LESSON 8: IDEAL TRANSFORMER THEORY AND OPERATION

ET 332b Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

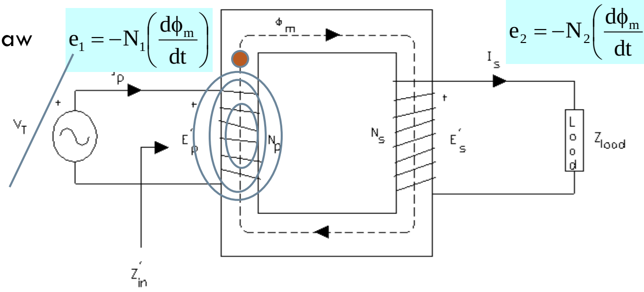
- Explain how an ideal transformer operates
- Find the voltages and currents on both sides of an ideal transformer using the turns ration
- Reflect impedances through a transformer
- Identify and compute the no-load currents that flow in a non-ideal transformer
- Draw the no-load circuit model of a non-ideal transformer.

Ideal Transformer Action

Lentz's Law

$$e_1 = -N_1\left(\frac{d\phi_m}{dt}\right)$$

$$e_2 = -N_2\left(\frac{d\phi_m}{dt}\right)$$



Induced voltage has opposite polarity from source

Principle: Stationary coils, time varying flux due to ac current flow. Flux produced by one coil must link to other coil to induce voltage

Ideal Transformer Action

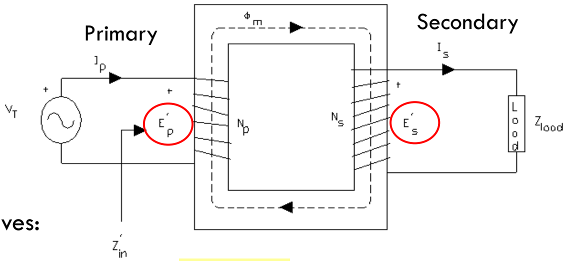
For sinusoidal sources

$$E'_p = 4.44 \cdot N_p \cdot f \cdot \phi_{max}$$

$$E'_s = 4.44 \cdot N_s \cdot f \cdot \phi_{max}$$

Primary

Secondary



Dividing the above equations gives:

$$\frac{E'_p}{E'_s} = \frac{\cancel{4.44} \cdot N_p \cdot \cancel{f} \cdot \cancel{\phi}_{max}}{\cancel{4.44} \cdot N_s \cdot \cancel{f} \cdot \cancel{\phi}_{max}}$$

$$\frac{E'_p}{E'_s} = \frac{N_p}{N_s}$$

Voltage relationship for Ideal transformer

Voltage ratio equals the turns ratio

Where: E'_p = voltage induced in the primary (V)
E'_s = voltage induced in the secondary (V)
N_p = turns in the primary coil
N_s = turns in the secondary coil

Assumptions for Ideal Transformer Operation

1) All flux produced in the primary coil links to the secondary coil
2) no core losses due to hysteresis or eddy currents
3) no power losses
4) permeability is infinite (no saturation no magnetizing ϕ)
5) windings have zero resistance
6) no current required to magnetize the iron core

For ideal transformer

$$a = \frac{E'_p}{E'_s} = \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

Where: a = turns ratio
V_p = nameplate rated primary voltage (higher V)
V_s = nameplate rated secondary voltage (lower V)
E'_p = induced primary voltage
E'_s = induced secondary voltage

