Lesson 12: Parallel Transformers and Autotransformers

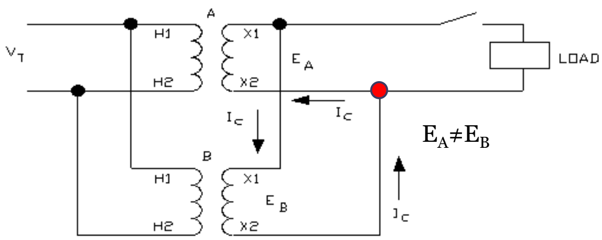
ET 332b Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Explain what causes circulating currents in parallel and compute its value.
- Compute the load division between parallel transformers.
- Explain how autotransformers operate
- Make calculation using ideal autotransformer model

Parallel Operation of Transformers



$E_A \neq E_B$

Currents circulate between A and B based on the voltage difference and transformer impedance even with no load

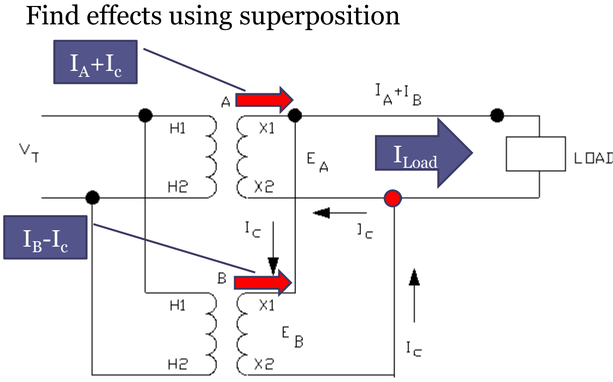
$$I_c = \frac{E_A - E_B}{Z_A + Z_B}$$

When voltage ratios are not equal, currents circulate between the windings of each transformer without a load connected. Circulating currents reduce the load capacity of transformer

Where: E_A = operating voltage of transformer A
E_B = operating voltage of transformer B
Z_A = series impedance of A
Z_B = series impedance of B

Capacity Loss Due to Circulating Currents

Find effects using superposition



Transformer A current

$I_{TA} = I_A + I_c$

Transformer B current

$I_{TB} = I_B - I_c$

I_c driven by $E_A - E_B$

Adding circulating current to transformer A increases total current in winding. Not seen in load current. Can cause overload

Circulating Current Example

Example 12-1: Two 100 kVA single phase transformer operated in parallel.

Nameplate data:

Transformer	V-ratio	%R	%X
A	2300-460	1.36	3.50
B	2300-450	1.40	3.32

Find I_c magnitude and I_c as percent of transformer secondary ratings

Example 12-1 Solution (1)

Use per unit method – V_{base} = secondary voltage

$V_b = 460\text{ V}$ $\quad V_{TxA} = \dfrac{V_R}{V_{bA}} = \dfrac{460\text{V}}{460\text{V}} = 1.0 \text{ p.u.}$

$$V_{TxB} = \frac{V_R}{V_{bB}} = \frac{450\text{V}}{460\text{V}} = 0.9783 \text{ p.u.}$$

Find $Z_A + Z_B$ in p.u.

$\bar{Z}_A = 1.36 + 3.50j \ \% / 100\%$ $\quad \bar{Z}_B = 1.40 + 3.32j \ \% / 100\%$

$\bar{Z}_A = 0.0136 + 0.035j$ p.u. $\quad \bar{Z}_B = 0.0140 + 0.0332j$ pu

$$\bar{Z}_A + \bar{Z}_B = (0.0136 + 0.035j) + (0.014 + 0.0332j) \text{ p.u.}$$

$$\bar{Z}_A + \bar{Z}_B = 0.0276 + 0.0682j \text{ p.u.}$$

$$\bar{Z}_A + \bar{Z}_B = 0.07357 \angle 67.97^\circ \text{ pu}$$

Example 12-1 Solution (2)

