1

INTRODUCTION TO PROCESS CONTROL

INSTRUCTIONAL OBJECTIVES

This chapter will consider the overall process-control loop, its function, and its description. After you have read this chapter, you should be able to

- Draw a block diagram of a process-control loop with a description of each element.
- List three typical controlled variables.
- Describe three criteria used to evaluate the response of a process-control loop.
- Define analog signal processing.
- Describe the two types of digital process control.
- Define accuracy, hysteresis, and sensitivity.
- List the SI units of measure for length, time, mass, and electric current.
- Convert a physical quantity from SI to English units and vice versa.
- Define the types of measurement time response.

1.1 INTRODUCTION

Human progress from a primitive state to our present complex, technological world is marked by learning new and improved methods to control the environment. Simply stated, the term *control* means methods to force parameters in the environment to have specific values. This can be as simple as making the temperature in a room stay at 21°C or as complex as manufacturing an integrated circuit or guiding a spacecraft to Jupiter. In general, all the elements necessary to accomplish the control objective are described by the term *control system*.

The purpose of this book is to examine the elements and methods of control system operation used in industry to control industrial processes (hence the term *process control*). This chapter will present an overall view of process-control technology and its elements, including important definitions. Later chapters will study the elements of process control in more detail.

1.2 CONTROL SYSTEMS

The basic strategy by which a control system operates is logical and natural. In fact, the same strategy is employed in living organisms to maintain temperature, fluid flow rate, and a host of other biological functions. This is natural process control.

The technology of artificial control was first developed using a human as an integral part of the control action. When we learned how to use machines, electronics, and computers to replace the human function, the term *automatic control* came into use.

1.2.1 Process-Control Principles

In process control, the basic objective is to regulate the value of some quantity. To regulate means to maintain that quantity at some desired value regardless of external influences. The desired value is called the *reference value* or *setpoint.*

The following paragraphs use the development of a control system for a specific process-control example to introduce some of the terms and expressions in the field.

The Process Figure 1.1 shows the process to be used for this discussion. Liquid is flowing into a tank at some rate Q_{in} and out of the tank at some rate Q_{out}. The liquid in the tank has some height or level h. It is known that the flow rate out varies as the square root of the height, so the higher the level the faster the liquid flows out. If the output flow rate is not exactly equal to the input flow rate, the tank will either empty, if $Q_{out} > Q_{in}$, or overflow, if $Q_{out} < Q_{in}$.

This process has a property called *self-regulation.* This means that for some input flow rate, the liquid height will rise until it reaches a height for which the output flow rate matches the input flow rate. A self-regulating system does not provide regulation of a variable to any particular reference value. In this example the liquid level will adopt some value for which input and output flow rates are the same and there it will stay. But if the input flow rate changed, then the level would change also, so it is not regulated to a reference value.

Suppose we want to maintain the level at some particular value H in Figure 1.1, regardless of the input flow rate. Then something more than self-regulation is needed.

Human-Aided Control Figure 1.2 shows a modification of the tank system to allow artificial regulation of the level by a human. To regulate the level so that it maintains the value H it will be necessary to employ a sensor to measure the level. This has been provided via a "sight tube" S as shown in Figure 1.2. The actual liquid level or height is called the *controlled variable.* In addition, a valve

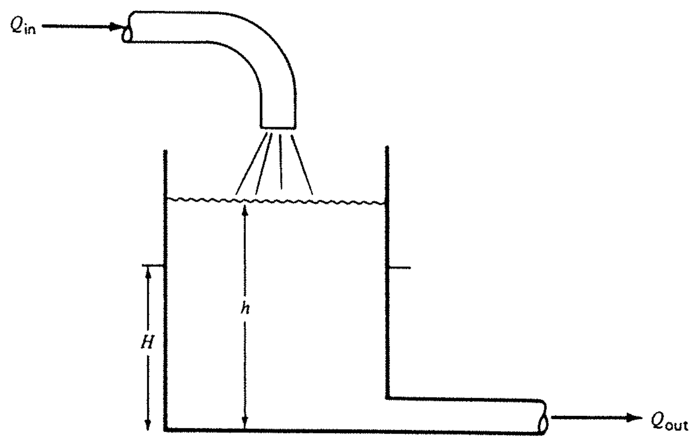


FIGURE 1.1
The objective is to regulate the level of liquid in the tank, h, to the value H.

has been added so the output flow rate can be changed by the human. The output flow rate is called the *manipulated variable* or *controlling variable.*

Now the height can be regulated apart from the input flow rate using the following strategy: The human measures the height in the sight tube and compares the value to the setpoint. If the measured value is larger, the human opens the valve a little to let the flow out increase, and thus the level lowers toward the setpoint. If the measured value is smaller than the setpoint, the human closes the valve a little to decrease the flow out and allow the level to rise toward the setpoint.

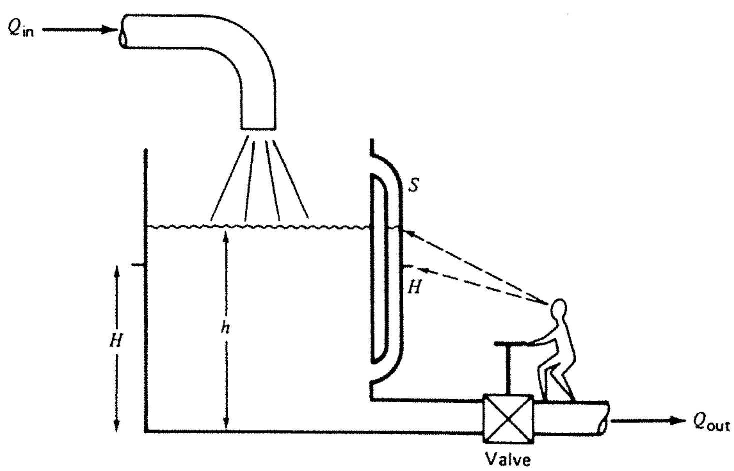


FIGURE 1.2
A human can regulate the level using a sight tube, S, to compare the level, h, to the objective, H, and adjust a valve to change the level.

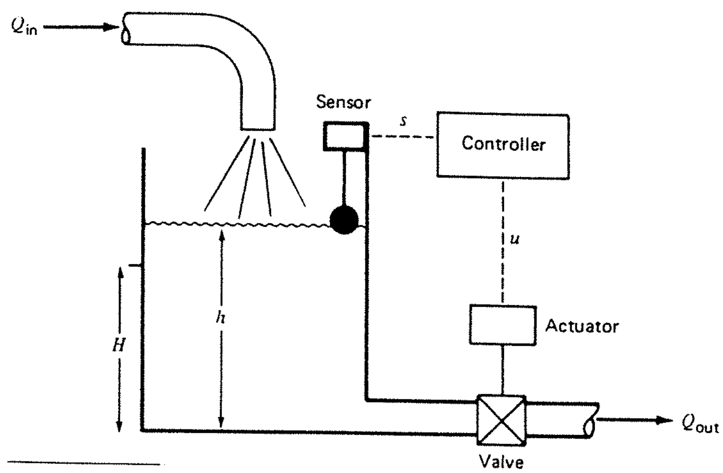


FIGURE 1.3
An automatic level-control system replaces the human by a controller and uses a sensor to measure the level.

By a succession of incremental opening and closing of the valve, the human can bring the level to the setpoint value H and maintain it there by continuous monitoring of the sight tube and adjustment of the valve. The height is regulated.

Automatic Control To provide automatic control, the system is modified as shown in Figure 1.3 so machines, electronics, or computers replace the operations of the human. An instrument called a *sensor* is added that is able to measure the value of the level and convert it into a proportional signal s. This signal is provided as input to a machine, electronic circuit, or computer, called the *controller.* This performs the function of the human in evaluating the measurement and providing an output signal u to change the valve setting via an *actuator* connected to the valve by a mechanical linkage.

When automatic control is applied to systems like the one shown in Figure 1.3, which are designed to regulate the value of some variable to a setpoint, it is called *process control.*

1.2.2 Servomechanisms

Another type of control system in common use, which has a slightly different objective from process control, is called a *servomechanism.* In this case the objective is to force some parameter to vary in a specific manner. This may be called a tracking control system. Instead of regulating a variable value to a setpoint, the servomechanism forces the controlled variable value to follow variation of the reference value.

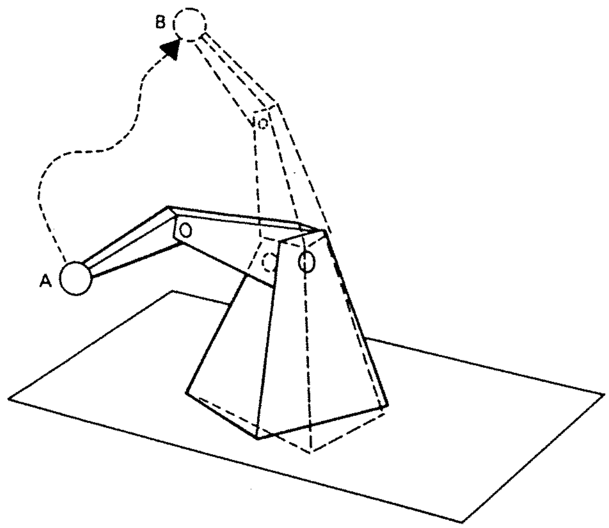


FIGURE 1.4
Servomechanism-type control systems are used to move a robot arm from point A to point B in a controlled fashion.

For example, in an industrial robot arm like the one shown in Figure 1.4, servomechanisms force the robot arm to follow a path from point A to point B. This is done by controlling the speed of motors driving the arm and the angles of the arm parts.

The strategy for servomechanisms is similar to process-control systems, but the dynamic differences between regulation and tracking result in differences in design and operation of the control system. This text is directed toward process-control technology.

1.3 PROCESS CONTROL BLOCK DIAGRAM

To provide a practical, working description of process control, it is useful to describe the elements and operations involved in more generic terms. Such a description should be independent of a particular application (such as the example presented in the previous section) and thus be applicable to *all* control situations. A model may be constructed using blocks to represent each distinctive element. The characteristics of control operation then may be developed from a consideration of the properties and interfacing of these elements. Numerous models have been employed in the history of process-control description; we will use one that seems most appropriate for a description of modern and developing technology of process control.

1.3.1 Identification of Elements

The elements of a process-control system are defined in terms of separate functional parts of the system. The following paragraphs define the basic elements of a process-control system and relate them to the example presented in Section 1.2.

