Lesson 1: Phasors and Complex Arithmetic

ET 332b

Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Write time equations that represent sinusoidal voltages and currents found in power systems.
- Explain the difference between peak and RMS electrical quantities.
- Write phasor representations of sinusoidal time equations.
- Perform calculations using both polar and rectangular forms of complex numbers.

Ac Analysis Techniques

Time function representation of ac signals

Time functions give representation of sign instantaneous values

$v(t) = V_{max} \cdot \sin(\omega \cdot t + \theta_v)$ Voltage drops

$e(t) = E_{max} \cdot \sin(\omega \cdot t + \theta_e)$ Source voltages

$i(t) = I_{max} \cdot \sin(\omega \cdot t + \theta_i)$ Currents

Where V_{max} = maximum (peak) value of voltage
E_{max} = maximum (peak) value of source voltage
I_{max} = maximum (peak) value of current
$\theta_v, \theta_e, \theta_i$ = phase shift of voltage or current
ω = frequency in rad/sec Note: $\omega=2\pi f$

Ac Signal Representations

Ac power system calculations use effective values of time waveforms (RMS values) Therefore:

$$V_{RMS} = \frac{V_{max}}{\sqrt{2}} \qquad E_{RMS} = \frac{E_{max}}{\sqrt{2}} \qquad I_{RMS} = \frac{I_{max}}{\sqrt{2}}$$

Where $\frac{1}{\sqrt{2}} = 0.707$

So RMS quantities can be expressed as:

$$V_{RMS} = 0.707 \cdot V_{max}$$
$$E_{RMS} = 0.707 \cdot E_{max}$$
$$I_{RMS} = 0.707 \cdot I_{max}$$

Ac Signal Representations

Ac power systems calculations use phasors to represent time functions

Phasor use complex numbers to represent the important information from the time functions (magnitude and phase angle) in vector form.

Phasor Notation

$$\mathbf{V} = V_{RMS}\angle\theta^{\circ}$$
or
$$\overline{V} = V_{RMS}\angle\theta^{\circ}$$

$$\mathbf{I} = I_{RMS}\angle\theta^{\circ}$$
or
$$\bar{I} = I_{RMS}\angle\theta^{\circ}$$

Where: V_{RMS}, I_{RMS} = RMS magnitude of voltages and currents
θ= phase shift in degrees for voltages and currents

Ac Signal Representations

Time to phasor conversion examples, Note all signal must be the same frequency

Time function-voltage

$$v(t) = 170 \cdot \sin(377 \cdot t + 30^{\circ})$$

Find RMS magnitude

$$V_{RMS} = 0.707 \cdot 170 = 120.2 \text{ V}$$

Phasor $\overline{V} = 120.2\angle 30^{\circ} \text{ V}$

Time function-current

$$i(t) = 25 \cdot \sin(377 \cdot t - 20^{\circ})$$

Find RMS magnitude

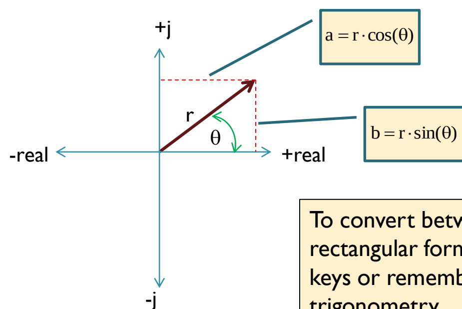
$$I_{RMS} = 0.707 \cdot 25 = 17.7 \text{ A}$$

Phasor $\bar{I} = 17.7\angle -20^{\circ} \text{ A}$

Phase shift can be given in either radians or degrees. To convert, this conversion: use 1 degree = $\pi/180$ radians.

Complex Number Representations

Polar to rectangular conversion



Rectangular form of complex number

$$z = a + j \cdot b$$

To convert between polar form and rectangular form, use calculator P-R and R-P keys or remember concepts from trigonometry

R-P conversion

Magnitude: $|z| = \sqrt{a^2 + b^2}$

Phase Angle: $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

P-R conversion

$$z = r \cdot (\cos(\theta) + j\sin(\theta))$$

Complex Number Representations

Example 1-1 Rectangular-to-polar (R-P) conversion using trigonometry. Find the polar equivalent of the complex number z.