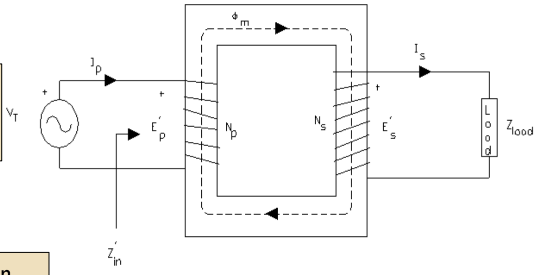
Ideal Transformer Equations

Voltage Ratio

$$a = \frac{E_p}{E_s} = \frac{N_p}{N_s}$$

$$E_p = a \cdot E_s$$

The turns ratio is a scalar. Introduces no phase shift



Apparent Power balance

$$E_p \cdot I_p = E_s \cdot I_s$$

$$S_p = S_s$$

No power losses in idea transformer

Current Ratio

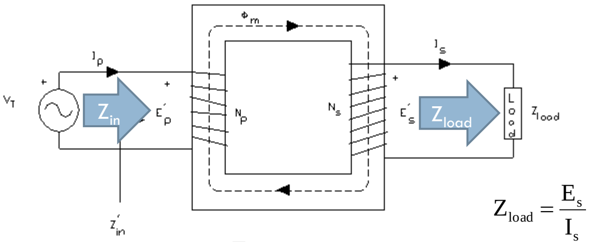
$$\frac{I_p}{I_s} = \frac{1}{a} \qquad I_p = \left(\frac{1}{a}\right) \cdot I_s$$

Current ratio is the inverse of the voltage ratio

Ideal Transformer Equations-Impedance Transforms

Impedances Reflected Through Ideal Transformers

Load impedance as seen from primary side of transformer



By Ohm's Law

$$Z_{in} = \frac{E_p}{I_p}$$

$$Z_{load} = \frac{E_s}{I_s}$$

Write E_s and I_s in terms of primary values

$$E_s = \frac{E_p}{a} \qquad I_s = a \cdot I_p$$

Load impedance is increased when viewed from primary side

$$\frac{E_s}{I_s} = \frac{\left(\frac{E_p}{a}\right)}{a \cdot I_p} = \left(\frac{E_p}{a}\right)\left(\frac{1}{a \cdot I_p}\right) = \left(\frac{E_p}{I_p}\right)\left(\frac{1}{a^2}\right)$$

$$Z_{load} = Z_{in} \cdot \left(\frac{1}{a^2}\right) \quad \Rightarrow \quad Z_{load} \cdot a^2 = Z_{in}$$

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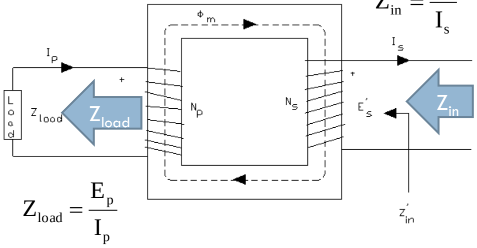
Ideal Transformer Equations-Impedance Transforms

Derive equation when impedances are connected to the primary side and viewed from the secondary side.



$$Z_{in} = \frac{E_s}{I_s}$$

$$Z_{load} = \frac{E_p}{I_p}$$

Write primary values in terms of secondary and substitute in the Z_{load} equation.

$$E_p = a \cdot E_s \qquad I_p = \frac{I_s}{a}$$

$$Z_{load} = \frac{E_p}{I_p} = \frac{a \cdot E_s}{\frac{I_s}{a}} = (a \cdot E_s)\left(\frac{a}{I_s}\right) = a^2 \cdot \left(\frac{E_s}{I_s}\right)$$

Generally : Moving impedance from secondary to primary multiply by a^2. Moving from primary to secondary, divide by a^2.

$$Z_{load} = Z_{in} \cdot a^2 \quad \Rightarrow \quad \frac{Z_{load}}{a^2} = Z_{in}$$

$$Z_p = Z_s \cdot a^2 \qquad \frac{Z_p}{a^2} = Z_s$$

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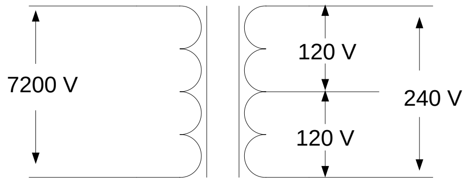
Ideal Transformer Calculations

Example 8-1: A 25 kVA, 7200 - 240/120 center-tap single phase transformer operates at rated voltage. It supplies a single phase load that has an equivalent impedance of 7.2 ∠+36.9° ohms. Assume Ideal operation and find:

- a.) turns ratio
- b.) secondary current
- c.) primary current
- d.) load Z as seen from primary side
- e.) P_T, S_T, Q_T, and F_p