Process In the previous example the flow of liquid in and out of the tank, the tank itself, and the liquid all constitute a process to be placed under control with respect to the fluid level. In general, a process can consist of a complex assembly of phenomena that relate to some manufacturing sequence. Many variables may be involved in such a process, and it may be desirable to control all these variables at the same time. There are *single-variable* processes, in which only one variable is to be controlled, as well as *multivariable* processes, in which many variables, perhaps interrelated, may require regulation.

Measurement Clearly, to effect control of a variable in a process, we must have information on the variable itself. Such information is found by measuring the variable. In general, a *measurement* refers to the conversion of the variable into some corresponding *analog* of the variable, such as a pneumatic pressure, an electrical voltage, or current. A sensor is a device that performs the initial measurement and energy conversion of a variable into analogous electrical or pneumatic information. Further transformation or *signal conditioning* may be required to complete the measurement function. The result of the measurement is a transformation of the variable into some proportional information in a useful form required by the other elements in the process-control operation.

In the system shown in Figure 1.3, the controlled variable is the level of liquid in the tank. The measurement is performed by some sensor which provides a signal s to the controller. In the case of Figure 1.2, the sensor is the sight tube showing the level to the human operator as an actual level in the tank.

The sensor is also called a *transducer*. The word *sensor* is preferred for the initial measurement device, however, because "transducer" represents a device that converts any signal from one form to another. Thus, for example, a device that converts a voltage into a proportional current would be a transducer. In other words, all sensors are transducers, but not all transducers are sensors.

Error Detector In Figure 1.2, the human looked at the difference between the actual level h and the setpoint level H and deduced an error. This error has both a magnitude and polarity. For the automatic control system of Figure 1.3, this same kind of error determination must be made before any control action can be taken by the controller. Although the error detector is often a part of the controller device, it is important to keep a clear distinction between the two.

Controller The next step in the process-control sequence is to examine the error and determine what action, if any, should be taken. This part of the control system has many names; however, *controller* is the most common. The evaluation may be performed by an operator (as in the previous example), by electronic signal processing, by pneumatic signal processing, or by a computer. Computer use is growing rapidly in the field of process control because computers are easily adapted to the decision-making operations and because of their inherent capacity to handle control of multivariable systems. The controller requires an input of both a *measured indication* of the controlled variable and a representation of the *reference value* of the variable, expressed in the same terms as the measured value. The reference value of the variable, you will recall, is referred to as the setpoint. Evaluation consists of determining action required to bring the controlled variable to the setpoint value.

Control Element The final element in the process-control operation is the device that exerts a direct influence on the process; that is, it provides those required changes in the controlled variable to bring it to the setpoint. This element accepts an input from the controller, which is then transformed into some proportional operation performed on the process. In our previous example, the control element is the valve that adjusts the outflow of fluid from the tank. This element is also referred to as the *final control element.*

1.3.2 Block Diagram

Figure 1.5 shows a block diagram constructed from the elements defined previously. The controlled variable in the process is denoted by c in this diagram, and the measured representation of the controlled variable is labeled b. The controlled variable setpoint is labeled r, for reference.

The error detector is a *subtracting-summing point* that outputs an *error signal* $e = r - b$ to the controller for comparison and action.

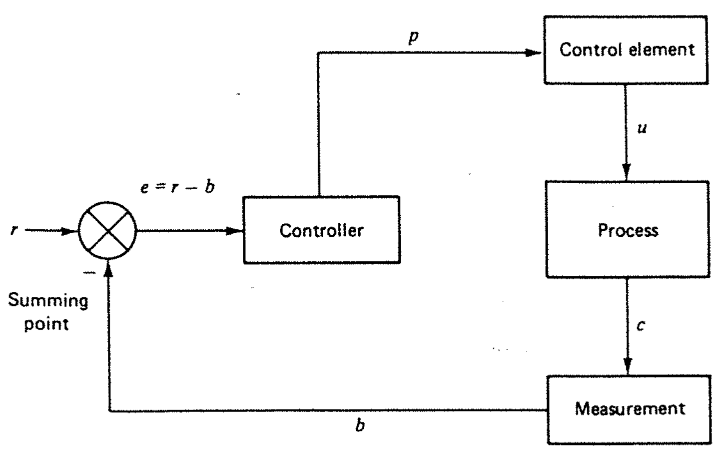
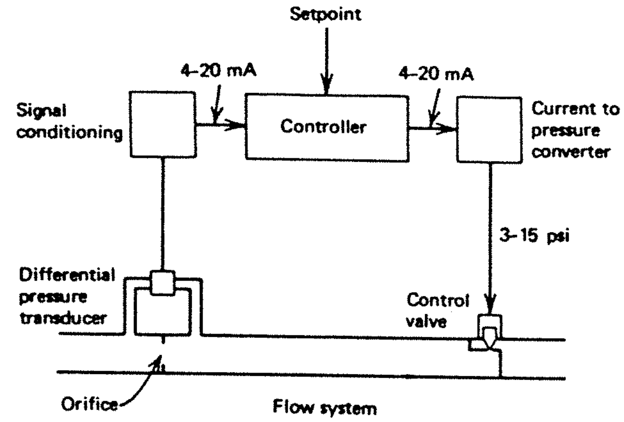
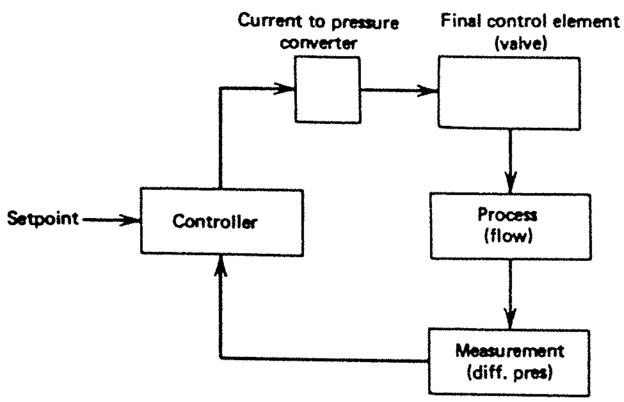


FIGURE 1.5
This block diagram of a control loop defines all the basic elements and signals involved.



a) Physical diagram of a process-control loop



b) Block diagram of a process-control loop

FIGURE 1.6
The physical diagram of a control loop and its corresponding block diagram look similar. Note the use of current- and pressure-transmission signals.

To illustrate further, Figure 1.6 shows a typical flow-control system and its representation by a block diagram. In this example, the controlled variable is the flow rate that is converted to electric current as an analog.

The purpose of a block diagram approach is to allow the process-control system to be analyzed as the interaction of smaller and simpler subsystems. If the characteristics of each element of the system can be determined, then the characteristics of the assembled system can be established by an analytical marriage of these subsystems. The historical development of the system approach in technology was dictated by this practical aspect: first, to specify the characteristics desired of a total system, and then, to delegate the development of subsystems that provide the overall criteria.

It becomes evident that the specification of a process-control system to regulate a variable c, within specified limits and with specified time responses, determines the characteristics the measurement system must possess. This same set of system specifications is reflected in the design of the controller and control element.

From this concept, we conclude that the analysis of a process-control system requires an understanding of the overall system behavior and the reflection of this behavior in the properties of the system elements. Most people find that an understanding of the parts leads to a better understanding of the whole. We will proceed with this assumption as a guiding concept.

The Loop Notice in Figure 1.5 that the signal flow forms a complete circuit from process through measurement, error detector, controller, and final control element. This is called a *loop*, and in general we speak of a process-control loop. In most cases this is called a *feedback loop*, because we determine an error and feed back a correction to the process.

1.4 CONTROL SYSTEM EVALUATION

A process-control system is used to regulate the value of some process variable. When such a system is in use, it is natural to ask, "How well is it working?" This is not an easy question to answer, because it is possible to adjust a control system to provide different kinds of response to errors. This section discusses some methods for evaluating how well the system is working.

The variable used to measure the performance of the control system is the error, which is the difference between the constant setpoint or reference value r and the controlled variable $c(t)$.

$$e(t) = r - c(t) \tag{1.1}$$

Since the value of the controlled variable may vary in time, so may the error. (Note that in a servomechanism, the value of r may be forced to vary in time also.)

Control System Objective In principle, the objective of a control system is to make the error in Equation (1.1) exactly zero. but the control system only responds to errors (i.e., when an error occurs, the control system takes action to drive it to zero). Conversely, if the error were zero and stayed zero, the control system would be doing nothing and was not neeeded in the first place. Therefore, this objective can never be perfectly achieved, and there will always be some error. The question of evaluation becomes one of how large the error is and how it varies in time.

A more practical statement of the objective is that the control system should (1) be stable and (2) provide the best possible regulation of the controlled variable. Let's consider what this objective means.

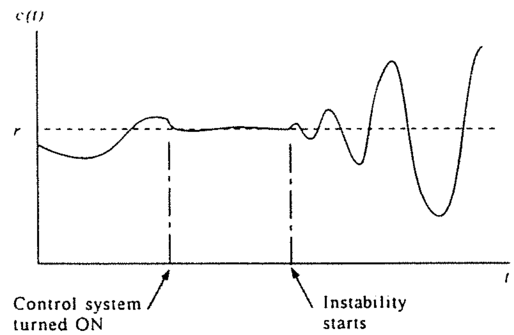


FIGURE 1.7
A control system can actually cause a system to become unstable.

1.4.1 Stability

The purpose of the control system is to regulate the value of some variable. This requires that action be taken on the process itself in response to a measurement of the variable. If this is not done correctly, the control system can cause the process to become unstable. In fact, the more tightly we try to control the variable, the greater the possibility of an instability.

Figure 1.7 shows that prior to turning on a control system the controlled variable drifted in a random fashion and was not regulated. After the control system was turned on, the variable was forced to adopt the setpoint value and all was well for awhile. Notice that some time later, however, the variable begins to exhibit growing oscillations of value, that is, an instability. This occurs even though the control system is still connected and operational; in fact, it occurs *because* the system is connected and operational.

The first objective, then, simply means that the control system must be designed and adjusted so the system is stable. Typically, as the control system is adjusted to give better control, the likelihood of instability also increases.

1.4.2 Steady-State Regulation

The objective of the best possible regulation simply means that the steady-state error should be a minimum. Generally, when a control system is specified there will be some allowable deviation, $\pm\Delta c$, about the setpoint. This means that variations of the variable within this band are expected and acceptable. External influences that tend to cause drifts of the value beyond the allowable deviation are corrected by the control system.