$$z = 30 + j \cdot 20$$

Solution

$a = 30 \quad b = 20$

$|z| = \sqrt{a^2 + b^2} = \sqrt{30^2 + 20^2} = 36.1$ Magnitude

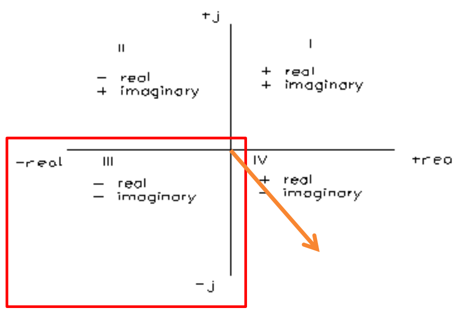
Phase angle $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$\theta = \tan^{-1}\left(\frac{20}{30}\right) = 33.7^\circ$$

Watch for the location of the phasor when making P-R or R-P conversions. Some calculators return $\tan^{-1}$ into 1st and 4th quadrants only.

Rectangular-to-Polar Conversion

Check the angle given from R-P conversion by geometric interpretation.



If real and imaginary parts both negative,

angle is in Quadrant III

Example 1-2: Find the polar form of z=-4+j5

Find r $|z| = \sqrt{(-4)^2 + 5^2} = 6.4$

Compute phase angle $\tan^{-1}\left(\frac{5}{-4}\right) = -51.3^\circ$

This $\tan^{-1}$ function computes the angle in Quadrant IV. The actual angle is 180 degrees from this value (- real, + imaginary is Quadrant II)

Polar-to-Rectangular Using Trig Functions

Conversion Equation

$$z = a + j \cdot b = r \cdot [\cos(\theta) + j \cdot \sin(\theta)]$$

Example 1-3: Convert $I = 50\angle 53.1^\circ$ to rectangular form

$$a + j \cdot b = r \cdot [\cos(\theta) + j \cdot \sin(\theta)]$$

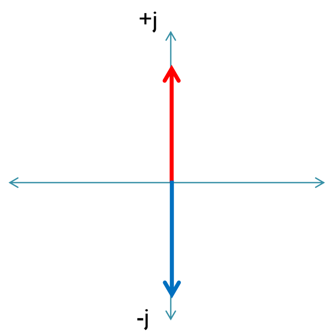
$$a + j \cdot b = 50 \cdot [\cos(53.1^\circ) + j \cdot \sin(53.1^\circ)]$$

$$a + j \cdot b = 30 + j \cdot 40$$

Ans

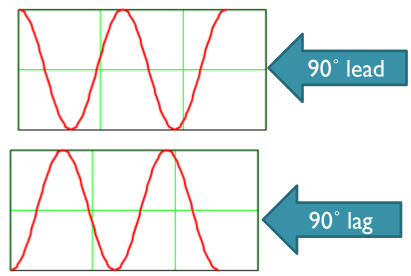
Complex Number Arithmetic

Properties of the imaginary operator $j=\sqrt{-1}$



The operator j translates physically into a 90° phase shift

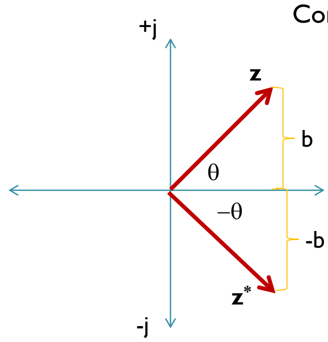
j = 90° ... a 90 degree phase lead
-j = -90° ... a 90 degree phase lag



Also
$1/j = -j$ and $1/-j = j$
$j(-j) = 1$

Complex Number Arithmetic

Complex Conjugate-reflection about the real axis



Conjugate (rectangular form)

$$(a + j\cdot b)^* = a - j\cdot b$$

Change sign on imaginary part

Conjugate (polar form)

$$(z\angle\theta^\circ)^* = z\angle-\theta^\circ$$

Change sign on angle

Complex Number Arithmetic

Addition and Subtraction of Complex Numbers

For calculators without complex number arithmetic

1) convert both numbers to rectangular form
2) add/subtract real parts of both numbers and imaginary parts of both numbers

Multiplication and Division of Complex Numbers

For calculators without complex number arithmetic

1) convert both numbers to polar form
2) multiply/divide magnitudes
3) add angles for multiplication, subtract angles for division

Inverting a Complex Number

1) convert number to polar form $z \angle \theta$
2) perform division $(1\angle 0^\circ) / (z \angle\theta) = 1/z \angle -\theta$

Complex Number Arithmetic

Example 1-4: Given the sinusoidal time functions and complex numbers below:

$$v_1(t) = 340 \cdot \sin(377 \cdot t + 10^\circ)$$

$$v_2(t) = 277 \cdot \sin(377 \cdot t - 30^\circ)$$

$$\bar{Z} = 70 + j20$$

$$\bar{I} = 3 - j2$$

Find $\mathbf{V}_2$+$\mathbf{V}_1$, $\mathbf{V}_2$-$\mathbf{V}_1$, $\mathbf{V}_1$/**Z**, **I**(**Z**) give the results in polar form for all calculations

