulse on-time, t_{on} .

$$t_{on} = \left[\frac{T_f}{2 \cdot V_{f\max}} \right] \cdot (V_{f\max} - V_{in}) \quad \text{For } -V_{f\max} \leq V_{in} \leq +V_{f\max} \quad (13)$$

This equation is valid for triangle waves input to a non-inverting comparator's negative input and control voltage applied to the positive terminal. This also assumes a final switching device connect so it conducts when the comparator output is high. Combining Equations (12) and (13) relates the input voltage of the PWM to the dc output.

$$V_o = \left[\frac{1}{2 \cdot V_{f\max}} \right] \cdot (V_{f\max} - V_{in}) \cdot V_{o\max} \quad (14)$$

PWM Drive Circuit Using Analog Devices

Figure 5 shows a PWM circuit for delivering a variable voltage to a small permanent magnet dc motor. The circuit uses common ICs to produce a triangle wave and compare it to a reference voltage. The dc motor directly couples to a tachogenerator and a small generator for this lab activity. This lab fixture is available for the lab instructor. The ICs U1, U2 and U3 form the triangle wave generator. The IC U1 is a 555 timer configured to operate as an astable multivibrator. In this configuration, the time produces a square wave output that has a widely variable duty cycle and a frequency set by resistors R1, R2 and capacitor C1.

