# Lesson 3: Ac Power in Single Phase Circuits 

ET 332b
Ac Motors, Generators and Power Systems

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## Learning Objectives

After this presentation you will be able to:
$>$ Identify the components of complex power.
> Compute complex power given ac voltage and current.
> Apply the correct sign convention for absorbed and delivered power.
> Draw power triangles for resistive/inductive and resistive/ capacitive loads.
> Compute reactive power and capacitance value necessary to achieve power factor correction to a specified value.

## Single Phase Ac Power Relationships

## General power system load

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{T}}=\text { terminal current }(\mathrm{RMS}) \\
& \mathrm{V}_{\mathrm{T}}=\text { terminal voltage }(\mathrm{RMS})
\end{aligned}
$$



Individual components of the
load are not known. They can be series-parallel combinations of devices

Complex power formula

$$
\overline{\mathrm{S}}=\mathrm{P}+\mathrm{j} \cdot \mathrm{Q}
$$

$$
\begin{aligned}
& S=\text { apparent power in volt-amperes }(\mathrm{VA}) \\
& P=\text { active, average, or real power in watts }(\mathrm{W}) \\
& \mathrm{Q}=\text { reactive power, volt-amps, reactive (vars) }
\end{aligned}
$$

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## Single Phase Ac Power

Computing Complex Power (volt-amperes VA)

$$
\overline{\mathrm{S}}=\overline{\mathrm{V}}_{\mathrm{T}} \cdot\left(\overline{\mathrm{I}}_{\mathrm{T}}\right)^{*}
$$

Where * is the complex conjugate of $\mathrm{I}_{\mathrm{T}}$. In polar form, * changes the sign on the current angle

$$
\begin{aligned}
\text { For example } & \overline{\mathrm{I}}_{\mathrm{T}}=20 \angle-30^{\circ} \quad\left(\overline{\mathrm{I}}_{\mathrm{T}}\right)^{*}=20 \angle 30^{\circ} \\
\text { In polar form: } & \overline{\mathrm{V}}_{\mathrm{T}}=\mathrm{V}_{\mathrm{T}} \angle \theta_{\mathrm{v}} \quad \overline{\mathrm{I}}_{\mathrm{T}}=\mathrm{I}_{\mathrm{T}} \angle \pm \theta_{\mathrm{I}} \\
& \overline{\mathrm{~S}}=\left(\mathrm{V}_{\mathrm{T}} \cdot \mathrm{I}_{\mathrm{T}}\right) \angle \theta_{\mathrm{v}} \mp \theta_{\mathrm{I}}
\end{aligned}
$$

Example: find the complex power delivered by the following voltage and current

$$
\overline{\mathrm{V}}_{\mathrm{T}}=120 \angle 0^{\circ} \mathrm{V} \quad \overline{\mathrm{I}}_{\mathrm{T}}=5 \angle-25^{\circ} \mathrm{A}
$$

$$
\begin{aligned}
& \overline{\mathrm{S}}=\left(\mathrm{V}_{\mathrm{T}} \cdot \mathrm{I}_{\mathrm{T}}\right) \angle \theta_{\mathrm{v}} \mp \theta_{\mathrm{I}} \\
& \left(\overline{\mathrm{I}}_{\mathrm{T}}\right)^{*}=\left(5 \angle-25^{\circ}\right)^{*}=5 \angle 25^{\circ}
\end{aligned}
$$

$$
\begin{aligned}
& \overline{\mathrm{S}}=(120 \cdot 5) \angle 0+25^{\circ} \\
& \overline{\mathrm{S}}=600 \angle 25^{\circ} \quad \text { Answer }
\end{aligned}
$$

## Single Phase Ac Power

Rectangular form of Complex Power


Angle between $\mathrm{V}_{\mathrm{T}}$
Active Power - Real Part Reactive Power-Imaginary Part and $\mathrm{I}_{\mathrm{T}}$

$$
\mathrm{P}_{\mathrm{T}}=\left(\mathrm{V}_{\mathrm{T}} \cdot \mathrm{I}_{\mathrm{T}}\right) \cdot \cos (\theta) \quad \mathrm{Q}_{\mathrm{T}}=\left(\mathrm{V}_{\mathrm{T}} \cdot \mathrm{I}_{\mathrm{T}}\right) \cdot \sin (\theta)
$$