Use formula

$$\bar{I}_c = \frac{\bar{V}_{TXA} - \bar{V}_{TXB}}{\bar{Z}_A + \bar{Z}_B}$$

$$\bar{I}_c = \frac{(1.0 - 0.9783)\angle 0^\circ \text{ p.u.}}{0.07357\angle 62.97^\circ \text{ p.u.}}$$

$$\bar{I}_c = 0.2955\angle -62.97^\circ \text{ p.u.}$$

Convert per unit to percent

$$\%\bar{I}_c = 0.2955\angle -62.97^\circ (100\%)$$

$$\%\bar{I}_c = 29.55\angle -62.97^\circ \%$$

Ans

29.55% of Transformer A's capacity is consumed by I_c.

Now convert this to amps using a base current

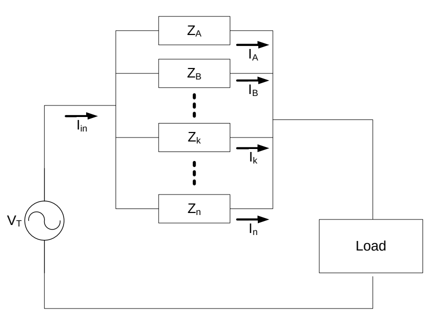
$$I_{base} = \frac{S_{base}}{V_{base}} = \frac{100,000 \text{ VA}}{460 \text{ V}}$$

$$I_{base} = 217.4 \text{ A}$$

$$\bar{I}_{cact} = \bar{I}_c I_{base} \qquad \bar{I}_{cact} = 0.2955\angle -62.97^\circ (217.4\text{A}) = 64.35\angle -62.97^\circ$$

Load Division Between Parallel Transformers

When turns ratios are equal, the load current divides following the winding impedance of the transformers. More current flows through the lowest impedance.



Circuit model

All Transformer Z's and Load Z referred to the same side of transformer or all per unit (%) quantities

$$Y_A = \frac{1}{Z_A}, \quad Y_B = \frac{1}{Z_B} \ldots \; Y_k = \frac{1}{Z_k} \ldots Y_n = \frac{1}{Z_n}$$

Use current divider rule

$$Y_p = Y_A + Y_B + \ldots Y_k + \ldots Y_n$$

$$I_k = I_{in} \cdot \left(\frac{Y_k}{Y_p}\right)$$

Finds the current in the k^{th} transformer

Parallel Transformer Example

Example 12-2: A 100 kVA transformer is to be paralleled with a 200 kVA transformer. Each transformer has rated voltages of 4160 - 240 V. Their percent impedances based on the ratings of each are:

$Z\% = 1.64+3.16j\ \%$ 100 kVA
$Z\% = 1.10+4.03j\ \%$ 200 kVA

Find: a) rated high side current of each transformer
b) % of total bank current drawn by each transformer
c) maximum bank load that can be handled without overloading either transformer

Example 12-2 Solution (1)

a) Rated current of both transformers

$$I_{ratedA} = \frac{S_{rated}}{V_{rated}} = \frac{100{,}000\ VA}{4160\ V} = 24.04\ A$$ Transformer A: 100 kVA

$$I_{ratedB} = \frac{200{,}000}{4160\ V} = 48.08\ A$$ Transformer B: 200 kVA

b) Percent current drawn by each transformer

Convert %Z to actual ohms. Need base impedances

$$Z_{baseA} = \frac{(V_{baseA})^2}{S_{baseA}} = \frac{(4160\ V)^2}{100{,}000\ VA} = 173.1\ \Omega$$

$$Z_{baseB} = \frac{(V_{baseB})^2}{S_{baseB}} = \frac{(4160\ V)^2}{200{,}000\ VA} = 86.53\ \Omega$$

Example 12-2 Solution (2)

p.u.

$\bar{Z}_A = 0.0164 + 0.0316j$ $\qquad \bar{Z}_A = 0.0356\angle 62.6°$ p.u.

$\bar{Z}_B = 0.011 + 0.0403j$ $\qquad \bar{Z}_B = 0.0418\angle 74.7°$ p.u.