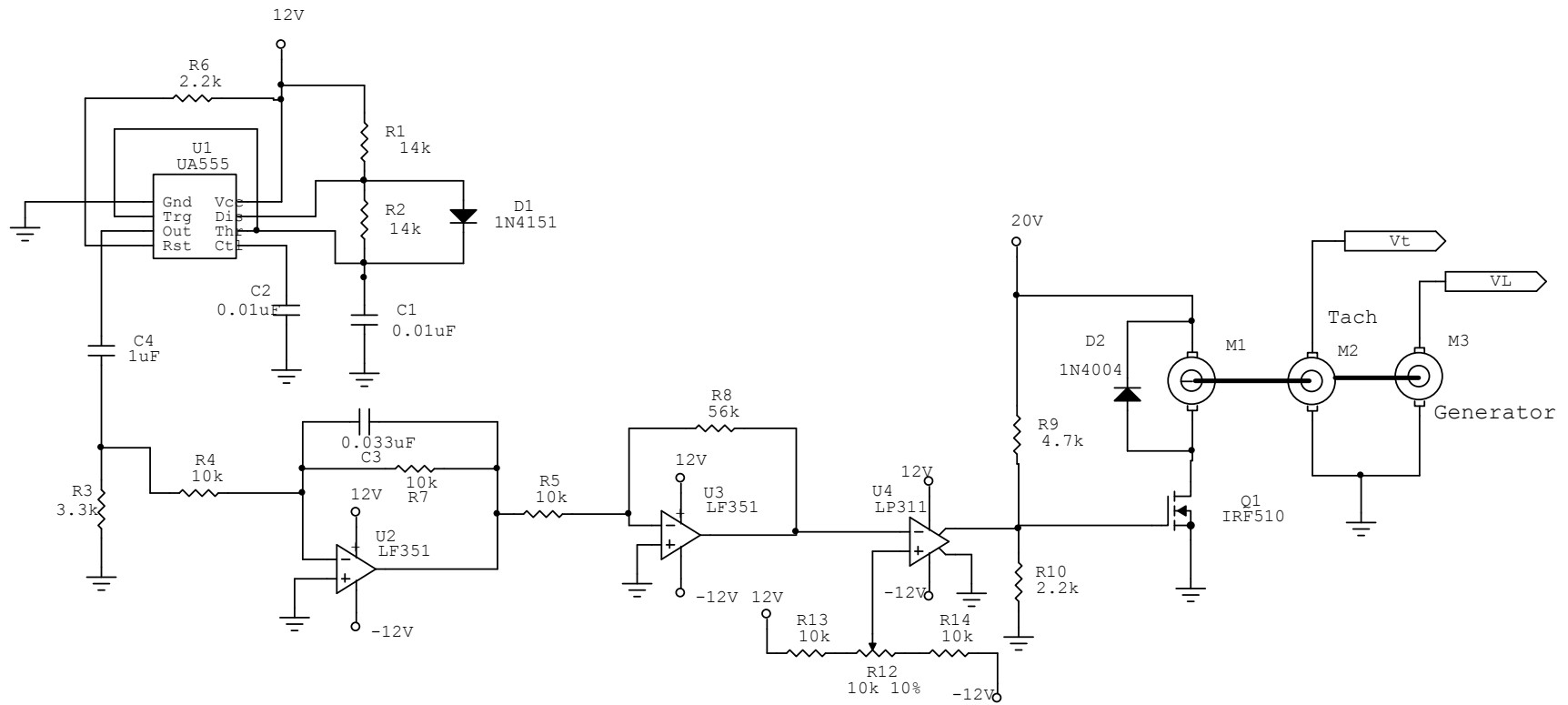


Figure 5. Typical Dc motor Current Dive Circuit Using a unity Gain Follower

The diode D1 bypasses R2 forming a wide range duty cycle circuit. The following design equations will find values of R1 and R2 given a value of C1 duty cycle and frequency. The design shown is set for 5000 Hz and 50% duty cycle.

$$f = \frac{1}{0.67 \cdot (R1 + R2) \cdot C1} \quad (15)$$
$$DC = \frac{R1}{R1 + R2}$$

The output of the timer couples to the OP AMP integrator circuit through a high pass filter formed by C4 and R3. This circuit removes the dc component of the square wave and centers it about ground. The high pass filter has a cut-off frequency of 50 Hz.

The OP AMP U2 is an integrator circuit with a cut-off frequency of 500 Hz and a dc gain of 1. Design this circuit by selecting a frequency well below the timer output, usually a factor of 1/10 is sufficient, selecting a value of input resistance R4 and computing a value of C3. The ICs U2 and U3 are high slew rated JFET input OP AMPs that can accurately reproduce higher frequency signals.

The circuit implemented using U3 is simply an inverting OP AMP with a gain set by resistors R5 and R8. This circuit has a dc gain of 5.6 and inverts the output with respect to the input signal.

The IC U4 is a non-inverting voltage comparator that acts as the pulse width modulator. The triangle wave enters the device on the inverting input and the control voltage, V_c , enters on the non-inverting terminal. The selected device has an open collector output that can be connected to a separate voltage source. Varying the value of V_c changes the duty cycle of the comparator output as shown in Figure 4.

The output of the comparator drives a MOSFET device that is in series with the permanent magnet dc motor. MOSFET devices are voltage controlled devices that draw negligible current. The resistors R9 and R10 form the bias network of the MOSFET. When the output of U4 is high the MOSFET acts as a closed switch and the full motor supply voltage appears across its terminals. Measuring the voltage across the MOSFET will show zero volts indicating that the source voltage appears across the motor armature.

When the comparator output is low the MOSFET acts as an open switch and the source dc voltage appears across the MOSFET. No voltage appears across the motor armature. The schematic shows a diode D2 connected across the motor armature so it is reverse biased. This diode suppresses inductive voltage spikes that appear when the armature current is switched off. This diode should have a voltage rating of several hundred volts and a current rating similar to the full load current of the motor.

The resistors and potentiometer R12, R13 and R14 form a voltage divider network that provides a control voltage for testing this circuit. Remove these components and connect this comparator input to the controller output.

