

◆ **Figure 6.12** A three-op-amp instrumentation amplifier consists of an input stage and a differential amplifier. The user can adjust the size of the external resistor to obtain a wide range of gains.

Equating the right side of the two equations for  $i$ , we have

$$\frac{v_b - v_a}{2R_1 + R_e} = \frac{v_{ib} - v_{ia}}{R_e}$$

$$v_b - v_a = \left( \frac{2R_1 + R_e}{R_e} \right) (v_{ib} - v_{ia})$$

Finally, replacing  $v_b - v_a$  in Equation (6.18) by the right side of the above equation gives us the following equation for the instrumentation amplifier:

INSTRUMENTATION AMPLIFIER  
 IN THE LINEAR OPERATING REGION

$$v_{out} = \left( \frac{2R_1 + R_e}{R_e} \right) \left( \frac{R_3}{R_2} \right) (v_{ib} - v_{ia}) \quad (6.19)$$

The user can adjust the size of external resistor  $R_e$  to obtain a wide range of gains without increasing the common-mode error signal.

**EXAMPLE 6.7** An instrumentation amplifier similar to Figure 6.12 has the following precision resistor values:

$$R_1 = 1000 \, \Omega, R_2 = 1000 \, \Omega, \text{ and } R_3 = 1000 \, \Omega$$

Determine the value of the external resistor,  $R_e$ , that will result in a gain of 1000.

*Solution*

$$\text{IA gain} = \left( \frac{2R_1 + R_e}{R_e} \right) \left( \frac{R_3}{R_2} \right)$$

The *two-wire bridge circuit* has the same lead-wire error as the two-wire direct method. However, the bridge circuit has a decided advantage. The RTD resistance is measured in terms of accurately calibrated bridge resistors, independent of the bridge voltage value. Refer to Section 6.4 for a detailed analysis of self-balancing and unbalanced bridge circuits.

The *three-wire bridge circuit* corrects most of the effect of the lead-wire resistance. The lead wires are matched so they have the same resistance. One lead wire is placed in the  $R_2$  side of the bridge, and the other is placed in the  $R_3$  side. When  $R_2$  and  $R_3$  are equal, the lead-wire effect is virtually eliminated. When  $R_2$  and  $R_3$  are not equal, there will be a small lead-wire error. Many RTDs are supplied with three-wire leads.

**EXAMPLE 8.2** Use the values in Table 8.1 to determine the average sensitivity of the platinum RTD over the range from 0 to 100°C. Then assume the two-wire direct method is used to measure the RTD resistance and determine the lead-wire error caused by 10 ft of each of the following sizes of copper wire:

| Wire Gage | Resistance ( $\Omega$ /ft) |
|-----------|----------------------------|
| 12        | 0.00162                    |
| 14        | 0.00258                    |
| 16        | 0.00409                    |
| 18        | 0.00651                    |
| 20        | 0.0104                     |
| 24        | 0.0262                     |
| 28        | 0.0662                     |
| 32        | 0.167                      |

*Solution*

$$\text{Sensitivity} = \frac{139.9 - 100.0}{100 - 0} = 0.393 \Omega/^{\circ}\text{C}$$

$$\text{Error} = \frac{\text{lead-wire resistance, } \Omega}{\text{sensitivity, } \Omega/^{\circ}\text{C}}, ^{\circ}\text{C}$$

$$\text{Error (12 gage)} = \frac{10(0.00162)}{0.393} = 0.041^{\circ}\text{C}$$

The results are summarized in the following table:

| Wire Gage | Ten-Foot Lead-Wire Error ( $^{\circ}\text{C}$ ) |
|-----------|---|
| 12        | 0.041   |
| 14        | 0.066   |
| 16        | 0.104   |
| 18        | 0.166   |
| 20        | 0.265   |
| 24        | 0.667   |
| 28        | 1.684   |
| 32        | 4.249   |