For example, a process-control technologist might be asked to design and implement a control system to regulate temperature at 150°C within ±2°C. This means the setpoint is to be 150°C, but the temperature may be allowed to vary in the range of 148 to 152°C.

1.4.3 Transient Regulation

What happens to the value of the controlled variable when some sudden event occurs that would otherwise cause a large variation? For example, there could be a setpoint change. Suppose the setpoint in the aforementioned temperature case were suddenly changed to 160°C. Transient regulation specifies how the control system reacts to bring the temperature to this new setpoint.

Another type of transient influence is a sudden change of some other process variable. The controlled variable depends on other process variables. If one of them suddenly changes value, the controlled variable may be driven to change also, so the control system acts to minimize the effect. This is called *transient response.*

1.4.4 Evaluation Criteria

The question of how well the control system is working is thus answered by (1) ensuring stability, (2) satisfying steady-state requirements, and (3) evaluating the response to setpoint changes and transient effects against certain standard criteria. There are many criteria for gauging the response. In general, the term *tuning* is used to indicate how a process-control loop is adjusted to provide the best control. This topic is covered in more detail in Chapter 12.

Damped Response One type of criterion requires that the controlled variable exhibit a response such as that shown in Figure 1.8 for excitations of both setpoint changes and transient effects. Note that the error is of only one polarity (i.e., it never oscillates about the setpoint). For this case, measures of quality are the duration t_D of the excursion and, for the transient, the maximum error e_{max} for a given input. The duration is the time from exceeding the allowable error to regaining the allowable error.

Different tuning will provide different values of e_{max} and t_D for the same excitation. It is up to the process designers to decide whether the best control is larger duration with smaller peak error, or vice versa, or something in between.

Cyclic Response Another type of criterion applies to those cases in which the response to a setpoint change or transient is as shown in Figure 1.9. Note that the controlled variable oscillates about the setpoint. In this case, the parameters of interest are the maximum error e_{max} and the duration t_D, also called the settling time. The duration is measured from the time the allowable error is first exceeded to the time when it falls within the allowable error and stays.

The nature of the response is modified by adjusting the control loop parameters, which is called *tuning*. There may be large maximum error but short duration or long duration with small maximum error, and everything in between.

A number of standard cyclic tuning criteria are used. Two common types are minimum area and quarter amplitude. In *minimum area,* the tuning is adjusted until the net area under the error-time curve is a minimum, for the same degree

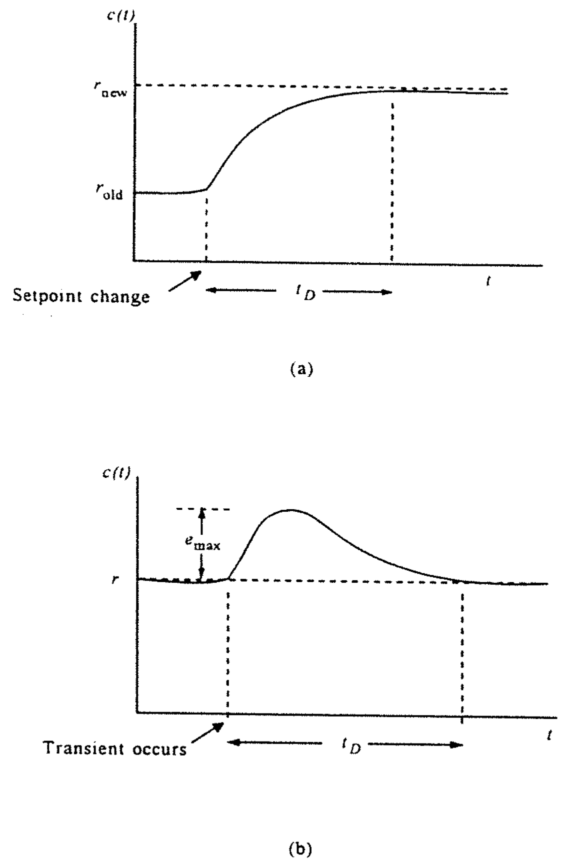


FIGURE 1.8
One of the measures of control system performance is how the system responds to changes of setpoint or a transient disturbance.

of excitation (setpoint change or transient). Figure 1.10a shows the area as a shaded part of the curve. Analytically, this is given by

$$A = \int |e(t)|\, dt = \text{minimum} \tag{1.2}$$

The *quarter-amplitude* criterion, shown in Figure 1.10b, specifies that the amplitude of each peak of the cyclic response must be a quarter of the preceding peak. Thus, $a_2 = a_1/4$, $a_3 = a_2/4$, and so on.

1.5 ANALOG AND DIGITAL PROCESSING

Until recently the functions of the controller in a control system were performed by sophisticated electronic circuits. Data were represented by the magnitude of voltages and currents in such systems. This is referred to as *analog processing.* Most modern control systems now employ digital computers to perform controller operations. In computers data are represented as binary numbers consisting of a

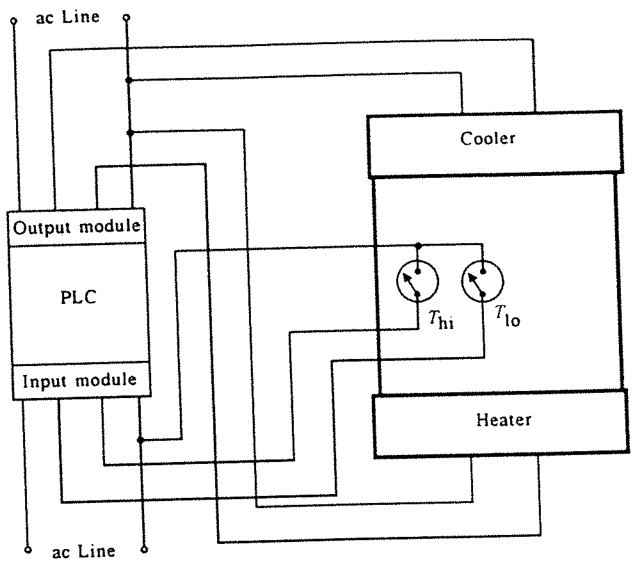


FIGURE 1.17
A programmable logic controller (PLC) is an outgrowth of ON/OFF-type control environments. In this case the heater and cooler are either ON or OFF.

are simply switches designed to open (or close) when the temperature reaches certain preset limits.

1.6 UNITS, STANDARDS, AND DEFINITIONS

As in any other technological discipline, the field of process control has many sets of units, standards, and definitions to describe its characteristics. Some of these are a result of historical use, some are for convenience, and some are just confusing. As the discipline grew, there were efforts to standardize terms so that professional workers in process control could effectively communicate among themselves and with specialists in other disciplines. In this section, we summarize the present state of affairs relative to the common units, standards, and definitions.

1.6.1 Units

To ensure precise technical communication among individuals employed in technological disciplines, it is essential to use a well-defined set of units of measurement. The metric system of units provides such communication and has been adopted by most technical disciplines. In process control, a particular set of metric units is used (which was developed by an international conference) called the International System (SI, Système International D'Unités). Because much technical work in the

United States is still done in the English system of units, it is necessary to perform transformations between these systems.

International System of Units The international system of units is maintained by an international agreement for worldwide standardization. The system is based on seven well-defined base units and two supplementary, dimensionless units. Everything else falls into the category of defined units, meaning defined in terms of the seven base and two supplementary units.

Quantity	Unit	Symbol
BASE		
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	A
Temperature	Kelvin	K
Amount of substance	Mole	mol
Luminous intensity	Candela	cd
SUPPLEMENTARY		
Plane angle	Radian	rad
Solid angle	Steradian	sr

All other SI units can be derived from these nine units, although in some cases a special name is assigned to the derived quantity. Thus, a force is measured by the newton (N), where 1 N = 1 kg m/s^2; energy is measured by the joule (J) or watt-second (W-s), given by 1 J = 1 kg m^2/s^2; and so on as shown in Appendix 1.

Other Units Although the SI system will be used in this text, other units remain in common use in some technical areas. The reader, therefore, should be able to identify and translate between the SI system and other systems. The centimeter-gram-second system (CGS) and English system also are given in Appendix 1. The following examples illustrate some typical translations of units.

EXAMPLE 1.1

Express a pressure of $p = 2.1 \times 10^3$ dyne/cm^2 in pascals. 1 Pa = 1 N/m^2.

Solution

From Appendix 1, we find 10^2 cm = 1 m and 10^5 dyne = 1 newton; thus,

$$p = (2.1 \times 10^3\,\text{dyne/cm}^2)\left(10^2\,\frac{\text{cm}}{\text{m}}\right)^2\left(\frac{1\ \text{N}}{10^5\,\text{dyne}}\right)$$

$$p = \mathbf{210}\ \text{pascals}$$

EXAMPLE 1.2
Find the number of feet in 5.7 m.

Solution
Reference to the table of conversions in Appendix 1 shows that 1 m = 39.37 in; therefore

$$(5.7\text{ m})\left(39.37\frac{\text{in}}{\text{m}}\right)\left(\frac{1\text{ ft}}{12\text{ in}}\right) = \mathbf{18.7}\text{ ft}$$

EXAMPLE 1.3
Express 6 ft in meters.

Solution
Using 39.37 in/m

$$(6\text{ ft})(12\text{ in/ft})\left(\frac{1\text{ m}}{39.37\text{ in}}\right) = \mathbf{1.829}\text{ m}$$

EXAMPLE 1.4
Find the mass in kg of a 2-lb object.

Solution
We first find the mass in slugs

$$m = \frac{2\text{ lb}}{32.17\text{ ft/s}^2}$$
$$m = 0.062\text{ slugs}$$

where

$$1\text{ slug} = 1\text{ lb ft/s}^2$$

Then, from Appendix 1, we have

$$m = (0.062\text{ slugs})\left(14.59\frac{\text{kg}}{\text{slug}}\right)$$
$$m = \mathbf{0.905}\text{ kg}$$

Metric Prefixes With the wide variation of variable magnitudes that occurs in industry, there is a need to abbreviate very large and small numbers. Scientific notation allows the expression of such numbers through powers of 10. A set of standard metric prefixes has been adopted by the SI to express these powers of 10, which are employed to simplfy the expression of very large or small numbers. These prefixes are given in Appendix 1.

EXAMPLE 1.5

Express 0.0000215 s and 3,781,000,000 W using decimal prefixes.