Separately Excited Dc Generator Model

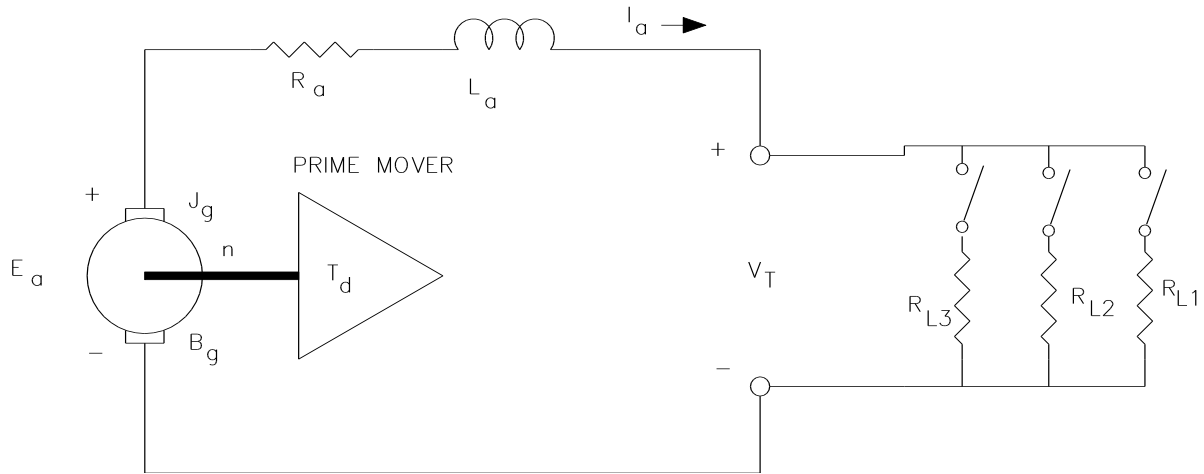


Figure 6. Separately Excited Dc Generator Model.

Figure 5 shows the schematic of a separately excited dc generator. A permanent magnet dc motor acts as a separately excited dc generator if it is coupled to a source of mechanical power. The source of mechanical power is called the prime mover of the generator. When the generator is not connected to an electrical load the prime mover supplies enough power to overcome the rotational losses of the generator. If the generator is driven at a constant speed then the rotational power losses can be expressed as a frictional torque just as in the motor case.

As electrical load is applied to the machine additional torque must be supplied by the prime mover or the speed of the system will decrease. The steady-state power balance at the armature of the generator is given as:

$$T_d \cdot n \cdot \left(\frac{\pi}{30} \right) = E_a \cdot I_a \quad (16)$$

where

- T_d = the torque developed at the armature (N-m)
- n = armature speed (RPM)
- E_a = induced armature voltage (V)
- I_a = armature current (A)

The left-hand side of the equation is the mechanical power input and the right-hand side is the total electrical power output. The power transferred to the load is the armature power given in Equation 12 minus the power losses of the armature resistance.

If I_a and V_t are measured and the value of armature resistance, R_a is known, the developed torque is given by Equation 13.

$$T_d = \left(\frac{30}{\pi} \right) \cdot \left(\frac{V_T \cdot I_a + I_a^2 \cdot R_a}{n} \right) \quad (17)$$

This relationship shows that if speed is held constant then the torque developed in the prime mover must increase. Increases in load occur when more resistors are connected in parallel across the generator terminals. This causes an increase in I_a which also increases the losses due to armature resistance.

The induced voltage is proportional to the armature speed for a separately excited generator. The emf constant of the generator gives the relationship between the prime mover speed and the induced voltage.

$$E_a = K_e \cdot n$$

The dynamic equations for the separately excited generator are shown in state variable form. These equations assume that the prime mover is a separately excited dc motor.

$$\begin{aligned} \frac{d\omega_g(t)}{dt} &= \frac{K_T}{J_g} \cdot i_{am}(t) - \frac{B_g}{J_g} \cdot \omega_g(t) - \frac{K_{Tg}}{J_g} \cdot i_a(t) & (a) \\ \frac{di_a(t)}{dt} &= -\frac{R_a + R_L}{L_a} \cdot i_a(t) + \frac{\omega_g(t) \cdot K_e}{L_a} & (b) \\ v_T(t) &= R_L \cdot i_a(t) & (c) \end{aligned} \quad (18)$$

where

- $\omega_g(t)$ = generator armature speed (rad/sec)
- $i_{am}(t)$ = motor armature current (A)
- $i_a(t)$ = generator armature current (A)
- R_a = generator armature resistance (Ω)
- L_a = generator armature inductance (H)
- R_L = load resistance (Ω)
- K_e = generator emf constant (V-s/rad)
- K_T = motor torque constant (N-m/A)
- J_g = generator rotational inertia (N-m-s²/rad)
- B_g = generator viscous friction coefficient (N-m-s/rad)
- $v_T(t)$ = load terminal voltage

Equation 14a describes the mechanical dynamics of the generator. The prime mover armature current is considered an input to the system. This set of equations can be combined with the model for the current driven dc motor to give an overall response of the system. Equation 14b represents the electrical dynamics of the generator armature

circuit. The load is considered to be a fixed resistance. Equation 14c is the output equation that relates the state variables $\omega_g(t)$ and $i_a(t)$ to the output.