If component values are known, compute power using the following formulas

$$
\mathrm{P}_{\mathrm{T}}=\frac{\left|\overline{\mathrm{V}}_{\mathrm{T}}\right|^{2}}{\mathrm{R}}=\left|\overline{\mathrm{I}}_{\mathrm{T}}\right|^{2} \cdot \mathrm{R} \quad \mathrm{Q}_{\mathrm{T}}=\frac{\left|\overline{\mathrm{V}}_{\mathrm{T}}\right|^{2}}{|\overline{\mathrm{X}}|}=\left|\overline{\mathrm{I}}_{\mathrm{T}}\right|^{2} \cdot\left|\overline{\mathrm{X}}^{2}\right| \quad \text { Where }|\overline{\mathrm{X}}|=\left|\overline{\mathrm{X}}_{\mathrm{L}}\right|-\left|\overline{\mathrm{X}}_{\mathrm{C}}\right|
$$

$$
|\mathrm{X}| \text { is net reactance of the load }
$$

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## Sign Conventions of Sources and Loads in Ac Power



Devices that deliver power have negative power values and are power sources

It is possible in ac power systems for a source or load to simultaneously absorb and deliver power. Active and reactive power can have different signs. This means one is being absorbed while the other is being delivered.


Devices that absorb power have positive power values and are loads


Device above absorbs Q and delivers P


## Power Triangle Relationship- Inductive Circuits



Power factor formulas: combining previous equations

$$
\mathrm{F}_{\mathrm{P}}=\cos (\theta)=\frac{\mathrm{P}_{\mathrm{T}}}{\sqrt{\mathrm{P}_{\mathrm{T}}{ }^{2}+\mathrm{Q}_{\mathrm{T}}{ }^{2}}}
$$

$\mathrm{F}_{\mathrm{p}}$ will have a value between 0 and -1 for
inductive circuits. The closer the value is to
$|1|$ the more desirable.
$\mathrm{F}_{\mathrm{p}}=1$ indicates that:
a) the circuit is totally resistive or
b) the net reactance of the circuit is 0 (i.e. there is enough capacitance to cancel the inductive effects)

[^0]
## Power Triangle Relationship- Capacitive

## Circuits

Capacitive circuits have leading power factor, $\mathrm{F}_{\mathrm{p}}$.

$\mathrm{S}_{\mathrm{T}}=\sqrt{\mathrm{P}_{\mathrm{T}}{ }^{2}+\mathrm{Q}_{\mathrm{T}}{ }^{2}}$
$\mathrm{F}_{\mathrm{P}}=\frac{\mathrm{P}_{\mathrm{T}}}{\mathrm{S}_{\mathrm{T}}}=\cos (\theta)$



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Capacitors deliver negative vars. $\mathrm{I}_{\mathrm{T}}{ }^{*}$ gives negative angle on complex power phasor

## Power Triangle Relationship- Capacitive Circuits



Measuring -VARs indicates capacitive or leading reactive power. Capacitive power factor called leading.

$$
\mathrm{F}_{\mathrm{P}}=\cos (\theta)=\frac{\mathrm{P}_{\mathrm{T}}}{\sqrt{\mathrm{P}_{\mathrm{T}}{ }^{2}+\mathrm{Q}_{\mathrm{T}}{ }^{2}}}
$$

$F_{p}$ will have a value between 0 and +1 for capacitive circuits.

Devices with leading power factor are considered to be
VAR generators. Capacitors are said to deliver VARs to a circuit.

## Complex Power Calculation

Example 3-1: The load shown in the phasor diagram has a measured terminal current of $\mathbf{I}_{\mathrm{T}}=125 \angle 30^{\circ} \mathrm{A}$ and a terminal voltage of $\mathbf{V}_{\mathrm{T}}=460 \angle 20^{\circ} \mathrm{V}$ Find:
a) apparent power delivered
b) active and reactive power delivered
c) determine if the circuit is acting as a capacitor or an inductor
d) power factor of the load


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## Example 3-1 Solution (1)

a) $\bar{S}_{T}=\bar{V}_{T} \bar{I}_{T}^{*}$
$\bar{S}_{T}=4<0 \angle 20^{\circ}\left(125 \angle-30^{T}\right)$
b) Total apparent power, expand into rectangular form to find P and Q
$\bar{S}_{7}=5 ?, 500 \angle-10^{\circ} \mathrm{VA}$

$$
S=57,500\left[\cos \left(-10^{\circ}\right)+y \sin \left(-10^{\circ}\right)\right]
$$


$S=56,626-y 9985 \mathrm{VA}$

$$
\begin{aligned}
\text { Real Port } P & =56,626 \mathrm{~W} \\
\text { Imaginary } Q & =9985 \mathrm{VAR}
\end{aligned}
$$

c) The sign on the reactive power above is negative so this device delivers reactive power-capacitive.
d) $\mathrm{F}_{\mathrm{P}}=\cos \left(20^{\circ}-30^{\circ}\right)=\cos \left(-10^{\circ}\right)=0.9848$ Leading

Note: The angular relationship between $\mathrm{V}_{\mathrm{T}}$ and $\mathrm{I}_{\mathrm{T}}$ determines $\mathrm{F}_{\mathrm{p}}$

## Power Factor Correction

## Utility companies prefer high power factor loads.

- These loads consume the least amount of capacity for the amount of billable power (active power).
- Lower power factors are penalized. Charge extra for low power factor. Usually 0.85 or less.