Solution

We first express quantities in scientific notation and then find the appropriate decimal prefix from Appendix 1.

$$0.0000215 \text{ s} = 21.5 \times 10^{-6} \text{ s} = \mathbf{215\ \mu s}$$

and

$$3{,}781{,}000{,}000 \text{ W} = 3.781 \times 10^{9} \text{ W} = \mathbf{3.781\ GW}$$

1.6.2 Analog Data Representation

For measurement systems or control systems, part of the specification is the range of the variables involved. Thus, if a system is to measure temperature, there will be a range of temperature specified, for example, 20°C to 120°C. Similarly, if the controller is to output a signal to a continuous valve, this signal will be designed to cover the range from fully closed to fully open, with all the various valve settings in between.

Two analog standards are in common use as a means of representing the range of variables in control systems. For electrical systems we use a range of electric current carried in wires, and for pneumatic systems we use a range of gas pressure carried in pipes. These signals are used primarily to transmit variable information over some distance, such as to and from the control room and the plant. Figure 1.18 shows a diagram of a process-control installation where current

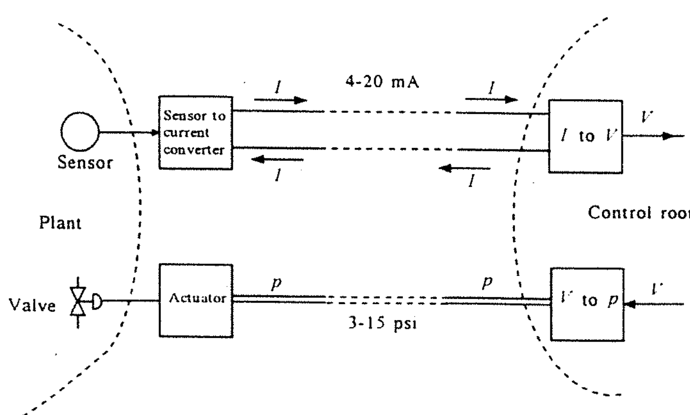


FIGURE 1.18
Electric current and pneumatic pressures are the most common means of information transmission in the industrial environment.

is used to transmit measurement data about the controlled variable to the control room, and gas pressure in pipes is used to transmit a feedback signal to a valve to change flow as the controlling variable.

Current Signal The most common current transmission signal is 4 to 20 mA. Thus, in the preceding temperature example, 20°C might be represented by 4 mA, and 120°C by 20 mA, with all temperatures in between represented by a proportional current.

EXAMPLE 1.6

Suppose the temperature range 20°C to 120°C is linearly represented by the standard current range of 4 to 20 mA. What current will result from 66°C? What temperature does 6.5 mA represent?

Solution

The easiest way to solve this kind of problem is to develop a linear equation between temperature and current. We can write this equation as $I = mT + I_0$ and we know from the given data that $I = 4$ mA when $T = 20°\text{C}$ and $I = 20$ mA when $T = 120°\text{C}$. Thus, we have two equations in two unknowns:

$$4 \text{ mA} = (20°\text{C})m + I_0$$
$$20 \text{ mA} = (120°\text{C})m + I_0$$

Subtracting the first from the second gives

$$16 \text{ mA} = (100°\text{C})m$$

so that $m = 0.16$ mA/°C. Then we find I_0:

$$I_0 = 4 \text{ mA} - (20°\text{C})\,(0.16 \text{ mA/°C})$$
$$I_0 = 0.8 \text{ mA}$$

Thus, the equation relating current and temperature is

$$I = (0.16 \text{ mA/°C})T + 0.8 \text{ mA}$$

Now answering the questions is easy. For 66°C we have

$$I = (0.16 \text{ mA/°C})66°\text{C} + 0.8 \text{ mA} = 11.36 \text{ mA}$$

For 6.5 mA we solve for T:

$$6.5 \text{ mA} = (0.16 \text{ mA/°C})T + 0.8 \text{ mA}$$

for which $T = 35.6°\text{C}$.

Current is used instead of voltage because the system is then less dependent on load. The sensor-to-current converter in Figure 1.18, also called a transmitter,

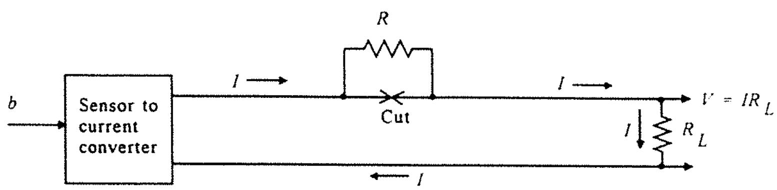


FIGURE 1.19
One of the advantages of current as a transmission signal is that it is independent of line resistance, to a point.

is designed to launch a current into the line regardless of load, to a degree. In Figure 1.19 a resistor R has been added to the lines connecting the plant to the control room. In the control room the incoming current has been converted to a voltage using a resistor R_L. Note that if the short around resistor R is cut so that R is now in the circuit, no change in current will occur. The transmitter is designed to adjust conditions (in this case output voltage) so that the current is held constant. Practically speaking, most current transmitters can work into any load from 0 to about 1000 Ω.

Voltage is not used for transmission because of its susceptibility to changes of resistance in the line.

Pneumatic Signals In the United States the most common standard for pneumatic signal transmission is 3 to 15 psi. In this case, when a sensor measures some variable in a range it is converted into a proportional pressure of gas in a pipe. The gas is usually dry air. The pipe may be many hundreds of meters long, but as long as there is no leak in the system the pressure will be propagated down the pipe. This English system standard is still widely used in the U.S., despite the move to the SI system of units. The equivalent SI range that will eventually be adopted is 20 to 100 kPa.

1.6.3 Definitions

This section presents definitions of some of the common terms and expressions used to describe process-control elements.

Error The most important quantity in control systems is the error. When used to describe the results of a measurement, error is the difference between the actual value of a variable and the measured indication of its value. Often, the actual value is not known. In that case, the *accuracy* of the measurement system places bounds on the possible error.

When used for a controlled variable in a control system, error is the difference between the actual value of the variable and the desired value, that is, the reference or setpoint value.

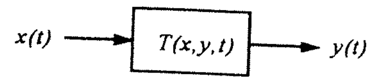


FIGURE 1.20
A transfer function shows how a system-block output variable varies in response to an input variable, as a function of both static input value and time.

Block Definitions As noted in Section 1.3.2, control systems are often described in terms of blocks. One block represents the measurement, another the controller, and so on. In order to work effectively in control systems one must understand the terms and expressions used to describe the characteristics of a block. Figure 1.20 shows a block that has an input of some variable $x(t)$ and an output of another variable $y(t)$. This model will be used in the following paragraphs to define the characteristics of the block.

Transfer Function The transfer function, $T(x, y, t)$ in Figure 1.20, describes the relationship between the input and output for the block. The transfer function is often described in two parts, the static part and the dynamic part. The static transfer function describes the input/output relationship when the input is not changing in time. The dynamic transfer function describes the input/output relationship when there is time variation of the input.

Static transfer functions may be presented in the form of equations, tables, or graphs. For example, a flow meter may relate flow, Q in gallons per minute, to a differential pressure Δp in psi, via an equation such as

$$Q = 119.5\sqrt{\Delta p}$$

On the other hand, an RTD temperature sensor usually is represented by a table of resistance versus temperature. Graphs are often used to display visually how input and output vary. Frequently, the transfer is valid only over a certain range of variable values.

The dynamic transfer function is often represented by a differential equation in time. Common examples of simple dynamic transfer functions are presented in Section 1.7.

Accuracy This term is used to specify the maximum overall error to be expected from a device, such as measurement of a variable. Accuracy usually is expressed as the *inaccuracy*, and can appear in several forms.

1. Measured variable; the accuracy is $\pm 2°C$ in some temperature measurement. Thus, there would be an uncertainty of $\pm 2°C$ in any value of temperature measured.
2. Percentage of the instrument full-scale (FS) reading. Thus, an accuracy of $\pm 0.5\%$ FS in a 5-volt full-scale range meter would mean the inaccuracy or uncertainty in any measurement is ± 0.025 volts.
3. Percentage of instrument span, that is, percentage of the range of instrument measurement capability. Thus, for a device measuring $\pm 3\%$ of span for a

20–50 psi range of pressure, the accuracy would be $(\pm 0.03)(50 - 20) = \pm 0.9$ psi.

4. Percentage of the actual reading. Thus, for a ±2% of reading voltmeter, we would have an inaccuracy of ±0.04 volts for a reading of 2 volts.

EXAMPLE 1.7

A temperature sensor has a span of 20°–250°C. A measurement results in a value of 55°C for the temperature. Specify the error if the accuracy is **(a)** ±0.5% FS, **(b)** ±0.75% of span, and **(c)** ±0.8% of reading. What is the possible temperature in each case?

Solution

Using the given definitions, we find

a. Error = (±0.005)(250°C) = **±1.25°C.** Thus, the actual temperature is in the range 53.75–56.25°C.

b. Error = (±0.0075)(250 − 20)°C = **±1.725°C.** Thus, the actual temperature is in the range 53.275–56.725°C.

c. Error = (±0.008)(55°C) = **±0.44°C.** Thus, the temperature is in the range 54.56°C–55.44°C.

EXAMPLE 1.8

A temperature sensor has a transfer function of 5 mV/°C with an accuracy of ±1%. Find the possible range of the transfer function.

Solution

The transfer function range will be (±0.01)(5 mV/°C) = ±0.05 mV/°C. Thus, the range is 4.95–5.05 mV/°C.

EXAMPLE 1.9

Suppose a reading of 27.5 mV results from the sensor used in Example 1.8. Find the temperature that could provide this reading.

Solution

Because the range of transfer function is 4.95–5.05 mV/°C, the possible temperature range that could be inferred from a reading of 27.5 mV is

$$(27.5\text{ mV})\left(\frac{1}{4.95\text{ mV/°C}}\right) = \mathbf{5.56°C}$$

$$(27.5\text{ mV})\left(\frac{1}{5.05\text{ mV/°C}}\right) = \mathbf{5.45°C}$$

Thus, we can be certain only that the temperature is between 5.45°C and 5.56°C.

The application of digital processing has necessitated an accuracy definition compatible with digital signals. In this regard we are most concerned with the error involved in the digital representation of analog information. Thus, the accuracy is quoted as the percentage deviation of the analog variable per bit of the digital signal. As an example, an A/D converter may be specified as 0.635 volts per bit $\pm 1\%$. This means that a bit will be set for an input voltage change of 0.635 ± 0.006 volts, or 0.629 to 6.41 V.