Design Project I – Proportional Control of a Dc Motor-Generator System

- 1.) Construct the PWM Driver circuit shown in Figure 5 using the values shown in the schematic. The IRP510 power MOSFET should be mounted on a heat sink. Use two power supplies to derive the voltage sources for the circuit. One supply will provide the 20 Vdc for the motor and MOSFET only. The other supply will provide the voltages for the controller and current converter circuits. Set the OP AMP supply for ± 12 Vdc.
- 2.) Test the circuit by connecting it to the motor-generator test setup provided. It consists of 3 identical permanent magnet motors with coupled shafts. The machine parameters are:

$$\begin{aligned}
 K_e &= 0.06398 \text{ V-sec/rad} \\
 K_T &= 0.06426 \text{ N-m/A} \\
 R_a &= 22.6 \text{ ohms} \\
 L_a &= 0.0144 \text{ H} \\
 B_m &= 10.745 \times 10^{-6} \text{ N-m-s/rad} \\
 J_m &= 3.39 \times 10^{-6} \text{ N-m-s}^2/\text{rad}
 \end{aligned}$$

Assume that all three machines are identical so the above parameters describe all machines mounted on the test setup.

One of the machines will be driven by the PWM driver circuit while another machine will function as a dc tachometer to measure the speed of the motor. The last machine will be connected as a dc generator. Initially the generator will operate with no-load. The functional block diagram of the desired system is shown in Figure 7.

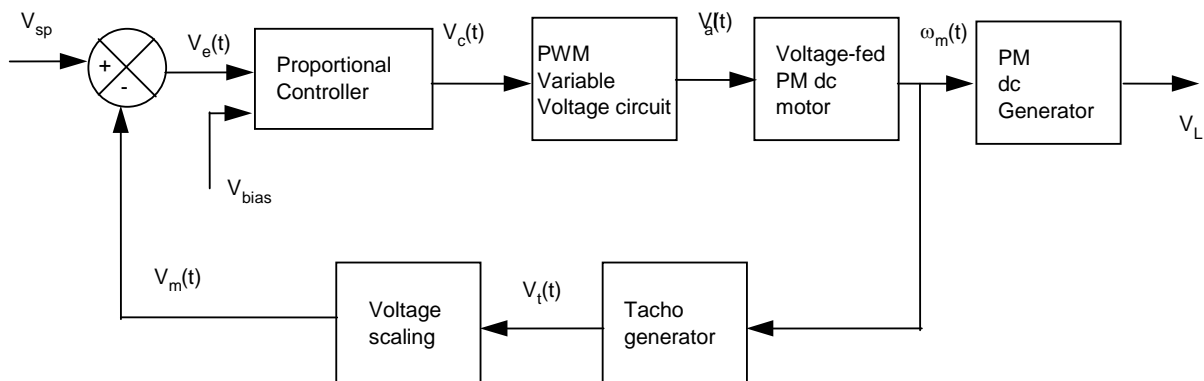


Figure 7. Block Diagram of the Motor-Generator Control System.

