ET 332b
Ac Motors, Generators and Power Systems

LESSON 13 INDUCTION MOTOR MODEL EXAMPLE

LEARNING OBJECTIVES

After this presentation you will be able to:

- Draw the per phase circuit model of an induction motor
- Correctly place motor parameters on the circuit model
- Reduce the circuit model and solve of stator and rotor currents
- Compute the motor power and losses
- Compute motor torques

Example 13-1: A 60 Hz, 15 HP, 460 V, 4-pole wye connected induction spins a mechanical load at 1778 RPM. The motor parameters given in ohms refered to the stator are:

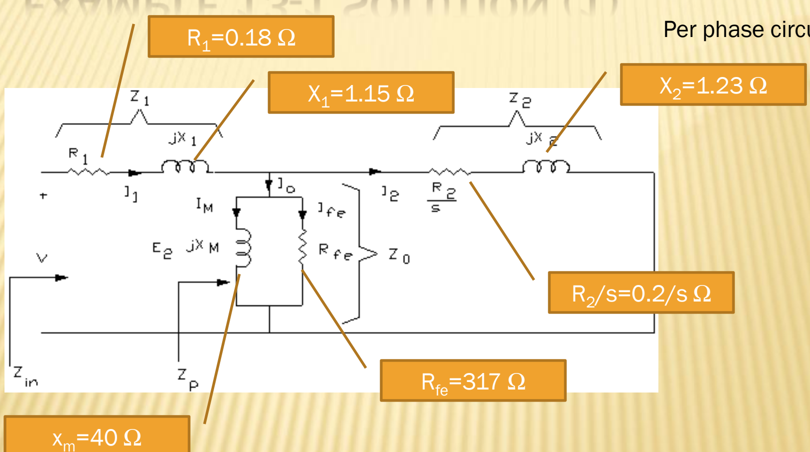
$R_1 = 0.18$ $R_2 = 0.20$
$X_1 = 1.15$ $X_2 = 1.23$
$X_M = 40$ $R_{fe} = 317$

Total mechanical power losses (friction, windage and stray) are 170 W
Find: a.) the motor slip; b.) the motor line current; c.) the apparent power the motor draws from the system; d.) active power drawn by the motor; e.) motor power factor; f.) total electric power losses of motor; g.) shaft power and torque; h.) efficiency

Use per phase circuit model and circuit analysis to find these quantities

EXAMPLE 13-1 SOLUTION (1)

Per phase circuit model



f = 60 Hz Number of poles : P=4 n_r = 1778 RPM
$P_{fw}+P_{stray}$ = 170 W

EXAMPLE 13-1 SOLUTION (2)

Find the motor slip

a.) MOTOR SLIP

$$s = \frac{n_s - n_r}{n_s} \qquad n_s = \frac{120f}{p} = \frac{120(60\,Hz)}{4}$$

$$n_s = 1800 \text{ RPM}$$

$$s = \frac{1800 - 1778}{1800} = \boxed{0.0122}$$

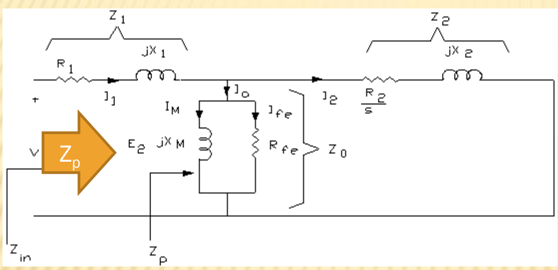
b.) MOTOR LINE CURRENT WYE CONNECTED $I_L = I_p = I_1$

REDUCE CIRCUIT

$$Z_2 = \frac{R_2}{s} + jX_2 = \frac{0.20}{0.0122} + j1.23\ \Omega$$

$$Z_2 = 16.364 + j1.23\ \Omega = 16.41\angle 4.3^\circ$$

EXAMPLE 13-1 SOLUTION (3)



$Z_p = Z_2 \| R_{fe} \| jX_m$ Use formulas shown previously OR

$$Z_p = \frac{1}{\frac{1}{Z_2} + \frac{1}{R_{fe}} + \frac{1}{jX_m}} = \frac{1}{\frac{1}{16.41\angle 4.3^\circ\Omega} + \frac{1}{317\Omega} + \frac{1}{40\angle 90^\circ}}$$

$$Z_p = \frac{1}{0.0609\angle -4.3^\circ + 0.00315\angle 0^\circ + 0.025\angle -90^\circ} = 14.207\angle 24.84^\circ$$

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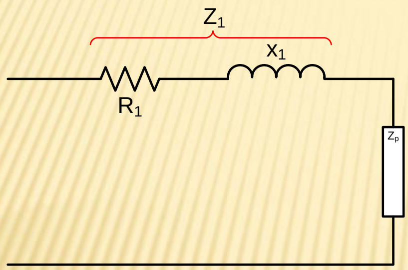
EXAMPLE 13-1 SOLUTION (4)