## Power factor of Industrial Plants

- Most industrial plants have lagging power factor (inductive due to motors and transformers).
- Adding capacitors improves power factor
- Capacitors deliver reactive power to inductive loads
- Inductors - +Q absorb reactive power
- Capacitors- -Q deliver reactive power


## Power Factor Correction

Remember the inductive power triangle


$$
\text { At } \mathrm{F}_{\mathrm{p}}=1 \mathrm{P}_{\mathrm{T}}=\mathrm{S}_{\mathrm{T}} \theta=0^{\circ} \text { and } \mathrm{Q}_{\mathrm{T}}=0
$$



To increase $F_{p}$ we must decrease $Q_{T}$ by adding -
Q from capacitors

Since $\cos (\theta)=F_{p}$ reducing $\theta$ by reducing $\mathrm{Q}_{\mathrm{T}}$ improves power factor

The length of the $\mathrm{P}_{\mathrm{T}}$ side of the triangle stays the same. This is the amount of active power that is consumed. The total apparent power, $\mathrm{S}_{\mathrm{T}}$ is reduced. This reduces the current that is necessary to supply the same amount of active power.

## Power Factor Example

Example 3-2: a $10 \mathrm{~kW}, 220 \mathrm{~V}, 60 \mathrm{~Hz}$ single phase motor operates at a power factor of 0.7 lagging. Find the value of capacitance that must be connected in parallel with the motor to improve the power factor to 0.95 lagging

Find the sides and angle of the initial power triangle


$$
\begin{gathered}
F_{p}=0.7 \text { lagging } \quad P_{T}=10 \mathrm{~kW} \quad \operatorname{Cos}(\theta)=0.7 \\
\cos (\theta)=\frac{P}{S} \quad \text { and } \quad S_{T}=\frac{P_{T}}{F_{p}} \\
S_{T i}=\frac{10 \mathrm{~kW}}{0.7}=14.286 \mathrm{kvA}
\end{gathered}
$$

$$
\text { Find } \begin{aligned}
\theta, & \theta=\cos ^{-1}\left(F_{p}\right) \\
& \left.\theta=\cos ^{-1}(0.2)=45.57^{\circ}\right)
\end{aligned}
$$

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## Example 3-2 Solution (2)



Construct the power triangle for $\mathrm{F}_{\mathrm{p}}=0.95$ lagging


## Example 3-2 Solution (3)

Find the power required from the capacitor
 $Q_{c}=Q_{T_{1}}-Q_{T_{2}}$


This is the Q that the capacitor must supply to correct $\mathrm{F}_{\mathrm{P}}$.

Find the capacitor value

$Q_{c}=\frac{V^{2}}{\left|\bar{x}_{c}\right|} \quad\left|\bar{x}_{c}\right|=\frac{1}{2 \pi f c}$
Combine these equations and solve for C

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$Q_{c}=\frac{V^{2}}{\frac{1}{2 \pi f c}} \Rightarrow 2 \pi f c V^{2}=Q_{c} \quad \Rightarrow C=\frac{Q_{c}}{2 \pi f V^{2}}$

## Example 3-2 Solution (4)

Compute the capacitor value using the formula derived

$$
\begin{aligned}
& C=\frac{Q_{C}}{2 \pi f \mathrm{~V}^{2}} \quad \begin{array}{l}
\mathrm{Q}=6913 \mathrm{VAR} \\
\mathrm{~V}=220 \mathrm{~V} \\
\mathrm{f}=60 \mathrm{~Hz}
\end{array} \\
& C=\frac{6913}{2 \pi 60(220)^{2}} \quad \begin{array}{l}
C=3.79 \times 10^{-4} \mathrm{~F} \\
C=\frac{3.79 \times 10^{-4} \mathrm{~F}}{1 \times 10^{-6} \mathrm{~F} / \mathrm{FF}} \\
C=379 \mu \mathrm{~F} \quad \mathrm{Ans}
\end{array}
\end{aligned}
$$

