Lesson 4: Three Phase Sources and Loads

ET 332b

ET 332b Ac Motors, Generators and Power Systems

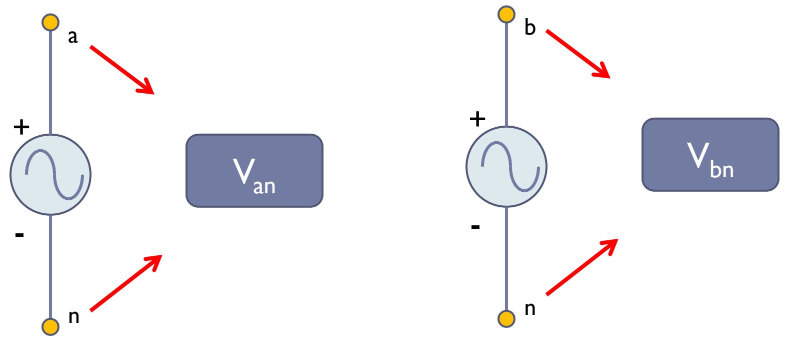
Learning Objectives

After this presentation you will be able to:

- Identify circuit impedances, voltages and currents using the double subscript notation.
- Perform calculations on wye connected three-phase sources and loads.
- Perform calculations on delta connected three-phase sources and loads.
- Construct phasor diagrams of three-phase sources and loads.
- Identify the time and phasor plots of a three phase set of voltages and currents.

Double Subscript Notation

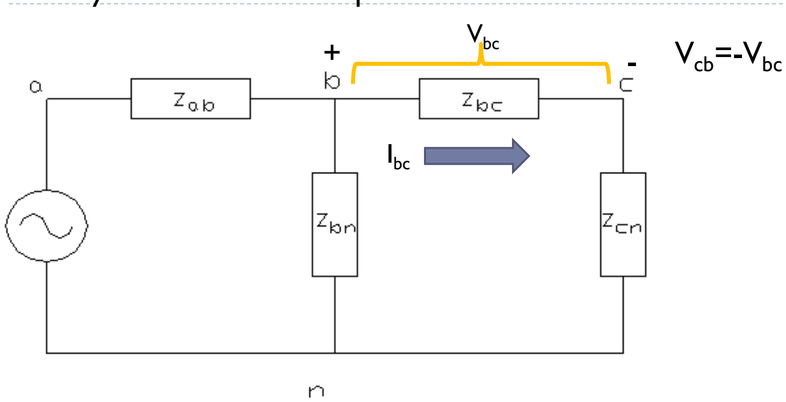
Sources and voltage drops are defined by the terminal letter. Voltage drop and polarity defined by order of subscripts



Voltages considered positive if first node subscript is higher potential than second node subscript

Double Subscript Notation

Identify current flow from point b to c



$$I_{bc} = \frac{V_{bc}}{Z_{bc}}$$

V_{bc} = difference in potential between points b and c. If voltage at point b is taken as the reference point, then the polarity is reversed.

Three-Phase Power Systems

Balanced three-phase voltage sources

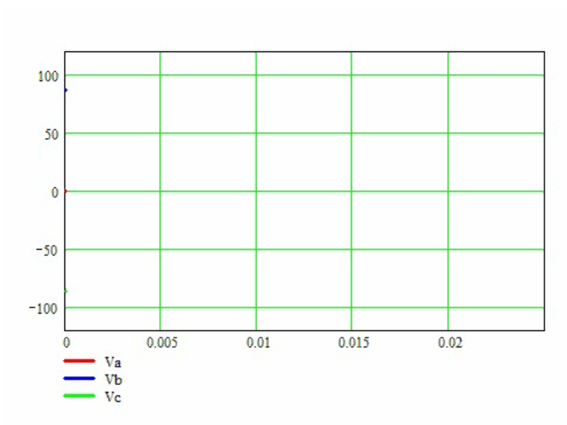
Characteristics:	Three phasor voltages Equal voltage magnitudes Phase shift equally spaced 120 degrees apart