Convert p.u. to ohms

$\bar{Z}_A = Z_{base}\bar{Z}_A = (173.1\ \Omega)(0.0356\angle 62.6°)$

$\bar{Z}_A = 6.161\angle 62.6°\ \Omega$

$\bar{Z}_B = Z_{base}\bar{Z}_B = (86.53\ \Omega)(0.0418\angle 74.7°)$

$\bar{Z}_B = 3.617\angle 74.7°\ \Omega$

Example 12-2 Solution (3)

Find the admittance $\qquad Y_p = \frac{1}{\bar{Z}_A} + \frac{1}{\bar{Z}_B}$

$Y_p = \frac{1}{6.161\angle 62.7°} + \frac{1}{3.615\angle 74.7°}$

$Y_p = 0.1623\angle -62.7° + 0.2766\angle -74.7°\ S$

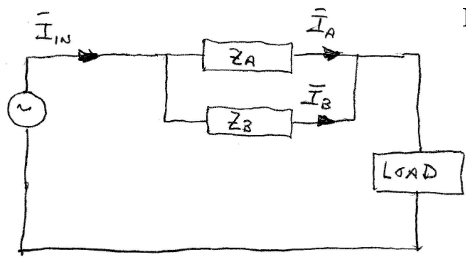
$Y_p = (0.0744 - j0.1442) + (0.0738 - j0.2668)\ S$

$Y_p = 0.1474 - j0.411\ S$

$Y_p = 0.4366\angle -70.3°\ S$ Total admittance.

Now use current divide rule to find flows through each transformer.

Example 12-2 Solution (4)



Find I_A and I_B in terms of I_{in}

$$\bar{Y}_A = \frac{1}{\bar{Z}_A} = 0.1623\angle -62.7° \text{ S}$$

$$\bar{Y}_B = \frac{1}{\bar{Z}_B} = 0.2766\angle -74.7° \text{ S}$$

$$|\bar{I}_A| = |\bar{I}_{IN}| \cdot \frac{|\bar{Y}_A|}{|\bar{Y}_P|} \qquad |\bar{I}_A| = I_{IN}\left[\frac{0.1623\angle -62.7°}{0.4366\angle -70.3}\right] \qquad I_A = I_{IN} \cdot 0.3717$$

$$|\bar{I}_B| = |\bar{I}_{IN}| \cdot \frac{|\bar{Y}_B|}{|\bar{Y}_P|} \qquad |\bar{I}_B| = I_{IN}\left[\frac{0.2766\angle -74.7°}{0.4366\angle -70.3°}\right] \qquad I_B = I_{IN} 0.6335$$

Example 12-2 Solution (5)

Transformer A carries 37.17% of the total load

$$I_A = I_{IN} \cdot 0.3717$$

Transformer B carries 63.35% of the total load

$$I_B = I_{IN} 0.6335$$

c) Find the maximum load of the parallel transformers without an overload

Let $I_A = I_{ratedA} = 24.04$ A and compute I_{in} using relationships above. Then find flow through other transformer

$$I_A = 0.3717 I_{IN} \qquad \frac{I_A}{0.3717} = I_{IN} \qquad \frac{24.04}{0.3717} = I_{IN} \qquad 64.68\text{A} = I_{IN}$$

Example 12-2 Solution (6)

$I_B = 0.6335 I_{IN}$

$I_B = 0.6335(64.68A)$

$I_B = 40.97A$

TX B not overloaded I_{ratedB}=48.08

Let $I_B=I_{ratedB}$=48.08 A. Find I_{in} and then compute the flow in transformer A

$I_B = 0.6335 I_{IN}$

$\frac{I_B}{0.6335} = I_{IN}$ $\quad \frac{48.08A}{0.6335} = I_{IN}$ $\quad 75.9A = I_{IN}$

Example 12-2 Solution (7)