## Example 3-2 Solution (5)

Find $Q_{c}$ using a one step formula

$$
\begin{aligned}
& Q_{C}=P_{T}\left(\tan \left[\cos ^{-1}\left(F_{p_{1}}\right)\right]-\tan \left[\cos ^{-1}\left(F_{p_{2}}\right)\right]\right) \\
& F_{p 1}=0.7 \quad F_{p 2}=0.95 \quad P_{T}=10,000 \mathrm{~W} \\
& Q_{C}=10,000\left(\tan \left[\cos ^{-1}(0.7)\right]-\tan \left[\cos ^{-1}(0.95)\right]\right) \\
& Q_{C}=10,000\left(\tan \left(45.52^{\circ}\right)-\tan \left(10.195^{\circ}\right)\right) \\
& Q_{C}=10000(1.02-0.32868) \\
& Q_{C}=6913 \mathrm{VAR}
\end{aligned}
$$

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## Power Factor and Load Current

Example 3-3: A $480 \mathrm{~V}, 60 \mathrm{~Hz}$, single phase load draws 50.25 kVA at a power factor of 0.87 lagging.
Find:
a) the current and the active power in kW that the load absorbs
b) the angle between the source voltage and the load current
c) the amount of reactive power necessary to correct the load power factor to 0.98 lagging
d) the current the load draws at 0.98 power factor

## Example 3-3 Solution (1)

a) the current and the active power in kW that the load absorbs
$|\bar{s}|=|\bar{v}| \cdot|\bar{x}| \Rightarrow|\bar{I}|=\frac{|s|}{|\vec{v}|}$

$$
\frac{50,250 \mathrm{VA}}{480 \mathrm{~V}}=104.7 \mathrm{~A}
$$

To find active power

$$
\begin{aligned}
F_{P}=\frac{P}{s} \Rightarrow F_{p} s & =P \quad F_{p}=0.87 \text { hagging } \\
P & =F_{p} s=50,250 \mathrm{VA}(0.82) \\
P & =43,7,8 \mathrm{~W} \text { Ans }
\end{aligned}
$$

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## Example 3-3 Solution (2)

b) the angle between the source voltage and the load current

$$
\begin{aligned}
& F_{p}=\cos \theta \quad \theta=\text { angle between } V \text { and } I \\
& \theta=\cos ^{-1}\left(F_{p}\right) \\
& \theta=\cos ^{-1}(0.87)=29.54^{\circ} \\
& F_{p} \text { is lagging } 50 \theta \text { is negutive } \\
& \theta=-29.59^{\circ} \quad \text { Ans }
\end{aligned}
$$



## Example 3-3 Solution (3)

c) the amount of reactive power necessary to correct the load power factor to 0.98 lagging

Find initial reactive power
New reactive power

$$
\begin{array}{ll}
Q_{T 1}=S_{T 1} \sin (\theta) & F_{P_{2}}=0.98 \\
Q_{T 1}=51,250 \sin \left(29.59^{\circ}\right) & S_{T 2}=\frac{p}{F_{P_{2}}} \\
Q_{T 1}=24,776 \text { VAR } & S_{T 2}=\frac{43,718 \mathrm{~W}}{0.98}=44,610 \mathrm{VA}
\end{array}
$$

NEW ANGLE

$$
\theta_{2}=\cos ^{-1}(0.90)=11.5^{\circ}
$$

$$
Q_{r i}=\varepsilon_{r 2} \sin \theta_{2}
$$

$$
Q_{+2}=44,610 \mathrm{VA} \sin \left(11.5^{\circ}\right)
$$

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$$
Q_{T_{2}}=8879 \mathrm{VAR}
$$

## Example 3-3 Solution (4)

Calculate the power required from the capacitor


Use one-step formula
$Q_{C}=P\left[\tan \left[\cos ^{-1}\left(F_{p_{1}}\right)\right] \cdot \tan \left[\cos ^{-1}\left(F_{p_{2}}\right)\right]\right]$
$Q_{c}=43,718\left[\tan \left[\cos ^{-1}(0.87)\right]-\tan \left[\cos ^{-1}(0.98)\right]\right]$
$Q_{c}=43,718\left[\tan \left[29,54^{\circ}\right)-\tan \left(11,5^{\prime}\right)\right]$
$\hat{Q}_{C}=\underset{24.718(0.56627-0.2035)}{4 \text { lesson 3_et332b.pptx }}$


## Example 3-3 Solution (5)

d) the current the load draws at 0.98 power factor

$$
S_{T 2}=V_{T} I_{T 2} \quad \frac{S_{T 2}}{V_{t}}=I_{T 2}
$$

$$
\frac{44,718 \mathrm{VA}}{480 \mathrm{~V}}=I_{T}
$$

$$
Q 2.94 A=I_{T}
$$

Current reduction due to $\mathrm{F}_{\mathrm{p}}$ increase

$$
104-7 A-92.94=11.7<A
$$

## End Lesson 3: Ac Power in Single Phase Circuits

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