Test the combination of the PWM driver and motor-generator-tachometer system by applying different values of input voltage, V_c , to the comparator U4 using the

- potentiometer R12. Record the values of motor armature current I_a , tachometer output voltage V_t , Q1 drain-to-source voltage, V_{DS} and the generator output voltage. Make the measurements for the following values of V_c shown in Table 1-A. Determine the value of V_c that just causes the armature to turn (between 160 and 180 mA of armature current) when there is no-load connected to the generator. Record this data in Table 1-B.
- 3.) Modify the difference amplifier and proportional controller from the Experiment 2 so that a bias voltage produces a V_c that just causes the motor armature to spin. The feedback signal to the difference amplifier and the set-point voltage signals should be disconnected and both inputs to the difference amplifier grounded when testing this circuit modification. Grounding the inputs is equivalent of having zero error.
 - 4.) Unground the set-point input to the difference amplifier and connect it to the wiper arm of a potentiometer that gives 0-5 Vdc output. With the motor armature current set to the value found in step 3 by the proportional controller bias, Adjust the output of the set-point potentiometer and measure the voltage output of the tachometer. Adjust the set-point value until the tachometer output reads 12.06 Vdc. Using a voltage divider circuit and an OP AMP voltage follower circuit, scale the tachometer output voltage to be equal to 2.5 Vdc.
 - 5.) De-energize the whole system and set the value of K_p to 2. Disconnect the inputs of the difference amplifier from ground and connect them to the set-point value and the feedback signal supplied from the scaling circuit. Set the set-point voltage to 2.5 Vdc and energize the whole system. Measure the motor armature current I_a , the controller output V_c , the feedback signal voltage, V_m , the error voltage V_e , and the tachometer output voltage V_t . For the set-point voltages shown in Table 2-A Estimate the motor speed, n_m , by dividing V_t by the tachometer constant, $K_e=0.0067$ V/RPM, and also record this in Table 2-A.
 - 6.) Set the set-point value to 2.5 V. Connect a 1000 ohm 1/2 watt resistor across the generator terminals with the proportional controller in operation. Measure I_a , V_c , V_m , V_e and V_t again. Record this data in Table 2-B. Parallel 1000 ohm resistors to get the remaining loads in Table 2-B and record the results for each value of R_L .
 - 7.) Repeat steps 5 and 6 with values of K_p set to 10 and 50. Enter the data for $K_p=10$ into Tables 3-A and 3-B. For $K_p=50$ enter the data into Tables 4-A and 4-B.

Include all the data collected from the tests on the system in the report. Also derive a closed loop transfer function for the system with the set-point voltage as the input and the generator terminal as the output using $K_p=2$ and $R_L=1000$ ohms. The block diagrams for this system and the derived equations are in Appendix A of this handout.

Using the data in Tables 2-A, 3-A, 4-A, produce three plots of the set-point voltage, V_{sp} , (x) vs. the error voltage, V_e , (y). In the laboratory report, comment on how increasing the proportional gain affects the speed error. Also, produce three plots using the data in Tables 2-B, 3-B, 4-B of the generator load resistance, R_L , (x) vs. the motor speed, n_m , (y). Discuss how the motors speed is affected by the load change with the controller in operation. Compare the performance with an uncontrolled system.

Design Project II – Proportional-Integral Control of a Dc Motor-Generator System

Proportional control gives fast response but requires high gains to achieve small values of steady-state error. Applications that need more precise control can employ a proportional-integral (PI) controller. Figure 7 shows an OP AMP implementation of a PI controller.

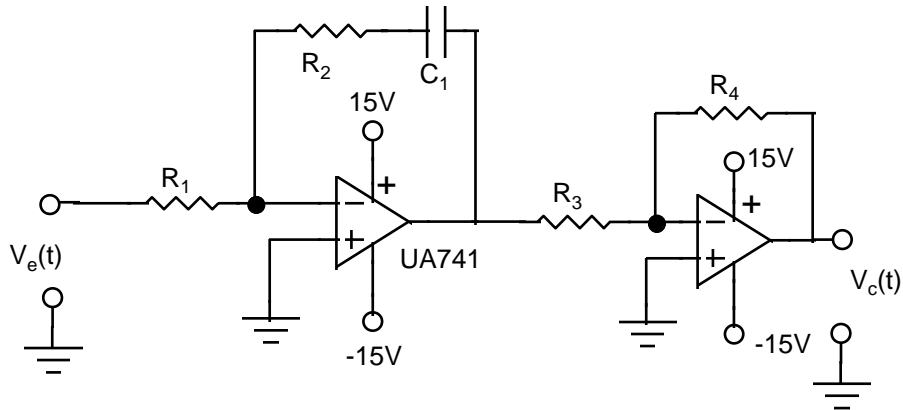


Figure 8. Proportional-Integral Controller Using a Single OP AMP

The transfer function of this controller is:

$$\frac{V_c(s)}{V_e(s)} = \left[\frac{R_2}{R_1} + \frac{1}{R_2 C_1 s} \right] \quad (19)$$