Time equations for balanced three-phase voltage sources

$$v_{an}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t + 0^\circ)$$
$$v_{bn}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t - 120^\circ)$$
$$v_{cn}(t) = V_p \cdot \sin(2\pi \cdot f \cdot t - 240^\circ)$$

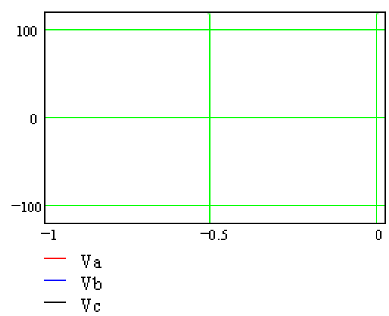
Three-Phase Power Systems

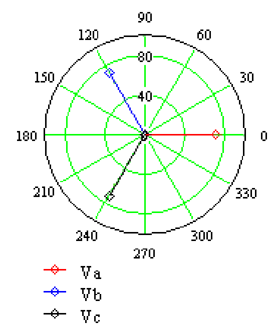
Time Plots of Three Phase Voltages



Three-Phase Three-Phase

Time Plots of Three Phase Voltages and Phasors

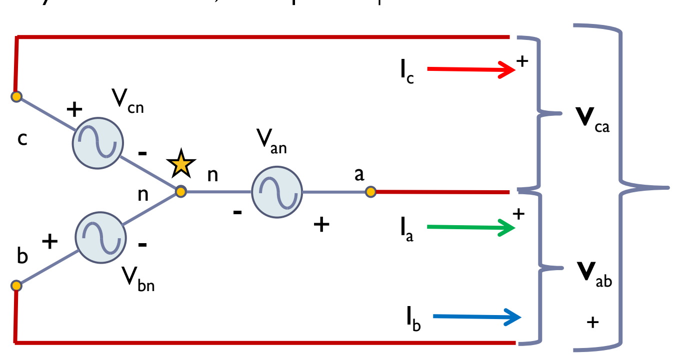




$\phi = 0 \cdot deg$

Three-Phase Source Connections

Wye – Connected, three-phase 3ϕ sources



In wye connection:

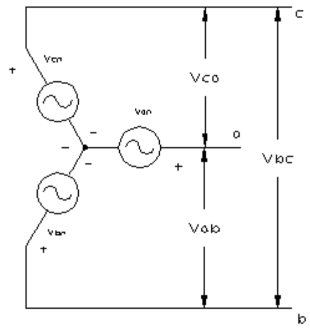
$I_L = I_p$

Where:
I_L = line current
I_p = phase current

Determine the relationship between the magnitude and phase shift of each source voltage and the current and voltage at the terminals of the connection

Wye Connected Sources

Voltage relationships



Perform phasor subtraction to find the values

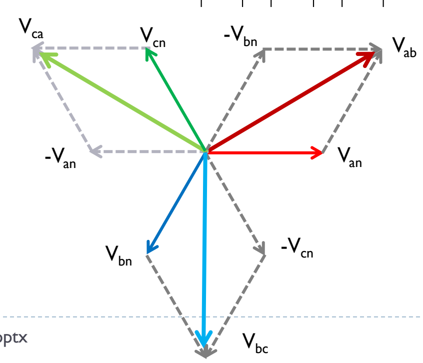
Line-to-line voltage phasors

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn}$$

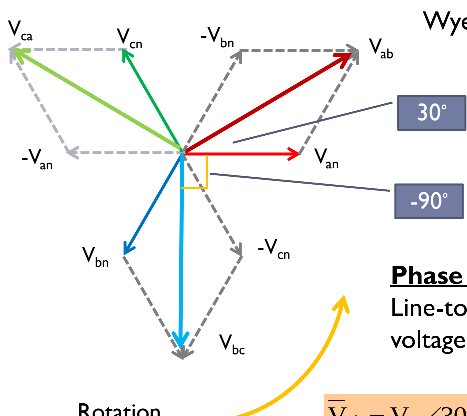
$$\overline{V}_{bc} = \overline{V}_{bn} - \overline{V}_{cn}$$

$$\overline{V}_{ca} = \overline{V}_{cn} - \overline{V}_{an}$$

In balanced systems $\left|\overline{V}_{an}\right| = \left|\overline{V}_{bn}\right| = \left|\overline{V}_{cn}\right|$