System Accuracy Often, one must consider the overall accuracy of *many* elements in a process-control loop to represent a process variable. Generally, the best way to do this is to express the accuracy of each element in terms of the transfer functions. For example, suppose we have a process with two transfer functions that act on the dynamic variable to produce an output voltage as shown in Figure 1.21. We can describe the output as

$$V \pm \Delta V = (K \pm \Delta K)(G \pm \Delta G)C \tag{1.3}$$

where V = output voltage
$\pm\Delta V$ = uncertainty in output voltage
K, G = nominal transfer functions
$\Delta K, \Delta G$ = uncertainties in transfer functions
C = dynamic variable

From Equation (1.3), we can find the output uncertainty to be

$$\Delta V = \pm GC\,\Delta K \pm KC\,\Delta G \pm \Delta K\,\Delta G C \tag{1.4}$$

Equation (1.4) can be further simplified by noting that the nominal output is $V = KGC$ and by ignoring second-order errors.

$$\frac{\Delta V}{V} = \pm\frac{\Delta K}{K} \pm \frac{\Delta G}{G} \tag{1.5}$$

where $\dfrac{\Delta V}{V}$ = fractional uncertainty in V

$\dfrac{\Delta K}{K}, \dfrac{\Delta G}{G}$ = fractional uncertainties in transfer functions

We can best interpret Equation (1.5) as stating that the worst-case accuracy would be the sum of the uncertainties of each transfer function.

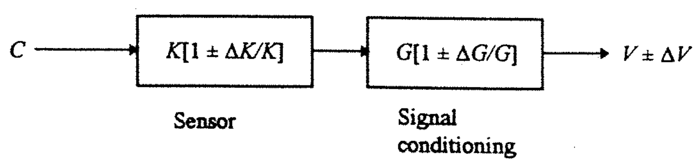


FIGURE 1.21
Uncertainties in block transfer functions build up as more blocks are involved in the transformation.

From a statistical point of view, it is more realistic to express the overall system accuracy as the root-mean-square (rms) of the individual element accuracy. This will give a system accuracy better than worst case but more likely to reflect the actual response. This is found from the relation

$$\left[\frac{\Delta V}{V}\right]_{\text{rms}} = \pm\sqrt{\left(\frac{\Delta K}{K}\right)^2 + \left(\frac{\Delta G}{G}\right)^2}$$

EXAMPLE 1.10

Find the system accuracy of a flow process if the transducer transfer function is 10 mV/(m³/s) ± 1.5% and the signal conditioning system transfer function is 2 mA/mV ± 0.05%.

Solution

Here we have a direct application of

$$\frac{\Delta V}{V} = \pm\left[\frac{\Delta K}{K} + \frac{\Delta G}{G}\right]$$

$$\frac{\Delta V}{V} = \pm[0.015 + 0.005]$$

$$\frac{\Delta V}{V} = \pm 0.02 = \pm 2\%$$

so that the net transfer function is 20 mA/(m³/s) ± 2%. If we use the more statistically correct rms approach, the system accuracy would be

$$\left[\frac{\Delta V}{V}\right]_{\text{rms}} = \pm\sqrt{(0.015)^2 + (0.005)^2}$$

$$= \pm 0.0158$$

So the accuracy is about ±1.6%.

Sensitivity Sensitivity is a measure of the change in output of an instrument for a change in input. Generally speaking, high sensitivity is desirable in an instrument because a large change in output for a small change in input implies that a measurement may be taken easily. Sensitivity must be evaluated together with other parameters, such as *linearity* of output to input, *range*, and *accuracy*. The value of the sensitivity is generally indicated by the transfer function. Thus, when a temperature transducer outputs 5 mV per degree Celsius, the sensitivity is 5 mV/°C.

Hysteresis and Reproducibility Frequently, an instrument will not have the same output value for a given input in repeated trials. Such variation can be due to inherent uncertainties that imply a limit on *reproducibility* of the device. This variation is random from measurement to measurement and is not predictable.

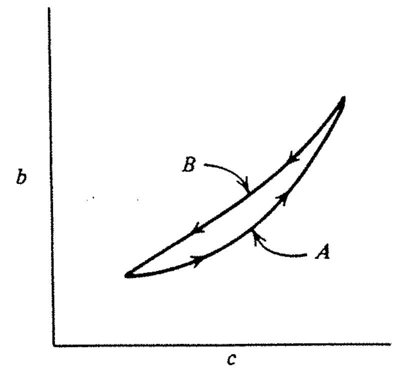


FIGURE 1.22
Hysteresis is a predictable error resulting from differences in the transfer function as the input variable increases or decreases.

A similar effect is related to the history of a particular measurement taken with an instrument. In this case, a different reading results for a specific input, depending on whether the input value is approached from higher or lower values. This effect, called *hysteresis*, is shown in Figure 1.22, where the output of an instrument has been plotted against input. We see that if the input parameter is varied from low to high, curve A gives values of the output. If the input parameter is decreasing, then curve B relates input to output. Hysteresis usually is specified as a percentage of full-scale maximum deviation between the two curves. This effect is predictable if measurement values are always approached from one direction, because hysteresis will not cause measurement errors.

Resolution Inherent in many measurement devices is a minimum measurable value of the input variable. Such a specification is called the *resolution* of the device. This characteristic of the instrument can be changed only by redesign. A good example is a wire-wound potentiometer where the slider moves across windings to vary resistance. If one turn of the winding represents a change of ΔR ohms, then the potentiometer cannot provide a resistance change *less* than ΔR. We say that the potentiometer resolution is ΔR. This is often expressed as a percentage of the full-scale range.

EXAMPLE 1.11
A force sensor measures a range of 0–150 N with a resolution of 0.1% FS. Find the smallest change in force that can be measured.

Solution
Because the resolution is 0.1% FS, we have a resolution of (0.001)(150 N) = 0.15 N, which is the smallest measurable change in force.

In some cases, the resolution of a measurement system is limited by the sensitivity of associated signal conditioning. When this occurs, the resolution can be improved by employing better conditioning.

EXAMPLE 1.12

A sensor has a transfer function of 5 mV/°C. Find the required voltage resolution of the signal conditioning if a temperature resolution of 0.2°C is required.

Solution

A temperature change of 0.2°C will result in a voltage change of

$$\left(5\,\frac{\text{mV}}{°\text{C}}\right)(0.2°\text{C}) = \mathbf{1.0}\ \text{mV}$$

Thus, the voltage system must be able to resolve 1.0 mV.

In analog systems the resolution of the system is usually determined by the smallest measurable change in the analog output signal of the measurement system. In digital systems, the resolution is a well-defined quantity that is simply the change in dynamic variable represented by a *one-bit change* in the binary word output. In these cases, resolution can be improved only by a different coding of the analog information or adding more bits to the word. (This will be discussed further in Section 3.4.)

Linearity In both sensor and signal conditioning, output is represented in some functional relationship to the input. The only stipulation is that this relationship be unique; that is, for each value of the input variable there exists one unique value of the output variable. For simplicity of design, a linear relationship between input and output is highly desirable. When a linear relationship exists, a straight-line equation can be used to relate the measured variable and measurement output.

$$c_m = mc + c_0 \tag{1.6}$$

where
c = variable to be measured
m = slope of straight line
c_0 = offset or intercept of straight line
c_m = output of measure

No simple relationship such as Equation (1.6) can usually be found for the nonlinear cases, although in some cases approximations of a linear or quadratic nature are fitted to portions of these curves, as will be shown in Chapter 4.

EXAMPLE 1.13

A sensor resistance changes linearly from 100 to 180 Ω as temperature changes from 20° to 120°C. Find a linear equation relating resistance and temperature.

Solution

Using Equation (1.6) as a guide, the desired equation would be of the form

$$R = mT + R_0$$

To find the two constants, m and R_0, we form two equations and two unknowns from the facts given.

$$100\ \Omega = (20°\text{C})m + R_0$$
$$180\ \Omega = (120°\text{C})m + R_0$$

Subtracting the first equation from the second gives

$$80\ \Omega = (100°\text{C})m \quad \text{or} \quad m = 0.8\ \Omega/°\text{C}$$

Then, from the first equation we find

$$100\ \Omega = (20°\text{C})(0.8\ \Omega/°\text{C}) + R_0$$

from which

$$R_0 = 84\ \Omega$$

The equation relating resistance and temperature is

$$R = 0.8T + 84$$

One of the specifications of sensor output is the degree to which it is linear with the measured variable and the span over which this occurs. A measure of sensor linearity is to determine the deviation of the sensor output from a best-fit straight line over a particular range. A common specification of linearity is the maximum deviation from a straight line expressed as percent of FS.

Consider a sensor that outputs a voltage as a function of pressure from 0 to 100 psi with a linearity of 5% FS. This means that at some point on the curve of voltage versus pressure, the deviation between actual pressure and linearly indicated pressure for a given voltage deviates by 5% of 100 psi, or 5 psi.

Figure 1.23 indicates this graphically. A straight line has been fitted to the slightly nonlinear sensor curve. One can either specify that for a given voltage there is a deviation between actual and linearly predicted pressure or that for a given pressure there is a deviation between actual and linearly predicted voltage.

1.6.4 Process-Control Drawings

A standard form and set of symbols are used to prepare drawings of process-control systems. These drawings are similar to a schematic of an electric circuit. In process control, the drawing is often referred to as a piping and instrumentation drawing (P&ID), although the symbols and diagram may be used in processes for which there is no piping. It is important to recognize the nature of the P&ID and the symbols. Appendix 5 presents a detailed account of the set of symbols and abbreviations. Here, we merely note the basic structure of such diagrams. Thus, Figure 1.24 shows some part of a typical P&ID. First, the P&ID contains a diagram of the entire process, that is, the reactants and products, as well as the instrumentation and signals that form the various process-control loops. Thus, someone with a good

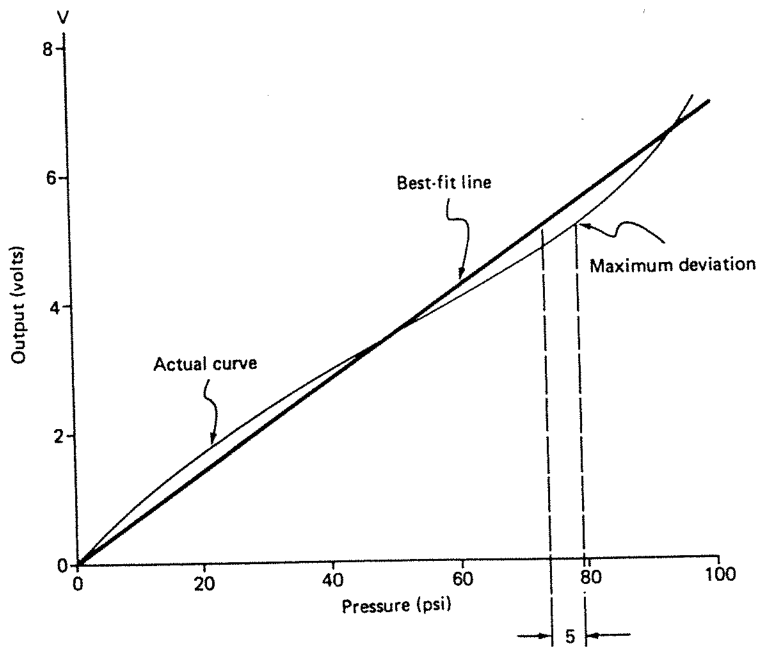


FIGURE 1.23
Comparison of an actual curve and its best-fit straight line, where the maximum deviation is 5% FS.

understanding of the P&ID will be able to see how the overall manufacturing sequence works, as well as being able to recognize the individual process-control loops. The actual process-flow lines, such as reactant flow and crackers on a conveyor, are shown as heavy, solid lines in the diagram.