Equation (15) assumes that $R_3=R_4$. The ratio of R_2 and R_1 sets the proportional gain and the capacitor and R_2 set the integral action rate. The reciprocal of the integral action rate is the time required for the integral mode to match the change in output produced by the proportional mode. The proportional gain is defined in terms of the OP AMP parameters as

$$K_p = \frac{R_2}{R_1} \quad (20)$$

$$K_i = \frac{1}{R_2 C_1} \quad (21)$$

Disconnect the proportional controller from the feedback loop and substitute a PI controller that has the following parameters: $K_p=2$ and $K_i=213 \text{ sec}^{-1}$. Let $C_1=0.01\mu\text{F}$ for this calculation and find values for R_1 , R_2 , R_3 , and R_4 . Figure 8 shows a block diagram of the motor-generator system with the PI controller added.

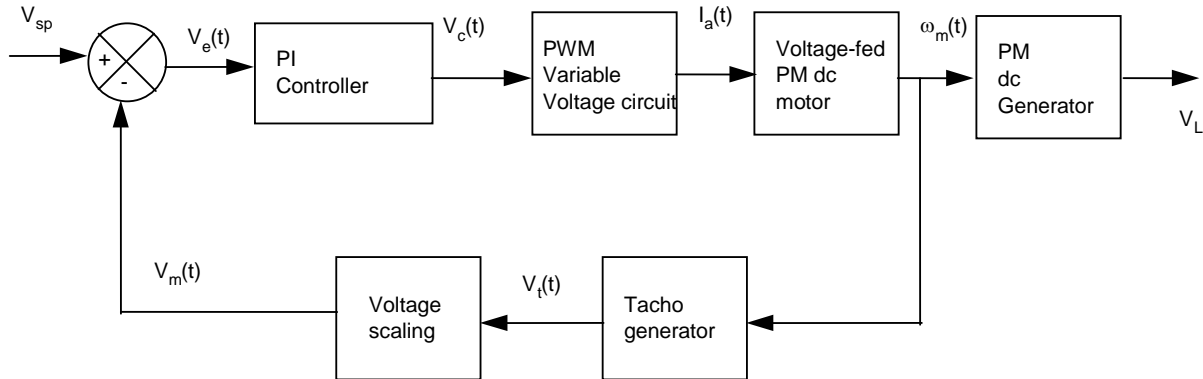


Figure 9. Motor Control Using Proportional Integral Controller.

- 1.) With the controller design from above installed in the control loop, vary the set-point voltage over the range shown in Table 5-A, and make all the necessary measurements to fill the table.
- 2.) Adjust the set-point to 2.5 V dc and add the generator load resistors listed in Table 5-B. Make all necessary measurements to fill the table.
- 3.) Set $K_p=10$ and maintain $K_I=213 \text{ sec}^{-1}$. Vary the set-point with no load resistor connected and record the data in Table 6-A.
- 4.) Adjust the set-point to 2.5 V dc and add the generator load resistors listed in Table 6-B. Make all necessary measurements to fill the table.

Using the data in Tables 5-A and 6-A, produce two plots of the set-point voltage, V_{sp} , (x) vs. the error voltage, V_e , (y). In the laboratory report, comment on how increasing the proportional gain affects the speed error and how the integral action changes the system performance. Also, produce two plots using the data in Tables 6-B, 6-B, of the generator load resistance, R_L , (x) vs. the motor speed, n_m , (y). Discuss how the motors speed is affected by the load change with the PI controller in operation. Compare the performance with an uncontrolled system and the proportional only controller.

