Lesson 20
Evaluating Economic Alternatives

Learning Objectives

After this presentation you will be able to:

- Formulate a present worth calculation to determine the best economic alternative
- Use time value of money formulas to compute present values.
- Evaluate and interpret the results of present worth analysis
- Consider the limitations of economic models and suggest improvements
Economic Comparison Techniques

Economic analysis selects the most financially beneficial solution to a technical problem from a number of viable technical solutions. In energy management, this is a number of energy management opportunities that must be ranked according to economic benefits.

Methods of comparison

- **Present Worth**: Bring all costs to time zero and compare
- **Future Worth**: Project all costs to future time and compare
- **Annual Worth**: Bring all costs to time 0; Project result across n as uniform series

**Benefit/Cost Ratio**

- BCR = PWB/PWC
- BCR > 1 attractive

**Internal ROR (IRR)**

- ROR that makes present worth of costs = present worth of benefits
- IRR > MARR

MARR = minimum attractive rate of return
Present Worth Analysis

Procedure: Step 1:

Establish lifetimes of projects or equipment. Projects must be over the same time horizon. If unequal, use least common multiple.

Example: Project 1 2-year life, Project 2 3-year life. Study period 6 years.

<table>
<thead>
<tr>
<th>Project 1</th>
<th>2 years</th>
<th>2 years</th>
<th>2 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Project 2</td>
<td>3 years</td>
<td>3 years</td>
<td></td>
</tr>
</tbody>
</table>

Study period = 6 years
Repeat all costs for repetitions of both projects

Procedure: Step 2: Use present worth formulas to bring all costs and benefits to time 0.

Present Worth Analysis

Procedure: Step 3: Compare present worth values of all alternatives.

Procedure: Step 4: Decision process: select alternative with least present worth cost, maximum benefits, or greatest income.

Example: A three injection molding machines are considered as energy saving alternatives. The present values of their costs are listed below.

<table>
<thead>
<tr>
<th>Molder 1</th>
<th>PW$_1$ = $17,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Molder 2</td>
<td>PW$_2$ = $19,000</td>
</tr>
<tr>
<td>Molder 3</td>
<td>PW$_3$ = $15,500</td>
</tr>
</tbody>
</table>

What is the most economical choice on a present worth basis?

The choice with the lowest present worth cost should be selected.
Present Worth Analysis Example

Selecting economic alternatives

The cost of gasoline is at an all-time high and older cars are becoming more expensive to own and operate. Consumers can buy new hybrid cars that combine internal combustion engines with electric drives and battery storage. The hybrid cars achieve gas mileage values of 70 mpg. High demand for these cars keeps their price high compared to standard vehicles. If a person drives an older car with lower gas mileage they have high operating and maintenance expenses but will not incur the cost of buying a new car and paying loan interest or paying for the car from savings. Consider the following problem data:

Annual average mileage: 21,000 miles
Hybrid purchase price: $23,000
Gas Mileage of hybrid: 55 mi/gal
Gas Mileage of present car: 21 mi/gal
Annual maintenance-hybrid: $450
Annual maintenance-old: $1500
Estimated hybrid Life: 7 years
Interest rate: 6.5%
Fuel cost: $2.50/gal

The lower gas mileage car has been paid for and is expected to last for 7 years.

Consider the following three alternatives: 1.) keep the old low gas mileage car for 7 more years, 2.) buy a hybrid car using a bank loan at 7.5% interest and 10% down. 3.) buy the hybrid using savings and take out no loan. Use Present Worth analysis to determine which of the above is the best economic choice.

Present Worth Analysis Example

Solution:

Make the following definitions:

- FC\_h = hybrid annual fuel cost
- MC\_h = hybrid annual maintenance cost
- AP\_h = annual payments on hybrid loan
- DP\_h = hybrid car down payment
- PC = hybrid car purchase cost
- FC = old car annual fuel cost
- MC = old car annual maintenance cost
- AM = average mileage per year
- M\_h = gas mileage of hybrid
- M = gas mileage of old car
- C\_F = cost of fuel
Present Worth Analysis Example

Solution 1: Find the Present Worth of owning the old car for 7 more years

The timeline below shows the costs associated with operating and maintaining the old car for 7 years.

<table>
<thead>
<tr>
<th>Year</th>
<th>Fuel Cost (FC)</th>
<th>Maintenance Cost (MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>1</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>2</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>3</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>4</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>5</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>6</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
<tr>
<td>7</td>
<td>$2,500</td>
<td>$1,500</td>
</tr>
</tbody>
</table>

Only fuel costs and maintenance costs are associated with owning the old car for 7 more years.

- Annual mileage: AM = 21,000 miles
- Fuel Cost per gallon: C_f = 2.500 $/gal
- Annual maintenance costs: MC = $1,500
- Old car gas mileage: M = 21 mi/gal

Compute the annual fuel cost of the older car using the following formula:

\[ FC = C_F \frac{AM}{M} \]

\[ FC = 2,500 \]

Both the fuel and maintenance cost series must be returned to time zero for a present worth analysis. Use formula for present worth of a series with the following parameter values.