Instrument Signal Lines As shown in Figure 1.24, the instrumentation signal lines are presented in a form that shows whether they are pneumatic or electric. A cross-hatched line is used to show all lines that are pneumatic, such as a 3–15 psi signal, for example. The common electric current line, such as 4–20 mA, is presented as a dashed line.

Instrumentation A balloon symbol with an enclosed two- or three-letter code is used to represent the instrumentation associated with the process-control loops. Thus, the balloon in Figure 1.24 with TT enclosed is a temperature sensor, and that with TC enclosed is the temperature controller associated with that loop. Special items such as control valves and in-line instruments (for example, flow meters) have special symbols, as shown in Figure 1.24 by the control valves and flow transducers.

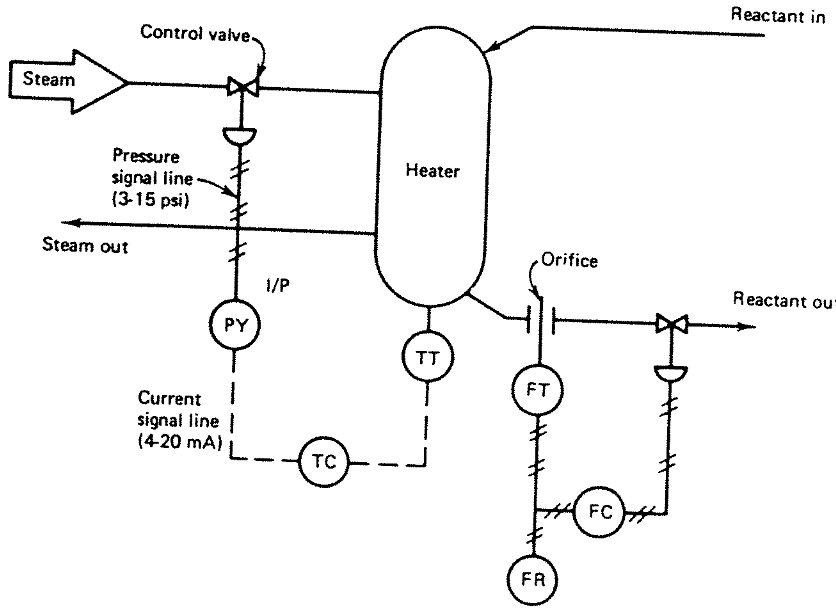


FIGURE 1.24
A P&ID uses special symbols and lines to show the devices and interconnections in a process-control system.

1.7 SENSOR TIME RESPONSE

The static transfer function of a process-control loop element specifies how the output is related to the input if the input is constant. An element also has a time dependence that specifies how the output changes in time when the input is changing in time. It is independent from the static transfer function. This dynamic transfer function is often simply called the *time response*. It is particularly important for sensors because they are the primary element for providing knowledge of the controlled variable value. This section will discuss the two most common types of sensor time responses.

Figure 1.25 shows a sensor that produces an output $b(t)$ as a function of the input $c(t)$. The static transfer function determines the output when the input is not changing in time. To specify the time response, the nature of the time variation in output, $b(t)$, is given when the input exhibits a step change as shown. Note that at $t = 0$ the input to the sensor is suddenly changed from an initial value c_i to a final value c_f. If the sensor were perfect, its output would be determined by the static transfer function to be b_i before $t = 0$ and b_f after $t = 0$. However, all sensors will exhibit some lag between the output and the input and some characteristic variation in time before settling on the final value.

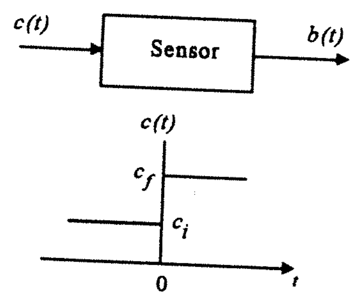


FIGURE 1.25
The dynamic transfer function specifies how a sensor output varies when the input changes instantaneously in time, i.e., a step change.

1.7.1 First-Order Response

The simplest time response is shown in Figure 1.26 as the output change in time following a step input as in Figure 1.25. This is called *first order* because for all sensors of this type the time response is determined by the solution of a first-order differential equation.

A general equation can be written for this response independent of the sensor, the variable being measured, or the static transfer function. The equation gives the sensor output as a function of time following the step input (i.e., it traces the curve of Figure 1.26 in time):

$$b(t) = b_i + (b_f - b_i)[1 - e^{-t/\tau}] \tag{1.7}$$

where b_i = initial sensor output from static transfer function and initial input
b_f = final sensor output from static transfer function and the final input
τ = sensor time constant

The sensor output is in error during the transition time of the output from b_i to b_f. The actual variable value was changed instantaneously to a new value at $t = 0$. Equation (1.7) describes transducer output very well except during the initial time period, that is, at the *start* of the response near $t = 0$. In particular, the actual transducer response generally starts the change with a *zero* slope, and Equation (1.7) predicts a finite starting slope.

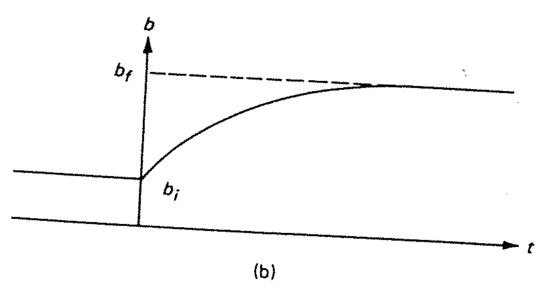


FIGURE 1.26
Characteristic first-order exponential time response of a sensor to a step change of input.

Time Constant The time constant, τ, is part of the specification of the sensor. Its significance can be seen by writing Equation (1.7) as follows:

$$b(t) - b_i = (b_f - b_i)[1 - e^{-t/\tau}] \tag{1.8}$$

In this equation the quantity on the left is the change in output as a function of time, whereas $(b_f - b_i)$ is the total change that will occur. Thus, the square-bracketed term in Equation (1.8) is the fraction of total change as a function of time.

Suppose we wish to find the change that has occurred at a time numerically equal to τ. Then we set $t = \tau$ in Equation (1.8) and find

$$\begin{aligned} b(\tau) - b_i &= (b_f - b_i)[1 - e^{-1}] \\ b(\tau) - b_i &= 0.6321(b_f - b_i) \end{aligned} \tag{1.9}$$

Thus, we see that one time constant represents the time at which the output value has changed by approximately 63% of the total change.

The time constant τ is sometimes referred to as the 63% time, the *response time,* or the e-folding time. For a step change, the output response has approximately reached its final value after five time constants, since, from Equation (1.8),

$$b(5\tau) - b_i = 0.993(b_f - b_i)$$

EXAMPLE 1.14

A sensor measures temperature linearly with a static transfer function of 33 mV/°C and has a 1.5-s time constant. Find the output 0.75 s after the input changes from 20°C to 41°C. Find the error in temperature this represents.

Solution

We first find the initial and final values of the sensor output.

$$\begin{aligned} b_i &= (33\ \text{mV/°C})(20\text{°C}) \\ b_i &= 660\ \text{mV} \\ b_f &= (33\ \text{mV/°C})(41\text{°C}) \\ b_f &= 1353\ \text{mV} \end{aligned}$$

Now,

$$\begin{aligned} b(t) &= b_i + (b_f - b_i)[1 - e^{-t/\tau}] \\ b(0.75) &= 660 + (1353 - 660)[1 - e^{-0.75/1.5}] \\ b(0.75) &= \mathbf{932.7\ mV} \end{aligned}$$

This corresponds to an indicated temperature of

$$T = \frac{932.7}{33\ \text{mV/°C}}$$

$$T = \mathbf{28.3°C}$$

so the error is 12.7°C.

In many cases, the transducer output may be inversely related to the input. Equation (1.7) still describes the time response of the element where the final output is less than the initial output.

Real-Time Effects The concept of the exponential time response and associated time constant is based on a sudden discontinuous change of the input value. In the real world such instantaneous changes occur rarely, if ever, and thus we have presented a worst-case situation in the time response. In general, a sensor should be able to track any changes in the physical dynamic variable in a time less than one time constant.

1.7.2 Second-Order Response

In some sensors, a step change in the input will cause the output to oscillate for a short period of time before settling down to a value that corresponds to the new input. Such oscillation (and the decay of the oscillation itself) is a function of the sensor. This output transient generated by the transducer is an error and must be accounted for in any measurement involving a transducer with this behavior.

This is called a *second-order response* because for this type of sensor the time behavior is described by a second-order differential equation. It is not possible to develop a universal solution, as it is for the first-order time response. Instead, we simply describe the general nature of the response.

Figure 1.27 shows a typical output curve that might be expected from a transducer having a second-order response for a discontinuous change in the input. It is impossible to describe this behavior by an analytic expression, as it is with the first-order response. However, the behavior can be described in time as

$$R(t) \propto R_0 e^{-at} \sin(2\pi f_n t) \tag{1.10}$$

where $R(t)$ = the transducer output
a = output damping constant
f_n = natural frequency of the oscillation
R_0 = amplitude

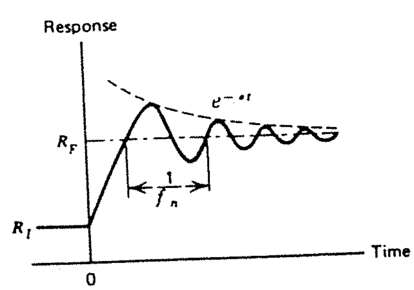


FIGURE 1.27
Characteristic second-order oscillatory time response of a sensor.