Appendix A Block Diagrams for Lab 5

The block diagram below describes the signal flows of torque-driven dc generator used in the lab.

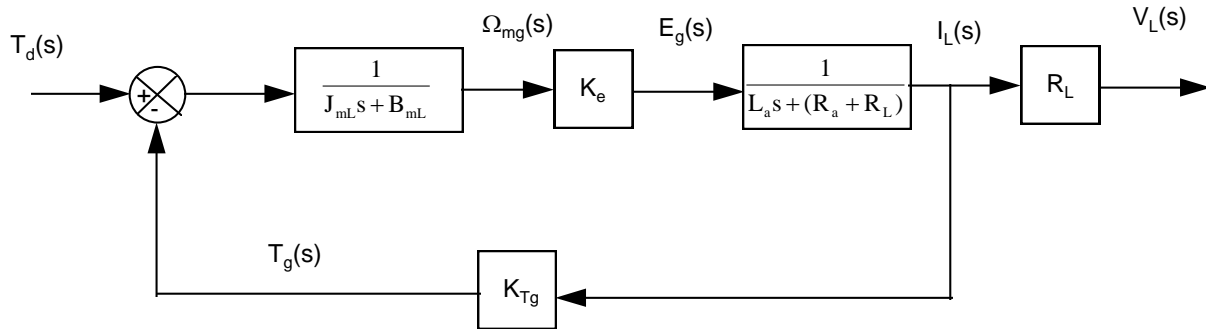


Figure 10. Block diagram of Generator.

The parameters have the following definitions:

- $T_d(s)$ = mechanical torque developed by the prime mover
- $T_g(s)$ = counter torque developed by the generator
- J_{mL} = the rotational inertia of the motor and generator
- B_{mL} = the viscous friction of the motor and generator
- K_e = generator emf constant
- L_a = generator armature inductance
- R_a = generator armature resistance
- R_L = load resistance
- K_{Tg} = generator torque constant
- $\Omega_{mg}(s)$ = shaft speed of motor-generator
- $I_L(s)$ = generator load current
- $V_L(s)$ = generator terminal voltage

When the speed feedback control is added the following block diagram results:

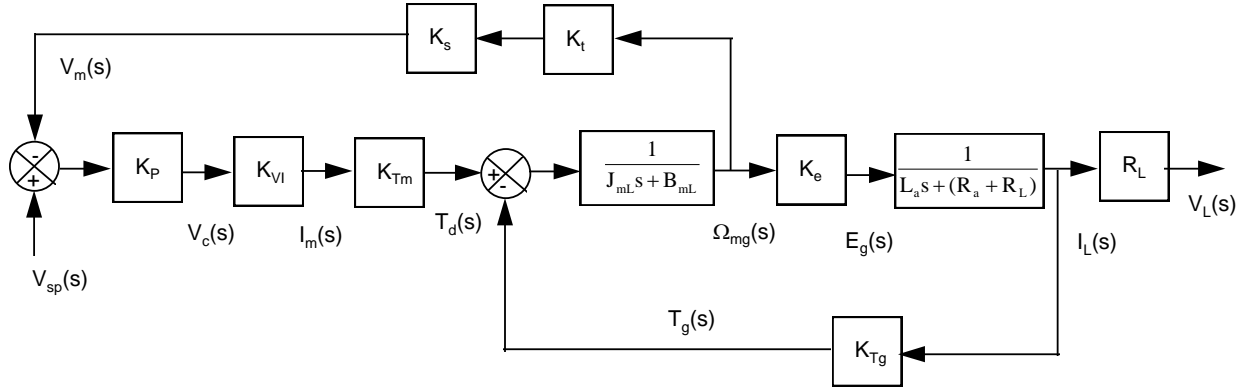


Figure 11. Overall system block diagram.

The parameters and signals are defined below.