Number of years \( n = 7 \)

Interest rate \( i = 0.065 \)

\[ PVS(n, i, A) = A \left( \frac{(1+i)^n - 1}{i(1+i)^n} \right) \]

\[ PW_1 := PVS(n, i, FC) + PVS(n, i, MC) \]

\[ PW_1 = 21,938.08 \]

Answer

Present worth of keeping the old car for the next 7 years.
Present Worth Analysis Example

Solution 2: Find the present worth of purchasing the hybrid with cash and owning it for 7 years.

Maintenance Cost-Hybrid: $450
Fuel Cost per gallon: \( C_f = 2.500 \frac{\text{$/gal}}{} \)
Annual mileage: \( AM = 21,000 \text{ miles} \)

Hybrid car gas mileage: \( \frac{AM}{M_h} = 55 \text{ mi/gal} \)

Purchase cost of Hybrid: \( PC = $23,000 \)

\[
FC_h = C_f \frac{AM}{M_h}
\]

Fuel cost of hybrid \( FC_h = 954.55 \)

The timeline below shows the costs associated with purchasing, operating and maintaining the hybrid for 7 years.

Present Worth Analysis Example

The timeline below shows the costs associated with purchasing, operating and maintaining the hybrid for 7 years.

- PC = car purchase cost
- \( FC_h \) = annual fuel cost of hybrid
- \( MC_h \) = annual maintenance of hybrid

Find the present worth of the two series

\[
PW_a = PV(\frac{n}{i}, FC_h) + PV(\frac{n}{i}, MC_h)
\]

Number of years: \( n = 7 \)
Annual interest rates: \( i = 0.065 \)

\( PW_a = $7,703.26 \)

Present worth of the fuel and maintenance costs

$954.55
$450
Present Worth Analysis Example

To complete this present worth analysis, add the hybrid purchase cost

\[ PW_2 = FC + PW_h \]

\[ PW_h = $30,703.26 \]  \hspace{1cm} \text{Answer}

**Solution 3:** Find the present worth of taking out a loan to buy the car and owning the hybrid for 7 years.

To simplify the calculations assume that the loan will be paid back in seven equal year end payments. Use the capital recovery function to find the annual payment using a loan rate of 7.5%.

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Present Worth Analysis Example

Define the variables for this solution

<table>
<thead>
<tr>
<th>Define the down payment fraction</th>
<th>( r_{DP} = 0.1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of years ( n )</td>
<td>( n = 7 )</td>
</tr>
<tr>
<td>Define the loan rate ( i_l )</td>
<td>( i_l = 0.075 )</td>
</tr>
<tr>
<td>Purchase Cost ( FC )</td>
<td>( FC = 23000 )</td>
</tr>
<tr>
<td>Down payment ( DP )</td>
<td>( DP = (1 - r_{DP}) \times FC = 2300 )</td>
</tr>
<tr>
<td>Define the loan amount ( LAM )</td>
<td>( LAM = FC - DP )</td>
</tr>
<tr>
<td>Calculate loan payment ( AP_h )</td>
<td>( AP_h = \frac{F(n, i_l, LAM)}{A} = 3,908.17 )</td>
</tr>
</tbody>
</table>

Capital Recovery Formula

\[ P(n, i, PA) = PA \frac{(1 + i)^n - 1}{(1 + i)^n - 1} \]

\[ n = 7 \]

\[ i = 0.075 \]

\[ AP_h = \frac{F(n, i_l, LAM)}{A} = 3,908.17 \]

\[ LAM = 20700 \]

\[ $20,700 \]
Present Worth Analysis Example

The timeline below shows the costs associated with purchasing, operating and maintaining the hybrid for 7 years. The car is financed with a 7.5% loan and a 10% down payment.

\[
\begin{array}{cccccccc}
\text{DP}_h = $2,300 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\text{FC}_h & \text{MC}_h & \text{AP}_h & \text{FC}_h & \text{MC}_h & \text{AP}_h & \text{FC}_h & \text{MC}_h \\
\end{array}
\]

The loan payment, fuel and maintenance cost series must be returned to time zero and then added to the down payment for a present worth analysis. The buyer's interest rate of 6.5% is used.

\[ n = 7 \quad i = 0.065 \]

Find the present worth of the loan series:

\[ \text{PVS}(n,i,\text{AP}_h) = 21,434.42 \]

Present Worth Analysis Example

The present worth of the loan series is greater than the loan amount because the buyer is investing money with a lower return. The lenders charge a higher rate of return. The difference is the lender's profit.

Add the present values of the maintenance, fuel cost and load payment series to the down payment:

\[ \text{PW}_2 := \text{PVS}(n,i,\text{MC}_h) + \text{PVS}(n,i,\text{FC}_h) + \text{PVS}(n,i,\text{AP}_h) + \text{DP} \]

Present Worth of the hybrid purchased with a loan \( \text{PW}_3 = 31,437.67 \)
Present Worth Analysis Example

Arrange the results of the present worth calculations in increasing order.

| PW1  | £2093.68 | Present worth of keeping the old car for the next 7 years |
| PW2  | £20793.26 | Present worth of buying the hybrid using cash at time zero. |
| PW3  | £3457.67 | Present value of the hybrid purchased with a loan |

Keeping the old car for the next 7 years produces the lowest present worth cost and should be selected based on the assumptions of the problem. A more realistic model would include the fuel costs that increase over the life of the cars. The maintenance costs of the old car would likely increase over time also and could be included in a more complex analysis.

End Lesson 20
Evaluating Economic Alternatives

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