**EXAMPLE 8.3** Given the following data for the circuit in Figure 8.5b:

$$i_s = 1 \text{ mA}$$

The sensor is a platinum RTD as defined in Table 8.1

$$R_s = 100 \text{ to } 139.3 \ \Omega \text{ when } T = 0 \text{ to } 100^\circ\text{C}$$

- (a) Determine  $v_s$  when  $T = 0^\circ\text{C}$  and when  $T = 100^\circ\text{C}$ .  
 (b) Design an op-amp circuit that will convert  $V_s$  into a 4- to 20-mA current signal as follows:

$$i_{\text{out}} = 4 \text{ mA when } T = 0^\circ\text{C}$$

$$i_{\text{out}} = 20 \text{ mA when } T = 100^\circ\text{C}$$

*Solution*

- (a) Current  $i_s$  passes through the sensor resistance,  $R_s$ , and produces the following range for  $v_s$ :

$$v_s(0^\circ\text{C}) = (0.001 \text{ A})(100 \ \Omega) = 0.1 \text{ V}$$

$$v_s(100^\circ\text{C}) = (0.001 \text{ A})(139.3 \ \Omega) = 0.1393 \text{ V}$$

- (b) The design, shown in Figure 8.6, uses four stages and a regulated voltage source that produces  $\pm 1.5 \text{ V}$ . The first stage provides the current source used by the four-wire direct method. It uses the 1.5-V source and a precision 1.5-k $\Omega$  resistor to produce the constant 1-mA source,  $i_s$  (see Exercise 6.15 and Figure 6.40.)

$$i_s = \frac{1.5 \text{ V}}{1.5 \text{ k}\Omega} = 1 \text{ mA}$$

The second stage is an instrumentation amplifier that isolates the sensor with its high input impedance and amplifies voltage  $v_s$  by a factor of 100 (see Figure 6.12 and Equation 6.19). The output of the instrumentation amplifier,  $v_1$ , has the following range:

$$v_1(0^\circ\text{C}) = 100v_s(0^\circ\text{C}) = 100(0.1) = 10.0 \text{ V}$$

$$v_1(100^\circ\text{C}) = 100v_s(100^\circ\text{C}) = 100(0.1393) = 13.93 \text{ V}$$

The third stage is a summing amplifier whose output,  $v_2$ , is equal to  $v_1$  minus a constant voltage value,  $v_0$ . The value of  $v_0$  is adjusted so that the ratio between the maximum and minimum values of  $v_2$  is that same as the ratio between the maximum and minimum values of the output current.

$$\frac{v_2(100^\circ\text{C})}{v_2(0^\circ\text{C})} = \frac{13.93 - v_0}{10.0 - v_0} = \frac{20 \text{ mA}}{4 \text{ mA}} = 5$$

$$13.93 - v_0 = 5(10.0 - v_0)$$

Solving the last equation gives the following value for  $v_0$  and range for  $v_2$ .