- $V_{sp}(s)$ = system control setpoint value
- $V_c(s)$ = controller output voltage
- $I_m(s)$ = motor armature current
- K_t = tachometer voltage constant
- K_s = voltage scaling circuit constant
- K_{VI} = voltage-to-current converter gain (h_{FE}/R_2)
- K_{TM} = motor torque constant
- K_P = controller proportional gain

The overall transfer function is found using the formula below:

$$\frac{V_L(s)}{V_{sp}(s)} = \frac{G_T(s)}{1 + G_1(s)G_2(s)H_1(s) - G_2(s)G_3(s)H_2(s)} \quad (22)$$

With:

$$G_1(s) = K_P K_{VI} K_{TM} \quad G_2(s) = \frac{1}{J_{mL}s + B_{mL}} \quad G_3(s) = \frac{1}{L_a s + (R_a + R_L)}$$

$$H_1(s) = K_s K_t \quad H_2(s) = K_{Tg}$$

$$G_T(s) = \frac{K_P K_{VI} K_{TM} K_e R_L}{(L_a s + (R_a + R_L))(J_{mL}s + B_{mL})}$$

Table 1-A

V_c (Vdc)	V_{DS} (Vdc)	V_t (Vdc)	I_a (mA)
0.0			
1.0			
1.5			
2.0			
2.5			
3.0			
3.5			
4.0			
5.0			
6.0			

Table 1-B Starting Current Values

I_a (mA)	V_{CE2} (Vdc)	V_t (Vdc)	V_c (Vdc)

Table 2-A

 $K_p = 2, R_L = \text{infinity}$

V_{sp} (Vdc)	V_t (Vdc)	V_m (Vdc)	V_e (Vdc)	V_c (Vdc)	n_m (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 2-B

 $K_p = 2, V_{sp} = 2.5$ Vdc

R_L (Ω)	I_a (mA)	V_t (Vdc)	V_m (Vdc)	V_c (Vdc)	V_L (Vdc)	n_m (RPM)
1000						
500						
250						
125						

Table 3-A
 $K_p = 10, R_L = \text{infinity}$

V_{sp} (Vdc)	V_t (Vdc)	V_m (Vdc)	V_e (Vdc)	V_c (Vdc)	n_m (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 3-B
 $K_p = 10, V_{sp} = 2.5 \text{ Vdc}$

R_L (Ω)	I_a (mA)	V_t (Vdc)	V_m (Vdc)	V_c (Vdc)	V_L (Vdc)	n_m (RPM)
1000						
500						
250						
125						

Table 4-A
 $K_p = 50, R_L = \text{infinity}$

V_{sp} (Vdc)	V_t (Vdc)	V_m (Vdc)	V_e (Vdc)	V_c (Vdc)	n_m (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 4-B
 $K_p = 50, V_{sp} = 2.5 \text{ Vdc}$

R_L (Ω)	I_a (mA)	V_t (Vdc)	V_m (Vdc)	V_c (Vdc)	V_L (Vdc)	n_m (RPM)
1000						
500						
250						
125						

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Table 5-A
 $K_p = 2, K_I = 213, R_L = \text{infinity}$

V_{sp} (Vdc)	V_t (Vdc)	V_m (Vdc)	V_e (Vdc)	V_C (Vdc)	n_m (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 5-B
 $K_p = 2, K_I = 213, V_{sp} = 2.5 \text{ Vdc}$

R_L (Ω)	I_a (mA)	V_t (Vdc)	V_m (Vdc)	V_C (Vdc)	V_L (Vdc)	n_m (RPM)
1000						
500						
250						
125						

Table 6-A
 $K_p = 10, K_I = 213, R_L = \text{infinity}$

V_{sp} (Vdc)	V_t (Vdc)	V_m (Vdc)	V_e (Vdc)	V_C (Vdc)	n_m (RPM)
0.5					
1.0					
1.5					
2.0					
2.5					
3.0					
3.5					
4.0					
4.5					
5.0					

Table 6-B
 $K_p=10, K_i=213, V_{sp}=2.5 \text{ Vdc}$

$R_L (\Omega)$	$I_a \text{ (mA)}$	$V_t \text{ (Vdc)}$	$V_m \text{ (Vdc)}$	$V_c \text{ (Vdc)}$	$V_L \text{ (Vdc)}$	$n_m \text{ (RPM)}$
1000						
500						
250						
125						