Wye Connected Sources



30°

-90°

Rotation

Wye connected line voltage magnitudes

$$\left|\overline{V}_{ab}\right| = \sqrt{3} \cdot \left|\overline{V}_{an}\right|$$

$$\left|\overline{V}_{bc}\right| = \sqrt{3} \cdot \left|\overline{V}_{bn}\right|$$

$$\left|\overline{V}_{ca}\right| = \sqrt{3} \cdot \left|\overline{V}_{cn}\right|$$

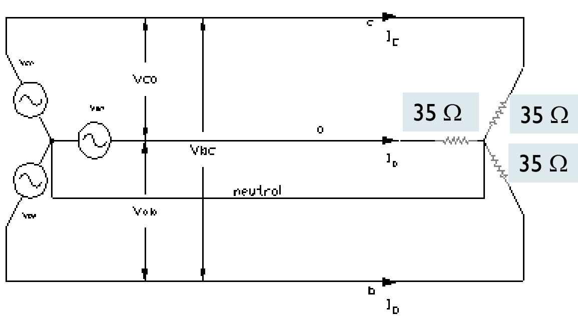
Phase shifts

Line-to-line (line) voltages lead phase voltages by 30 degrees for CCW rotation

$\overline{V}_{ab} = V_{ab}\angle 30^\circ$ $\overline{V}_{bc} = V_{bc}\angle -90^\circ$ $\overline{V}_{ca} = V_{ca}\angle -210^\circ$

Wye Connect Systems

Example 4-1: a) Find the line-to-line voltage phasors for the wye connected source. b) Find the line current phasors in each of the three phases of the resistive load. c) Find the current flowing in the neutral .
V_{an} = 100∠ 0° V_{bn} = 100 ∠ -120° V_{cn} = 100 ∠ -240°



Example 4-1 Solution (1)

Balanced 3ϕ systems can analyzed using a single phase to neutral and the other quantities determined by the appropriate phase shifts.

a)

$$\bar{V}_{an} = 100\angle 0^\circ \qquad V_{ab} = \sqrt{3}\,V_{an}$$

$$\bar{V}_{bn} = 100\angle -120^\circ \qquad V_{ab} = \sqrt{3}(100\,V) = 173.2\,V$$

$$\bar{V}_{cn} = 100\angle -240^\circ \qquad |\bar{V}_{ab}| = |\bar{V}_{bc}| = |\bar{V}_{ca}| = 173.2\,V$$

For this sequence of phasors, line-to-line voltages lead phase voltages by 30 degrees

$$\bar{V}_{ab} = 173.2\angle 30^\circ$$

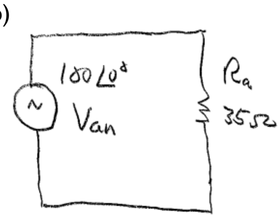
$$\bar{V}_{bc} = 173.2\angle -90^\circ$$

$$\bar{V}_{ca} = 173.2\angle -210^\circ$$

Ans

Example 4-1 Solution (2)

b)



In wye connection line current equals phase current

For "a" phase

$$\bar{I}_a = \frac{\bar{V}_{an}}{R_a} = \frac{100\angle 0^\circ}{35\angle 0^\circ} = \boxed{2.857\angle 0^\circ\ A}$$ Ans