Example 8-1 Solution (1)

a) For ideal transformers

$$\frac{N_p}{N_s} = \frac{V_p}{V_s} \qquad V_p = 7200\ V \quad V_s = 240V \qquad a = \frac{N_p}{N_s} = \frac{7200V}{240V} = 30$$

Ans

b) Secondary current

$$\frac{I_p}{I_s} = \frac{1}{a} \Rightarrow I_p = \frac{1}{a} I_s$$

Use Ohm's law to find I_s

$$\bar{E}_p = 7200\angle 0^\circ\ V$$

$$\bar{E}_s = 240\angle 0^\circ\ V$$

$$\bar{I}_s = \frac{240\angle 0^\circ V}{7.2\angle 36.9^\circ \Omega} \qquad \bar{I}_s = 33.33\angle -36.9^\circ A$$

Ans

Example 8-1 Solution (2)

c) Find the primary current

$\bar{I}_p = \frac{1}{a} \bar{I}_s \quad \bar{I}_p = \left(\frac{1}{30}\right)(33.33\angle -36.9^\circ)$

$\bar{I}_p = 1.11\angle -36.9^\circ \text{ A}$ Ans

d) Find the input impedance as seen from the primary side

$\bar{Z}_{in} = a^2 Z_{Load}$

$\bar{Z}_{in} = (30)^2(7.2\angle 36.9^\circ)$

$\bar{Z}_{in} = 6480\angle 36.9^\circ\ \Omega$ Ans

Example 8-1 Solution (3)

e) Find the power and the power factor of the load

Using secondary side quantities

$\bar{S}_{Ts} = \bar{I}_s^* \bar{V}_s$

$\bar{S}_{Ts} = (240\angle 0^\circ)(33.33\angle -36.9^\circ)^*$

$\bar{S}_{Ts} = (240\angle 0^\circ)(33.33\angle 36.9^\circ)$

$\bar{S}_{Ts} = 8000\angle 36.9^\circ \text{ VA}$ Ans

Using primary side quantities

$\bar{S}_{TP} = \bar{I}_p^* \bar{V}_p$

$\bar{S}_{TP} = (1.111\angle -36.9^\circ)^*(7200\angle 0^\circ)$

$\bar{S}_{TP} = (1.111\angle 36.9^\circ)(7200\angle 0^\circ)$

$\bar{S}_{TP} = 8000\angle 36.9^\circ \text{ VA}$ Ans

Power equal on both sides of ideal transformer

Example 8-1 Solution (4)

Now find the power factor and the active and reactive powers

$P_T = S_T \cos\theta \qquad \theta = 36.9^\circ \qquad Q_T = S_T \sin(\theta)$

$P_T = 8000 \cos(36.9^\circ) \qquad Q_T = 8000 \sin(36.9^\circ)$

$P_T = 6397.5\ \text{W}$ Ans $\qquad Q_T = 4803\ \text{VAR}$ Ans

$$F_p = \frac{P_T}{S_T}$$

$$F_T = \frac{6397.5}{8000} = 0.80\ \text{Lagging}$$ Ans

Ideal Transformer Calculations

Example 8-2: 300 kVA 2400-120, 60 Hz single phase transformer operates at 2300 volts on the primary side. It supplies 115 kVA to a load that has a power factor of 0.723 lagging. Assume idea operation and find:

a.) secondary voltage at operating voltage
b.) secondary current
c.) impedance of the load as seen on the secondary side
d.) impedance of the load as seen on the primary side

Example 8-2 Solution (1)

a) Find secondary voltage at operating voltage

$a = \frac{V_{pr}}{V_{sr}} \quad a = \frac{2400\,V}{120\,V} = 20$

Use rated values to find turns ratio

$V_p = a V_s \Rightarrow \frac{V_p}{a} = V_s$

Transformer operates at $2300\,V = V_p$

$V_s = \frac{2300\,V}{20} = 115\,V$

Ans

b) Find secondary current at operating voltage

$S_p = S_s$

Power is equal on both sides of ideal transformers

$I_s = \frac{S_s}{V_s} \quad I_s = \frac{115{,}000\,VA}{115\,V}$

$I_s = 1000\,A$

Ans

Example 8-2 Solution (2)

c) Find load impedance seen on secondary side

$|\bar{Z}_s| = \frac{V_s}{I_s} \quad |\bar{Z}_s| = \frac{115\,V}{1000\,A} = 0.115\,\Omega$