$I_A = 0.3717 I_{IN}$

$I_A = 0.3717(75.9A)$

$I_A = 28.2\ A$

$I_A > I_{rated}\ A$

$28.2 > 24.04\ A$

Max load, I_{in}=64.68 A

Find bank power $S_{BANK} = V_{rated} I_{IN}$

$V_{rated} = 4160\ V$

$I_{IN} = 64.68A$

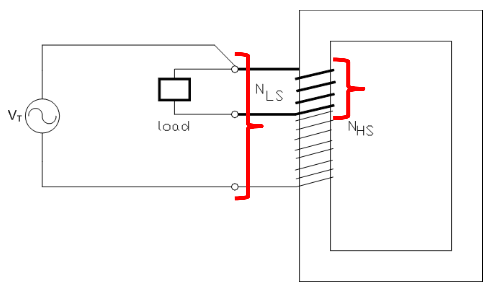
$S_{BANK} = (4160V)(64.68A)$

$S_{BANK} = 269,069\ VA$

$\approx 269\ kVA$

Autotransformers

Autotransformers use a single taped coil to change voltage levels and current levels – They provide **no electrical isolation**



N_{LS} = number of turns "embraced" by low side

N_{HS} = number of turns on high side

Polarity of induced voltages determined by direction of current and winding wraps.

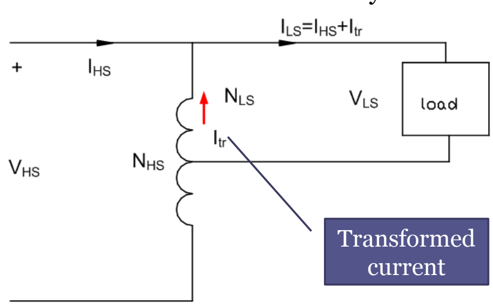
If N_{LS} = 20 and N_{HS} = 80

Step-down action

$$a = \frac{N_{HS}}{N_{LS}} = \frac{V_{HS}}{V_{LS}} \qquad a = \frac{80}{20} = 4 \quad V_{HS} = 120\text{ V} \quad \text{so} \quad V_{LS} = \frac{V_{HS}}{a} = \frac{120}{4} = 30\text{ V}$$

Autotransformers: Step-Down Operation

Some load is transferred via conduction from one side to the other and some is transferred by transformer action



Autotransformer connected in step-down mode. Note direction of I_{tr}

Like two winding transformers

$$S_{HS} = S_{LS}$$

$$V_{HS} \cdot I_{HS} = V_{LS} \cdot I_{LS}$$

I_{tr} = the current from transformer action

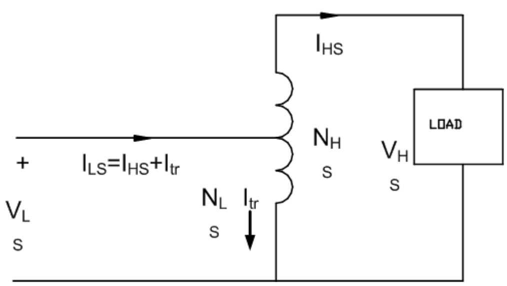
Low side current must increase to maintain power balance so:

$$I_{LS} = I_{HS} + I_{tr}$$

Autotransformer Current Ratio and Step-Up Operation

Current ratio of autotransformer $\frac{I_{HS}}{I_{LS}} = \frac{1}{a}$ Where $a = \frac{N_{HS}}{N_{LS}}$

Autotransformer In Step-up Mode

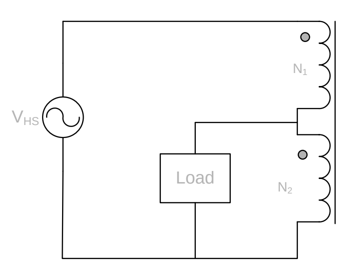


Coils in these diagrams are series aiding (induced voltages add)

Note: direction of I_{tr} reversed to maintain power balance

Autotransformers from Two-Winding Transformers

Autotransformer action can be obtained by proper connection of two winding transformer coils