Find Z_{in} to find the phase current in the stator



$$Z_1 = 0.18 + j1.155\Omega = 1.164\angle 81.1^\circ$$

$$Z_{in} = 1.164\angle 81.1^\circ + 14.207\angle 29.89^\circ$$

$$Z_{in} = 14.885\angle 28.57^\circ$$

$$Z_{in} = Z_1 + Z_p$$

I_1 = STATOR CURRENT

$$I_1 = \frac{V}{Z_{in}} \qquad V = \frac{V_{LL}}{\sqrt{3}} = \frac{460V}{\sqrt{3}} = 265.6V$$

$$I_1 = 17.843\angle -28.57^\circ$$

$$I_1 = \frac{265.6\angle 0^\circ}{14.885\angle 28.57^\circ}$$

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EXAMPLE 13-1 SOLUTION (5)

c.) Apparent Power S_{in}

$$\bar{S}_{in} = \sqrt{3}\,\bar{V}_{LL}\bar{I}_L^* = \sqrt{3}(460\angle 0^\circ)(17.843\angle +28.57^\circ)\ VA$$

$$S_{IN} = 14216\angle 28.57^\circ\ VA$$

d.) Active Power Motor Draws P_{in}

$$P_{in} = \sqrt{3}\,V_{LL}I_L\cos\theta \qquad \theta = \text{angle between V and I}$$

$$\theta = 28.57^\circ$$

$$P_{in} = \sqrt{3}(460)(17.843)\cos(28.57^\circ)$$

$$P_{in} = 12{,}486\ W$$

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EXAMPLE 13-1 SOLUTION (6)

Find the power factor and the total power losses

$$e)\ F_P = \frac{P_{in}}{S_{in}} = \frac{12,486\,W}{14,216\,VA} = 0.878\ Lag$$

f.) Total P_{Loss} Rotor and Stator

$$P_{rcl} = 3I_2^2 R_2 \qquad P_{scl} = 3I_1^2 R_1 \qquad P_{core} = \frac{3(E_2)^2}{R_{FE}}$$

NEED I_2 AND E_2

$$E_2 = I_1 Z_p \qquad Z_p = 14.207\angle 24.84^\circ\ \Omega$$

$$I_1 = 17.843\angle -28.57^\circ\ A$$

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EXAMPLE 13-1 SOLUTION (7)

Compute the rotor induced voltage

$$E_2 = (17.843\angle -28.57^\circ)(14.207\angle 24.84^\circ)$$

$$E_2 = 253.5\angle -3.73^\circ\ V$$

Value almost equal to phase voltage

Find the rotor current from the value of E_2 and the rotor impedance, Z_2

$$I_2 = \frac{E_2}{Z_2} = \frac{253.5\angle -3.73^\circ}{16.41\angle 4.3^\circ}$$

$$I_2 = 15.447\angle -8.03^\circ\ A$$

Power loss formula $P_{Loss} = P_{rcl} + P_{scl} + P_{core}$

EXAMPLE 13-1 SOLUTION (8)

$$P_{Loss} = 3I_2^2 R_2 + 3I_1^2 R_1 + \frac{3(E_2)^2}{R_{fe}}$$

$$P_{Loss} = 3(15.447)^2(0.20) + 3(17.843)^2(0.18) + \frac{3(253.5)^2}{317\,\Omega}$$

$$P_{Loss} = 143.2\,W + 171.9\,W + 608\,W$$

$$\boxed{P_{Loss} = 923.1\,W}$$

g.) Shaft Power and Torque

$$P_{mech} = \frac{3I_2^2(1-s)R_2}{s} = \frac{3(15.447)^2(1-0.0122)(0.20\,\Omega)}{0.0122}$$

$$P_{mech} = 11{,}298.4\,W$$

$$P_{shaft} = P_{mech} - P_{fw} - P_{stray} = 11298.4 - 170 = 11{,}128.4\,W$$

$$P_{shaft} = \frac{11{,}128.4\,W}{746\,W/HP} = \boxed{14.92\,HP}$$ NEAR RATED 15 HP

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EXAMPLE 13-1 SOLUTION (9)

g.) SHAFT TORQUE (lb-ft)

$$P_{shaft} = 11{,}128.4\,W$$

$$T_{shaft} = \frac{7.04\,(P_{shaft})}{n_r} = \frac{7.04(11{,}128.4\,W)}{1778\,RPM}$$

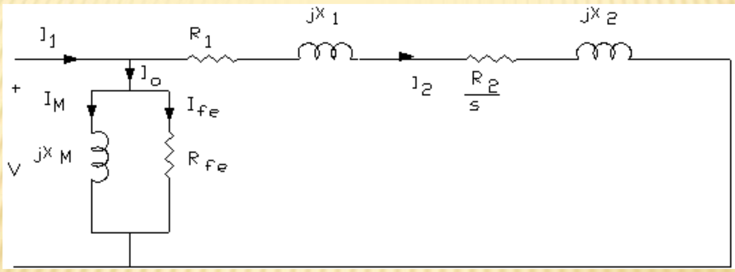
$$\boxed{T_{shaft} = 44.06\,lb\text{-}ft}$$

h.) $\eta = \frac{P_o}{P_{in}} \times 100\%$ $\quad P_o = P_{shaft} = 11128.4\,W$, $\quad P_{in} = 12{,}486\,W$

$$\eta = \frac{11128.4\,W}{12{,}486\,W} \times 100\% = 89.13\%$$

APPROXIMATE EQUIVALENT CIRCUIT

Approximate Equivalent Circuit - move magnetizing branch



Stator impedance is usually small so there is little voltage drop across Z_1

APPROXIMATE EQUIVALENT CIRCUIT

Adjusting the values of R_1 and R_2 affect T_D

$$T_D = \frac{21.12}{n_s} \cdot \left[\frac{\frac{R_2}{s} \cdot V^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2} \right]$$

Above equation derived from approximate equivalent circuit

T_D proportional to V^2 Reducing V reduces torques

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END LESSON 13 INDUCTION MOTOR MODEL EXAMPLE