Example 1-4 Solution (1)

Convert $v_1(t)$ and $v_2(t)$ into phasors

Find magnitudes $|\mathbf{V}_1| = \frac{340}{\sqrt{2}} = 240.4 \quad |\mathbf{V}_2| = \frac{277}{\sqrt{2}} = 195.9$

$$\mathbf{V}_1 = 240.4\angle 10^{\circ} \qquad \mathbf{V}_2 = 195.9\angle -30^{\circ}$$

Find $\mathbf{V}_1+\mathbf{V}_2$ Convert phasors to rectangular form

$$\mathbf{V}_1 = 240.4\cdot[\cos(10^{\circ}) + j\cdot\sin(10^{\circ})]$$
$$\mathbf{V}_1 = 236.75 + j\cdot 41.745$$

$$\mathbf{V}_2 = 195.9\cdot[\cos(-30^{\circ}) + j\cdot\sin(-30^{\circ})]$$
$$\mathbf{V}_2 = 169.65 - j\cdot 97.95$$

Example 1-4 Solution (2)

Add real and imaginary parts $\mathbf{V}_s = \mathbf{V}_1 + \mathbf{V}_2$

$$\mathbf{V}_s = (236.75 + 169.65) + j(41.745 + (-97.95))$$
$$\mathbf{V}_s = 406.4 - j56.2$$

Convert to polar form

$$|\mathbf{V}_s| = \sqrt{406.4^2 + (-56.2)^2} = 410.3$$
$$\theta_s = \tan^{-1}\left(\frac{-56.2}{406.4}\right) = -7.87^{\circ}$$

$$\mathbf{V}_s = 410.3\angle -7.87^{\circ}$$ ← Ans

Example 1-4 Solution (3)

Find $\mathbf{V}_1$-$\mathbf{V}_2$ subtract real and imaginary parts $\mathbf{V}_d = \mathbf{V}_1 - \mathbf{V}_2$

$$\mathbf{V}_d = (236.75 - 169.65) + j(41.745 - (-97.95))$$
$$\mathbf{V}_d = 67.1 + j139.7$$

Convert to polar form

$$\mathbf{V}_d = 67.1 + j139.7$$
$$|\mathbf{V}_d| = \sqrt{67.1^2 + 139.7^2} = 154.9$$

$$\theta_d = \tan^{-1}\left(\frac{139.7}{67.1}\right) = 64.3^\circ$$
$$\mathbf{V}_d = 154.9\angle 64.3^\circ$$

Ans

Example 1-4 Solution (4)

Compute the quantity $\mathbf{V}_1/\mathbf{Z}$ and give the results in polar form

$\mathbf{Z} = 70 + j \cdot 20$ Convert **Z** to polar form

$$|\mathbf{Z}| = \sqrt{70^2 + 20^2} = 72.8$$
$$\theta_Z = \tan^{-1}\left(\frac{20}{70}\right) = 15.95^\circ$$

To compute the quotient, divide magnitudes and subtract phase angles

$$\frac{\mathbf{V}_1}{\mathbf{Z}} = \frac{240.4\angle 10^\circ}{72.8\angle -15.95^\circ}$$
$$\frac{\mathbf{V}_1}{\mathbf{Z}} = \left(\frac{240.4}{72.8}\right)\angle 10 - 15.95^\circ = 3.3\angle -5.95^\circ$$

Ans

Example 1-4 Solution (5)

Compute the quantity I(Z) and give the results in polar form

Convert I to polar form $\mathbf{I} = 3 - j \cdot 2$

$$|\mathbf{I}| = \sqrt{3^2 + 2^2} = 3.61$$

$$\theta_I = \tan^{-1}\left(\frac{-2}{3}\right) = -33.7^\circ$$

Multiply magnitudes and add phase angles to get result

$$\mathbf{I} \cdot \mathbf{Z} = (3.61\angle -33.7^\circ) \cdot (72.8\angle 15.95^\circ)$$

$$\mathbf{I} \cdot \mathbf{Z} = 3.61 \cdot 72.8\angle -33.7^\circ + 15.95^\circ$$

$$\mathbf{I} \cdot \mathbf{Z} = 262.8\angle -17.75^\circ$$

Ans

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END LESSON 1: PHASORS AND COMPLEX ARITHMETIC