This equation shows the basic damped oscillation output of the device. The damping constant a and natural frequency f_n are characteristics of the transducer itself and must be considered in many applications.

In general, such a transducer can be said to track the input when the input changes in a time that is *greater* than the period represented by the natural frequency. The damping constant defines the time one must wait after a disturbance at $t = 0$ for the transducer output to be a true indication of the transducer input. Thus, we see that in time of $(1/a)$ the amplitude of the oscillations would be down to e^{-1}, or approximately 37%. More will be said of the effects of natural frequency and damping in the treatment of specific transducers that exhibit this behavior.

1.8 SIGNIFICANCE AND STATISTICS

Process control is vitally concerned with the value of variables, as the stated objective is to regulate the value of selected variables. It is therefore very important that the true significance of some measured value be understood. We have already seen that inherent errors may lend uncertainty to the value indicated by a measurement. In this section we need to consider another feature of measurement that may be misleading about the actual value of a variable, as well as a method to help interpret the significance of measurements.

1.8.1 Significant Figures

In any measurement we must be careful not to attach more significance to a variable value than the instrument can support. This is particularly true with the growing use of digital reading instruments and calculators with 8–12-digit readouts. Suppose, for example, that a digital instrument measures a resistance as 125 kΩ. Even if we ignore the instrument accuracy, this does not mean that the resistance is 125,000 Ω. Rather, it means that the resistance is closer to 125,000 than it is to 124,000 or 126,000 Ω. We can use the 125 kΩ number in subsequent calculations, but we cannot draw conclusions about results having more than three numbers, that is, three significant figures. The significant figures are the digits (places) actually read or known from a measurement or calculation.

Significance in Measurement When using a measuring instrument, the number of significant figures is indicated either by readability, in the case of analog instruments, or by the number of digits, in a digital instrument. This is not to be confused with accuracy, which supplies an uncertainty to the reading itself. The following example illustrates how significant figures in measurement and accuracy are treated in the same problem.

EXAMPLE 1.15

A digital multimeter measures the current through a 12.5-kΩ resistor as 2.21 mA, using the 10-mA scale. The instrument accuracy is ±0.2% FS. Find the voltage across the resistor and the uncertainty in the value obtained.

Solution
First, we note that current is given to three significant figures, so no result we find can be significant to more than three digits. Then we see that the given accuracy becomes an uncertainty in the current of ±0.02 mA. From Ohm's law we find the voltage as

$$V = IR = (2.21 \text{ mA})(12.5 \text{ k}\Omega) = 27.625 \text{ volts}$$

But in terms of significant figures we give this as $V = 27.6$ volts. The accuracy means the current could vary from 2.19 to 2.23 mA, which introduces an uncertainty of ±0.25 volts. Thus, the complete answer is 27.6 ± 0.3 volts, because we must express the uncertainty so that our significance is not changed.

Significance in Calculations In calculations one must be careful not to obtain a result that has more significance than the numbers employed in the calculation. The answer can have no more significance than the last of the numbers used in the calculation.

EXAMPLE 1.16
A transducer has a specified transfer function of 22.4 mV/°C for temperature measurement. The measured voltage is 412 mV. What is the temperature?

Solution
Using the values given, we find

$$T = (412 \text{ mV})/(22.4 \text{ mV/°C}) = 18.392857\text{°C}$$

This was found using an eight-digit calculator, but the two given values are significant to only three places. Our result can be significant to only three places; the answer is **18.4°C.**

Significance in Design The reader should be aware of the difference in significant figures associated with measurement and conclusions drawn from measurement and significant figures associated with design. A design is a hypothetical development that makes implicit assumptions about selected values in the design. If the designer specifies a 1.1-kΩ resistor, the assumption is that it is exactly 1100 Ω. If the designer specifies that there are 4.7 volts across the resistor, then there are exactly 4.7 volts and the current can be calculated as 4.2727272 mA. Now, suppose we measure the resistor when the design is built and find it to be 1.1 kΩ (two significant figures) and measure the voltage and find it to be 4.7 volts (two significant figures). In this case, we report the calculated current of 4.3 mA, because we are dealing with two significant figures.

In the examples and problems in this book, we will try to maintain the distinction between design values and measurement values. Whenever a problem or example involves design, perfect values are assumed. Thus, if a design specifies a 4.7-kΩ resistor, then it is assumed that the value is exactly 4.7 kΩ (i.e., 4700.00 Ω). Whenever measurement is suggested, the figures given are assumed to be the significant figures. If a problem specifies that the measured voltage is 5.0, it is assumed that it is known to only two significant figures.

1.8.2 Statistics

Often, confidence in the value of a variable can be improved by the use of elementary statistical analysis of measurements. This is particularly true where random errors in measurement cause a distribution of readings of the value of some variable.

Arithmetic Mean If many measurements of some variable are taken, the arithmetic mean is calculated to obtain an average value for the variable. There are many instances in process control when such an average value is of interest. For example, one may wish to control the average temperature in a process. The temperature might be measured in 10 locations and averaged to give a controlled variable value for use in the control loop.

Another common application is the calibration of transducers and other process instruments. In such cases, the average gives information about the transfer function. In digital or computer process control, it is often easier to use the average value of process variables. The arithmetic mean of a set of n values, given by x_1, $x_2, x_3, \cdots, x_n$ is defined by the equation

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \tag{1.11}$$

where $\bar{x}$ = arithmetic mean
n = number of values to be averaged
$x_1, x_2, \cdots, x_n$ = individual values

We often use the symbol Σ to represent a sum of numbers such as that used in Equation (1.11). Here we would write the equation as

$$\bar{x} = \frac{\Sigma x_i}{n} \tag{1.12}$$

where Σx_i = symbol for a sum of the values $x_1, x_2, \cdots, x_n$

Standard Deviation It often is insufficient to know the value of the arithmetic mean of a set of measurements. To interpret the measurements properly, it may be necessary to know something about how the individual values are spread out about the mean. Thus, although the mean of the set (50, 40, 30, 70) is 47.5 and the mean of the set (5, 150, 21, 14) is also 47.5, the second group of numbers is obviously far more spread out. The standard deviation is a measure of this spread. Given a set of n values $x_1, x_2, \cdots, x_n$, we first define a set of deviations by the difference between the individual values and the arithmetic mean of the values, $\bar{x}$. The deviations are

$$d_1 = x_1 - \bar{x}$$
$$d_2 = x_2 - \bar{x}$$

and so on, until

$$d_n = x_n - \bar{x}$$

The set of these n deviations is now used to define the standard deviation according to the equation

$$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \cdots + d_n^2}{n-1}} \tag{1.13}$$

or, using the summation symbol

$$\sigma = \sqrt{\frac{\Sigma d_i^2}{n-1}} \tag{1.14}$$

Of course, the larger the standard deviation, the more spread out the numbers from which it is calculated.

EXAMPLE 1.17

Temperature was measured in eight locations in a room, and the values obtained were 21.2°, 25.0°, 18.5°, 22.1°, 19.7°, 27.1°, 19.0°, and 20.0°C. Find the arithmetic mean of the temperature and the standard deviation.

Solution

Using Equation (1.11), we have

$$\bar{T} = \frac{21.2 + 25 + 18.5 + 22.1 + 19.7 + 27.1 + 19 + 20}{8}$$

$\bar{T}$ = **21.6°C** (remember significant figures)

The standard deviation is found from Equation (1.13):

$$\sigma = \sqrt{\frac{(21.2 - 21.6)^2 + (25 - 21.6)^2 + \cdots + (20 - 21.6)^2}{(8 - 1)}}$$

σ = **3.04°C**

Interpretation of Standard Deviation A more quantitative evaluation of spreading can be made if we make certain assumptions about the set of data values used. In particular, we assume that the errors are truly random and that we have taken a large sample of readings. We then can claim that the standard deviation and data are related to a special curve called the *normal probability curve* or *bell curve*. If this is true, then

1. 68% of all readings lie within $\pm 1\sigma$ of the mean.
2. 95.5% of all readings lie within $\pm 2\sigma$ of the mean.
3. 99.7% of all readings lie within $\pm 3\sigma$ of the mean.

values. Figure 1.5 shows a block diagram in which the elements of measurement, error detector, controller, and control element are connected to provide the required regulation.

2. Numerous criteria have been discussed that allow the evaluation of process-control loop performance, of which the settling time, peak error, and minimum area are the most indicative of loop characteristics.
3. Both analog and digital processing are used in process-control applications. The current trend is to make analog measurements of the controlled variable, digitize them, and used a digital controller for evaluation. The basic technique of digital encoding allows each bit of a binary word to correspond to a certain quantity of the measured variable. The arrangement of "0" and "1" states in the word then serves as the encoding.
4. The SI system of units forms the basis of computations in this text as well as in the process-control industry in general. However, it is still necessary to understand conversions to other systems, notably the English system (see Appendix 1).
5. A standard, adopted for analog process-control signals, is the 4–20 mA current range to represent the span of measurements of the dynamic variable.
6. The definitions of accuracy, resolution, and other terms used in process control are necessary and are similar to those in related fields.
7. The concept of transducer time response was introduced. The time constant becomes part of the dynamic properties of a transducer.
8. The use of significant figures is important to properly interpret measurements and conclusions drawn from measurements.
9. Statistics can help interpret the validity of measurements through the use of the arithmetic mean and the standard deviation.
10. P&ID drawings and symbols are the typical representation used to display process-control systems.

PROBLEMS

Section 1.2

1.1 Explain how the basic strategy of control is employed in a room air-conditioning system. What is the controlled variable? What is the manipulated variable? Is the system self-regulating?

1.2 Is the driving of an automobile best described as a servomechanism or a process-control system? Why?

Section 1.3

1.3 Construct a block diagram of a refrigerator control system. Define each block in terms of the refrigerator components. (If you do not know the components, look them up in an encyclopedia.)

This gives us the added ability to make quantitative statements about how the data is spread about the mean. Thus, if one set of pressure readings has a mean of 44 psi with a standard deviation of 14 psi and another a mean of 44 psi with a standard deviation of 3 psi, we know the latter is much more peaked about the mean. In fact, 68% of all the readings in the second case lie from 41 to 47 psi, whereas in the first case 68% of readings lie from 30 to 58 psi.