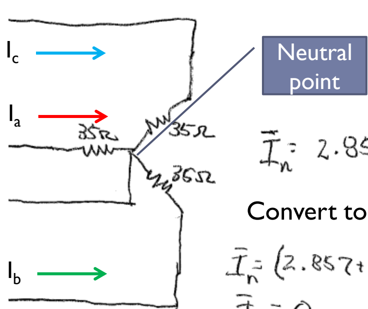
For other phases

$$\bar{I}_b = \frac{100\angle -120^\circ\ V}{35\angle 0^\circ\ \Omega} = 2.857\angle -120^\circ\ A$$ Ans

$$\bar{I}_c = \frac{100\angle -240^\circ}{35\angle 0^\circ\ \Omega} = 2.857\angle -240^\circ\ A$$ Ans

Example 4-1 Solution (3)

c) To find the neutral current, sum the currents at the load neutral point



$$\bar{I}_n = \bar{I}_a + \bar{I}_b + \bar{I}_c$$

$$\bar{I}_n = 2.857\angle 0^\circ + 2.857\angle -120^\circ + 2.867\angle -240^\circ$$

Convert to rectangular form to add

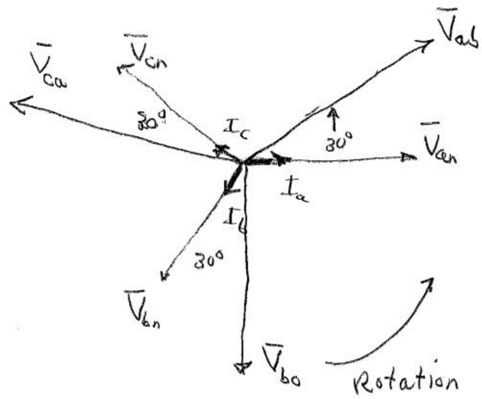
$$\bar{I}_n = (2.857 + j0) + (-1.4285 - j2.474) + (-1.4285 + j2.474)\ A$$

$$\bar{I}_n = 0$$

In balanced 3ϕ system, neutral conductor carries no current. It is used for safety and to handle unbalances

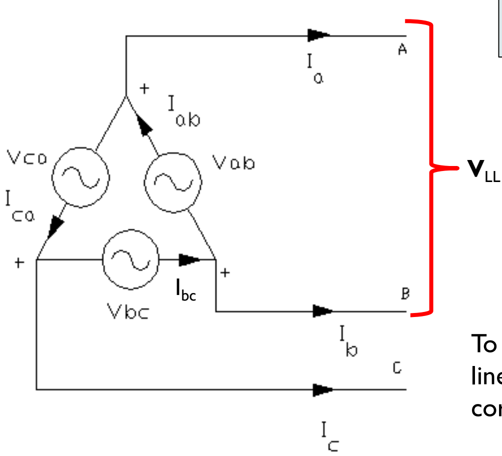
Example 4-1 Solution (4)

Phasor diagram



All line-to-line voltages lead the phase voltages by 30 degrees. For resistive load, the phase current is in phase with V_{pn}, the phase-to-neutral voltage.

Delta Connected Sources



Phase voltages are equal to line-to-line voltages in delta connections

$$|\overline{V}_{LL}| = |\overline{V}_{P}|$$

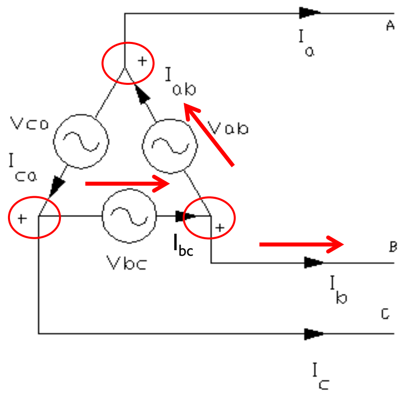
Where: V_{LL} = line-to-line voltage
V_p = phase voltage

So $$|\overline{V}_{ab}| = |\overline{V}_{bc}| = |\overline{V}_{ca}|$$

To find relationship between phase and line currents, perform KCL at every corner node