Next find impedance angle

$F_p = 0.723$ lagging $\quad \theta = \cos^{-1}(F_p)$

$\theta = \cos^{-1}(0.723)$

$\theta = -43.7^{\circ}$

Angle between V and I. Change sign for impedance angle

$\theta = 43.7^{\circ}$

$\bar{Z}_s = 0.115\angle 43.7^{\circ}\,\Omega$

Ans

Example 8-2 Solution (3)

d) Find load impedance seen on primary side of transformer

Reflecting impedance from secondary to primary-multiply by a^2.

$$\bar{Z}_p = a^2 \bar{Z}_s$$

$$\bar{Z}_p = (20)^2 (0.115\angle 43.7^\circ)$$

$$\bar{Z}_p = 46 \angle 43.7^\circ \Omega$$

Ans

Non-Ideal Operation-No-load

Practical transformers draw current with no load connected to secondary winding. Current caused by two non-ideal conditions: power losses and core magnetization

Active power losses

Hysteresis losses - power losses due to repeated change in magnetic polarity. It takes more mmf (NI) to demagnetize core in one direction than the other.

Eddy currents - ac currents induced in iron core due to changing magnetic field

Active power loss Control

Control hysteresis losses - use alloy steels designed for magnetic circuits

Control eddy current losses - laminate core, insulate laminates

Non-Ideal Operation-No-load

A finite amount of current is necessary to drive mutual flux between coils. Permeability is finite so reluctance is finite Some NI = $\mathfrak{F}$ needed.

$$\Re = \frac{\ell}{\mu \cdot A} \qquad \phi_m = \frac{N \cdot I}{\Re}$$

ϕ_m = mutual flux $\Re$ = reluctance

In terms of inductance $L = \frac{N^2}{\Re}$ so core has inductance with associated inductive reactance

Define above as the magnetizing inductance with associated magnetizing reactance X_m

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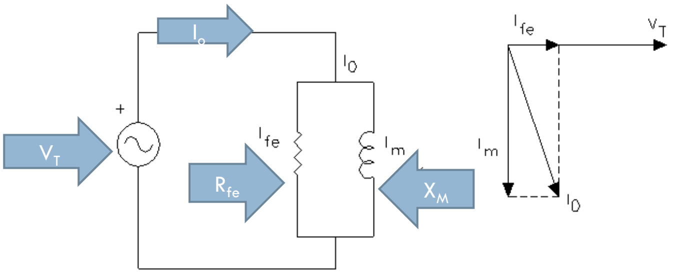
No-Load Circuit Model



V_T = the primary voltage
I_{fe} = core-loss component

I_0 = exciting current
I_M = magnetizing component

R_{fe} = resistance that represents the core losses
X_m = inductive reactance that represents the core magnetizing L

No-Load Circuit Model

Model equation using phasors

$\bar{I}_{fe} = \dfrac{\bar{V}_T}{R_{fe}}$ $\bar{I}_M = \dfrac{\bar{V}_T}{jX_M}$ $\bar{I}_o = \bar{I}_{fe} + \bar{I}_M$

$\bar{I}_o = \left|\bar{I}_{fe}\right| + j \cdot \left|\bar{I}_M\right|$ Add current magnitudes at 90 degrees

No-load apparent power $S_M = V_T \cdot I_o$

Model parameter formulas $P_{fe} = \dfrac{V_T^{\;2}}{R_{fe}} \Rightarrow R_{fe} = \dfrac{V_T^{\;2}}{P_{fe}}$ $X_M = \dfrac{V_T}{I_M}$

Core loss resistance

Magnetizing reactance

No-Load Transformer Example

Example 8-3: Computing the values of magnetizing reactance and core loss resistance. A 50 kVA 7200-240 V, 60 Hz single phase transformer is operating with no load. With the primary connected to a 7200 V system, it draws 248 W and has a power factor of 0.187 lagging. Find:

a) the exciting current and its components
b) the magnitudes of magnetizing reactance, X_M and core loss R
c) Repeat parts a and b if the transformer is energized from the secondary (low voltage) side.