EXAMPLE 1.18

A control system was installed to regulate the weight of potato chips dumped into bags in a packaging operation. Given samples of 15 bags drawn from the operation before and after the control system was installed, evaluate the success of the system. Do this by comparing the arithmetic mean and standard deviations before and after. The bags should be 200 g.

Samples before: 201, 205, 197, 185, 202, 207, 215, 220, 179, 201, 197, 221, 202, 200, 195

Samples after: 197, 202, 193, 210, 207, 195, 199, 202, 193, 195, 201, 201, 200, 189, 197

Solution

In the before case, we use Equations (1.12) and (1.14) to find the mean and standard deviation, and get

$$\overline{W}_b = 202 \text{ g}$$
$$\sigma_b = 11 \text{ g}$$

Now the mean and standard deviations are found for the after case.

$$\overline{W}_a = 199 \text{ g}$$
$$\sigma_a = 5 \text{ g}$$

Thus, we see that the control system has brought the average bag weight closer to the ideal of 200 g and that it has cut the spread by a factor of 2. In the before case, 99% of the bags weighed 202 ± 33 g, but with the control system, 99% of the bags weighed in the range of 199 ± 15 g.

SUMMARY

This chapter presents an overview of process control and its elements. Subsequent chapters will examine the topics discussed in more detail and provide a more quantitative understanding.

The following list will help the reader master the key points of the chapter.

1. Process control itself has been described as suitable for application to any situation where some variable is regulated to some desired value or range of

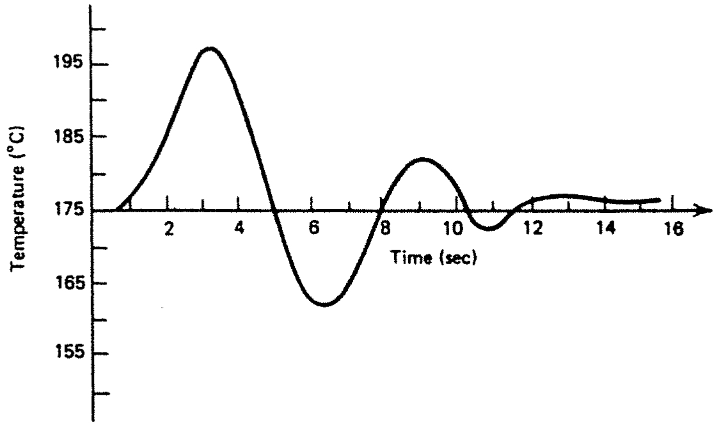


FIGURE 1.28
Figure for Problem 1.4.

Section 1.4

1.4 A process-control loop has a setpoint of 175°C and an allowable deviation of ±5°C. A transient causes the response shown in Figure 1.28. Specify the maximum error and settling time.

1.5 Two different tunings of a process-control loop result in the transient responses shown in Figure 1.29. Estimate which would be preferred to satisfy the minimum area criteria.

1.6 The second cyclic transient error peak of a response test measures 4.4%. For the quarter-amplitude criteria, what error should be the third peak value?

1.7 Does the response of Figure 1.28 satisfy the quarter-amplitude criterion?

Section 1.5

1.8 An analog sensor converts flow linearly so that flow from 0 to 300 m^3/hr becomes a current from 0 to 50 mA. Calculate the current for a flow of 225 m^3/hr.

1.9 What binary word would represent the decimal number 14 if Table 1.1 were continued?

1.10 Suppose each bit change in a 4-bit ADC represents a level of 0.15 m.

a. What would the four bits be for a level of 1.7 m?

b. Suppose the four bits were $\mathbf{1000_2}$. What is the range of possible levels?

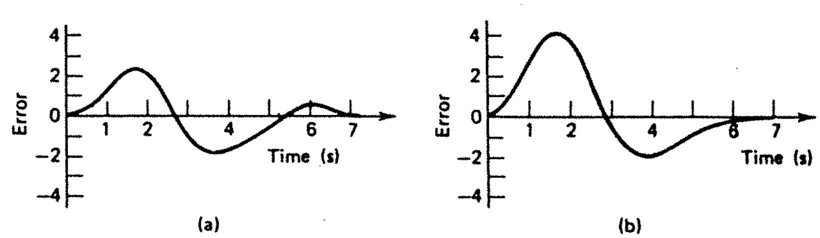


FIGURE 1.29
Figure for Problem 1.5.

1.11 For the process-control system of Figure 1.13, suppose that the relays close at |1.5| volts and open at |1.1| volts. This means that as the voltage on the relay reaches ±1.5 volts, it closes, and does not open again until the voltage drops to 1.1 volts (i.e., there is a deadband). The amplifier has a gain of 10, the reference is 3 V, and the sensor outputs 150 mV/°C. Calculate the temperatures at which the heater turns on and off and at which the cooler turns on and off.

1.12 Show how the control system of Figure 1.6 would be modified to use **(a)** supervisory computer control and **(b)** DDC or computer control.

1.13 Think about how you adjust the water temperature coming from a single nozzle using the hot and cold hand valves in a kitchen sink. Construct the block diagram of an automatic system as follows: The desired water temperature is selected by the user, perhaps by a knob and LCD readout. One hand valve turns on the cold water. The hot water valve is automatically set to keep the temperature at the selected value. Describe what elements would be necessary to do this using **(a)** analog control and **(b)** computer control.

Section 1.6

1.14 What is your mass in kilograms? What is your height in meters?

1.15 Atmospheric pressure is about 14.7 lb/in^2 (psi). What is this pressure in pascals?

1.16 An accelerometer is used to measure the constant acceleration of a race car that covers a quarter mile in 7.2 s.

a. Using $x = at^2/2$ to relate distance x, acceleration a, and time t, find the acceleration in ft/s^2.

b. Express this acceleration in m/s^2.

c. Find the car speed, v, in m/s at the end of the quarter mile using the relation $v^2 = 2ax$.

d. Find the car energy in joules at the end of the quarter mile if it weighs 2000 lb, where the energy $W = mv^2/2$.

1.17 Suppose a liquid level from 5.5 to 8.6 m is linearly converted to pneumatic pressure from 3 to 15 psi. What pressure will result from a level of 7.2 m? What level does a pressure of 4.7 psi represent?

1.18 A controller output is a 4- to 20-mA signal that drives a valve to control flow. The relation between current and flow is $Q = 45[I - 2\text{ mA}]^{1/2}$ gal/min. What is the flow for 12 mA? What current produces a flow of 162 gal/min?

1.19 An instrument has an accuracy of ±0.5% FS and measures resistance from 0 to 1500 Ω. What is the uncertainty in an indicated measurement of 397 Ω?

1.20 A sensor has a transfer function of 0.5 mV/°C and an accuracy of ±1%. If the temperature is known to be 60°C, what can be said with absolute certainty about the output voltage?

1.21 The sensor of Problem 1.20 is used with an amplifier with a gain of 15 ± 0.25 and displayed on a meter with a range of 0 to 2 volts at ±1.5% FS. What is the worst-case and rms uncertainty for the total measurement?

1.22. Using the nominal transfer function values, what is the maximum measurable temperature of the system in Problems 1.20 and 1.21?

1.23 A temperature sensor transfer function is 44.5 mV/°C. The output voltage is measured at 8.86 volts on a three-digit voltmeter. What can you say about the value of the temperature?

1.24 A level sensor inputs a range from 4.50 to 10.6 ft and outputs a pressure range from 3 to 15 psi. Find an equation such as Equation (1.6) between level and pressure. What is the pressure for the level of 9.2 ft?

1.25 Draw Figure 1.6a in the standard P&ID symbols.

Section 1.7

1.26 A temperature sensor has a static transfer function of 0.15 mV/°C and a time constant of 3.3 s. If a step change of 22°C to 50°C is applied at $t = 0$, find the output voltages at 0.5 s, 2.0 s, 3.3 s, and 9 s. What is the *indicated* temperature at these times?

1.27 A pressure sensor measures 44 psi just before a sudden change to 70 psi. The sensor measures 52 psi at a time 4.5 seconds after the change. What is the sensor time constant?

1.28 A photocell with a 35-ms time constant is used to measure light flashes. How long after a sudden dark to light flash before the cell output is 80% of the final value?

1.29 An alarm light goes ON when a pressure sensor voltage rises above 4.00 volts. The pressure sensor outputs 20 mV/kPa and has a time constant of 4.9 seconds. How long after the pressure rises suddenly from 100 kPa to 400 kPa does the light go ON?

1.30 A pressure sensor has a resistance that changes with pressure according to $R = (0.15\ \text{k}\Omega/\text{psi})p + 2.5\ \text{k}\Omega$. This resistance is then converted to a voltage with the transfer function

$$V = \frac{10R}{R + 10k}\ \text{volts.}$$

The sensor time constant is 350 ms. At $t = 0$ the pressure changes suddenly from 40 psi to 150 psi.

a. What is the voltage output at 0.5 seconds? What is the indicated pressure at this time?
b. At what time does the output reach 5.0 volts?

1.31 At $t = 0$ a temperature sensor was suddenly changed from 25°C to 100°C. The sensor outputs voltage given by the expression $V = (0.06\ \text{V/°C})\ [T - 20°\text{C}]$. The table below gives the voltages measured and the times. Determine the average time constant of the sensor.

t (seconds)	0	0.1	0.2	0.3	0.4	0.5
V (volts)	0.3	1.8	2.8	3.4	3.9	4.2

Section 1.8

1.32 A circuit design calls for a 1.5-kΩ resistor to have 4.7 volts across its terminals. What would be the expected current? The circuit is built and the resistance is measured at 1500 Ω and the voltage at 4.7 V. What is the current through the resistor?

1.33 Flow rate was monitored for a week, and the following values were recorded as gal/min: 10.1, 12.2, 9.7, 8.8, 11.4, 12.9, 10.2, 10.5, 9.8, 11.5, 10.3, 9.3, 7.7, 10.2, 10.0, 11.3. Find the mean and the standard deviation for these data.

1.34 A manufacturer specification sheet lists the transfer function of a pressure sensor as 45 ± 5% mV/kPa with a time constant of 4 ± 10% seconds. A highly accurate test system applies a step change of pressure from 20 kPa to 100 kPa.

a. What is the range of sensor voltage outputs initially and finally?
b. What range of voltages would be expected to be measured 2 seconds after the step change is applied?