For step-down mode

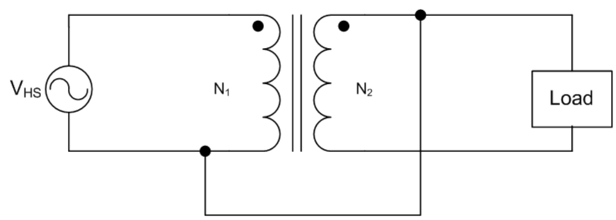
$$N_{HS} = N_1 + N_2$$
$$N_{LS} = N_2$$

$$a = \frac{N_{HS}}{N_{LS}} = \frac{N_1 + N_2}{N_2} = \frac{V_{HS}}{V_{LS}}$$

Where: N1 = number of turns in primary (HV)
N2 = number of turns in secondary (LV)

Autotransformers from Two-Winding Transformers

Step-Down Connections



Find V_{LS} with V_{HS}=120 V, N_1=500 and N_2 =100

$$a = \frac{V_{HS}}{V_{LS}} \quad \text{Where } V_{HS} = 120\text{ V}$$

$$a = \frac{N_{HS}}{N_{LS}} = \frac{N_1 + N_2}{N_2} = \frac{V_{HS}}{V_{LS}} \qquad a = \frac{500+100}{100} = 6$$

$$V_{LS} = \frac{V_{HS}}{a} = \frac{120\text{ V}}{6} = 20\text{ V}$$

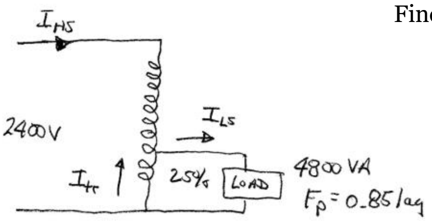
Autotransformer Example

Example 12-3: 400 turn autotransformer operating at a 25% tap supplies a 4.8 kVA load at 0.85 lagging P.F. V_{HS} = 2400 V

Find:

a) load current b) incoming line current c) I_{tr} d) apparent power transformed and conducted

Example 12-3 Solution (1)



Find turns ratio

$N_{HS} = 400t \quad N_{LS} = 0.25(N_{HS})$

$N_{LS} = 0.25(400t)$

$N_{LS} = 100t$

$a = \dfrac{N_{HS}}{N_{LS}} = \dfrac{400t}{100t} = 4$

Find secondary voltage

$\dfrac{V_{HS}}{V_{LS}} = a \qquad V_{HS} = a V_{LS}$

$\dfrac{V_{HS}}{a} = V_{LS} \qquad \dfrac{2400\text{V}}{4} = 600\text{V}$

a) Find I_{LS}

$S_{Load} = 4800 \text{ VA} \quad V_{LS} = 600\text{ V}$

$I_L = \dfrac{S_{Load}}{V_{LS}} \qquad I_L = \dfrac{4800 \text{ VA}}{600\text{ V}}$

$I_L = 8\text{ A}$ Ans

Example 12-3 Solution (2)

b) Find high-side current

Current must decrease to maintain power balance

$I_{HS} + I_{tr} = I_{LS}$

Ans

$\dfrac{I_{HS}}{I_{LS}} = \dfrac{1}{a} \Rightarrow I_{HS} = \dfrac{1}{a} I_{LS} \qquad I_{LS} = I_L = 8\text{A} \qquad I_{HS} = \dfrac{1}{4}(8\text{A}) = 2\text{A}$

c) Find transformed current

$I_{tr} = I_{LS} - I_{HS} \qquad I_{tr} = 8 - 2\text{ A}$

$I_{tr} = 6\text{ A}$ Ans

d) Find transformed and conducted apparent powers

$S_{tr} = V_{LS} I_{tr} \qquad S_{tr} = 600\text{V}(6\text{A}) = 3600\text{ VA}$ Ans

$S_{cond} = V_{LS} I_{HS} \qquad S_{cond} = 600\text{V}(2) = 1200\text{ VA}$ Ans

End Lesson 12: Parallel Transformers and Autotransformers

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