Delta Connected Sources

Current relationships between line and phase



For node B

$$\bar{I}_{bc} - \bar{I}_{ab} - \bar{I}_{b} = 0$$

$$\bar{I}_{bc} - \bar{I}_{ab} = \bar{I}_{b}$$

For node A

$$\bar{I}_{ab} - \bar{I}_{ca} - \bar{I}_{a} = 0$$

$$\bar{I}_{ab} - \bar{I}_{ca} = \bar{I}_{a}$$

For node C

$$\bar{I}_{ca} - \bar{I}_{bc} - \bar{I}_{c} = 0$$

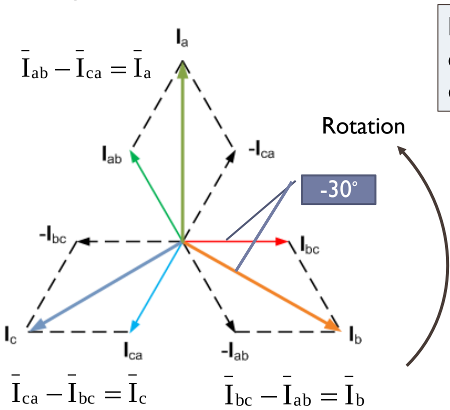
$$\bar{I}_{ca} - \bar{I}_{bc} = \bar{I}_{c}$$

Phasor subtraction gives the current magnitude $\left|\bar{I}_{L}\right| = \sqrt{3}\cdot\left|\bar{I}_{p}\right|$

Where:
I_L = line current
I_p = phase current

Delta Connected Sources

Phasor diagram of delta currents



Line current phasors lag phase currents by 30 degrees in balanced delta connection

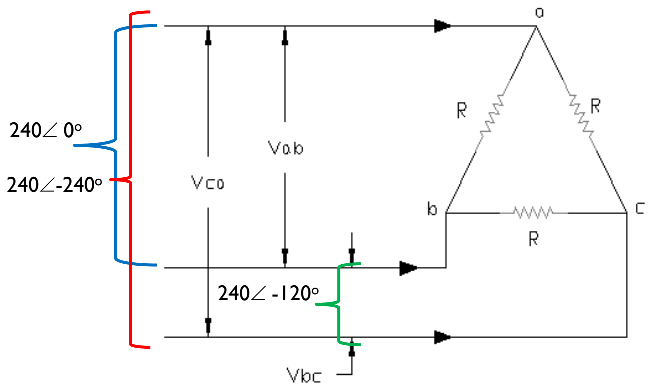
$$\bar{I}_{L} = \sqrt{3}\cdot\left|\bar{I}_{p}\right|\angle -30^{\circ}$$

Above hold for all phases with I_p as reference phasor

Phase rotation is ABC in this case

Delta Connection Example

Example 4-2: For the delta connected load shown, a) find the phasor values of phase and line currents for the circuit.: b) draw a phasor diagram of the computed currents and given voltages R = 15 ohms in each phase.



$V_{ab} = 240\angle 0°$
$V_{bc} = 240 \angle -120°$
$V_{ca} = 240 \angle -240°$

Example 4-2 Solution (1)

For this balanced system, compute the values for single phase and then shift the angles for the other values

$$\bar{I}_{ab} = \frac{\bar{V}_{ab}}{R} = \frac{240\angle 0°}{15\angle 0°} = 16\angle 0° \text{ A}$$

Now compute the line current

$$\bar{I}_a = \sqrt{3}\, \bar{I}_{ab} \angle -30°$$

$$\bar{I}_a = \sqrt{3}(16)\angle 0° - 30°$$

$$\bar{I}_a = 27.72 \angle -30° \text{ A}$$

$\bar{I}_{bc}$ and $\bar{I}_{ca}$ $-120°$ and $-240°$ from $\bar{I}_{ab}$ so

$$\bar{I}_{bc} = 16\angle -120° \quad \bar{I}_{ca} = 16\angle -240°$$

Example 4-2 Solution (2)