Example 8-3 Solution (1)

a) Find current I_o $P_c = 248\ \text{W}$ $F_p = 0.187$ Lagging

S_M = magnetizing apparent power $S_M = \frac{P_c}{F_p}$ $S_M = \frac{248\ \text{W}}{0.187} = 1326.2\ \text{VA}$

$S_M = V_T I_o$ $\frac{S_M}{V_T} = \frac{1326.2\ \text{VA}}{7200\ \text{V}} = I_o$

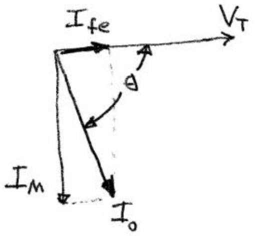
$I_o = 0.1842\ \text{A}$

$\theta = \cos^{-1}(F_p)$

$\theta = \cos^{-1}(0.187)$

$\theta = 79.2^\circ$ $\bar{I}_o = 0.1842\angle 79.2^\circ$ Ans

Example 8-3 Solution (2)



$I_M = 0.1842\sin(79.2^\circ)$

$I_M = 0.1809\ \text{A}$ Ans

$I_{fe} = 0.1842\cos(79.2^\circ)$

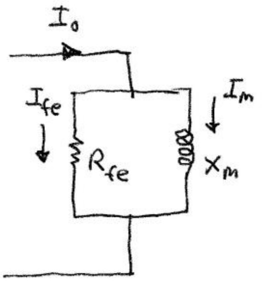
$I_{fe} = 0.0345\ \text{A}$ Ans

b) Find the value of core loss resistance and magnetizing reactance

$P_c = \frac{V_P^2}{R_{fe}} \Rightarrow R_{fe} = \frac{V_P^2}{P_c}$ $R_{fe} = \frac{(7200)^2}{248\ \text{W}} = 209{,}032\ \Omega$ Ans

Example 8-3 Solution (3)

$$X_m = \frac{V_P}{I_m} = \frac{7200V}{0.1809A} = 39,801\,\Omega$$



c) Find same parameters on secondary side

Power constant through transformer

$P_c = 248\,W$ $\quad S_m = 1326.2\,VA$

$V_s = 240V$ $\quad S_m = V_s I_s$

$$\frac{S_m}{V_s} = I_s \quad \frac{1326.2VA}{240\,V} = I_s \quad 5.525A = I_s$$

Ans

Example 8-3 Solution (4)

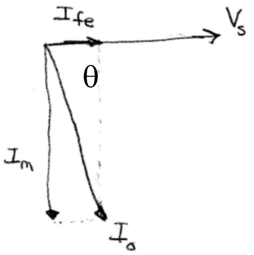
θ = angle between V_s and I_o

$\theta = \cos^{-1}(0.187)$

$\theta = 79.2°$

$I_{fe} = 5.525 \cos(79.2°)$

$I_{fe} = 1.0353\ A$



Compute R_{fe} and X_M

$I_m = 5.525 \sin(79.2°)$

$I_m = 5.427\ A$

$$R_{fe} = \frac{V_s^2}{P_c} \quad R_{fe} = \frac{(240)^2}{248\,W} = 232.2\,\Omega$$

Ans

Example 8-3 Solution (5)

$X_m = \frac{V_s}{I_m}$ $X_m = \frac{240V}{5.427A}$ $X_m = 44.2\ \Omega$ Ans

Compare using turns ratio transfer

$a = \frac{V_p}{V_s}$

$a = \frac{7200V}{240V}$

$a = 30$

$R_{fep} = R_{fe} a^2$

$R_{fep} = (232.2\Omega)(30)^2$

$R_{fep} = 208,980\Omega$

209,032 on primary

$X_{mp} = X_m a^2$

$X_{mp} = 44.2(30)^2$

$X_{mp} = 39,780\Omega$

39,801 on primary

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End Lesson 8: Ideal Transformer Theory and Operation