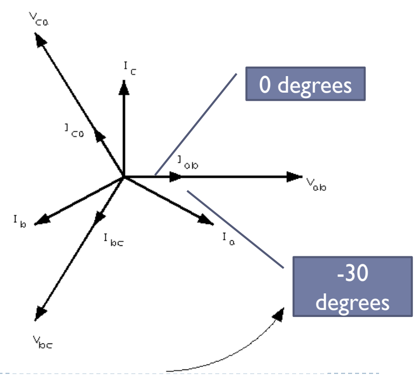
Similarly for the line current

$$\bar{I}_b = 27.72\angle -150^\circ \text{ A}$$
$$\bar{I}_c = 27.72\angle -270^\circ \text{ A}$$

Phasor diagram for delta connected load Example 4-2

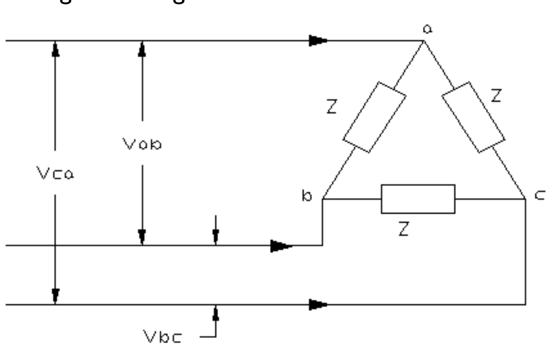
Resistive circuit, so phase currents are in phase with voltages that produce it.

Line currents lags the phase currents by the 30 degrees



Delta Connected Load-General Impedances

Example 4-3: a) Find the phasor values of phase and line currents for the circuit shown: b) draw a phasor diagram of the computed currents and given voltages. Z = 30∠-39° ohms.



$V_{ab} = 240\angle 0^\circ$
$V_{bc} = 240\angle -120^\circ$
$V_{ca} = 240\angle -240^\circ$

Example 4-3 Solution (1)

Phase current of V_{ab}

$$\bar{I}_{ab} = \frac{\bar{V}_{ab}}{\bar{Z}} = \frac{240\angle 0°}{30\angle -39°}$$

$$\bar{I}_{ab} = 8\angle 39°\ A$$

Line current $\bar{I}_{a} = \sqrt{3}\, I_{ab} \angle -30°+\theta$ $\quad \bar{I}_{a} = 13.85\angle 9°$

Other phases

$$\bar{I}_{bc} = 8\angle -81°\ A \qquad \bar{I}_{ca} = 8\angle -201°\ A$$

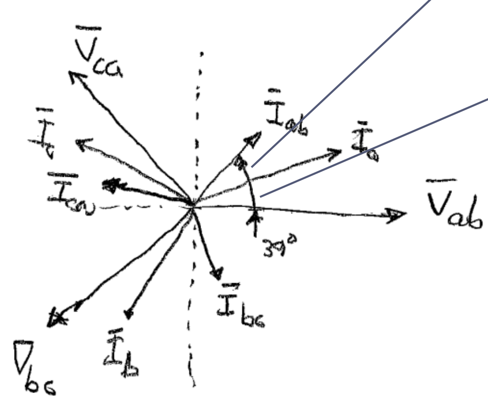
$$\bar{I}_{b} = \sqrt{3}\, I_{bc}\angle -30°+\theta \qquad \bar{I}_{c} = \sqrt{3}\, I_{ca}\angle -30+\theta$$

$$\bar{I}_{b} = 13.85\angle -30-81° \qquad \bar{I}_{c} = 13.85\angle -30-201°\ A$$

$$\bar{I}_{b} = 13.85\angle -111° \qquad \bar{I}_{c} = 13.85\angle -231°\ A$$

Example 4-3 Solution (2)

Example 4-3 phasor diagram



Phase current leads voltage by 39 degrees

Line current leads voltage by 9 degrees

End Lesson 4: Three Phase Sources and Loads

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