$$v_0 = 9.0175 \text{ V}$$

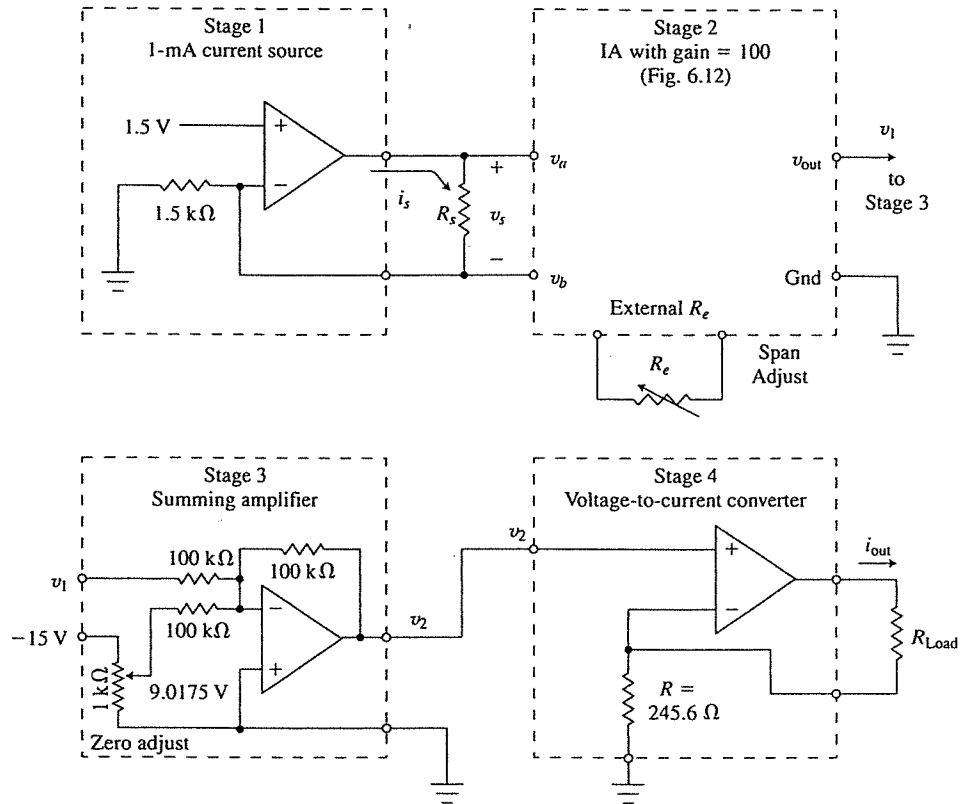
$$v_2(100^\circ\text{C}) = 13.93 - 9.0175 = 4.9125 \text{ V}$$

$$v_2(0^\circ\text{C}) = 10 - 9.0175 = 0.9825 \text{ V}$$

You may wish to verify that  $v_2(100^\circ\text{C})/v_2(0^\circ\text{C}) = 5.000$ .

The fourth stage is a voltage-to-current converter (see Figure 6.14b). Resistor  $R$  is sized to produce the required 4- to 20-mA output current. Using the maximum values,  $R$  has the following value:

$$R = \frac{v_2}{i_{\text{out}}} = \frac{4.9125}{0.020} = 245.6$$



◆ **Figure 8.6** This four-stage signal conditioner converts the resistance of an RTD sensing element into a 4- to 20-mA output signal. The circuit uses the four-wire direct method for measuring resistance (see Example 8.3).

You may wish to verify that the minimum values result in the same value for resistor  $R$ .

One final observation. The adjustable resistor in stage 2 provides a means of span adjustment, and the adjustable value of  $v_0$  provides the zero adjustment of the transmitter. ◆

The current used to measure the value of a resistance element also dissipates power in the form of heat. The power dissipated in the element raises the temperature of the element, resulting in an error called *the self-heating error*. The size of the self-heating error depends on three factors: (1) the power dissipated in the element, (2) the internal thermal resistance of the element, and (3) the thermal resistance of the film between the probe and the surrounding fluid.

The self-heating error of an RTD is given by the dissipation constant, a number that specifies the amount of power required to raise the temperature of the sensing element by 1°C (or 1°F). The specification of the dissipation constant includes one of the following environmental conditions: still air, air moving at 1 m/s, still water, water moving at 1 m/s, still oil, or oil moving at 1 m/s. Commercial RTDs with a diameter of 1.5 to 5 mm have dissipation constants on the order of 1 to 10 mW/°C in still air, on the order of 3 to 30 mW/°C in air mov-

ing at 1 m/s, and on the order of 14 to 140 mW/°C in water moving at 1 m/s. The self-heating error is given by the following equation:

$$\text{Self-heating error} = \frac{i_s^2 R_s}{P_{\text{diss}}}, \text{ } ^\circ\text{C}$$

where  $i_s$  = sensor current, A  
 $R_s$  = sensor resistance,  $\Omega$   
 $P_{\text{diss}}$  = dissipation constant, W/°C

**EXAMPLE 8.4** An RTD is used to measure the temperature of air flowing in a duct at 1 m/s. The dissipation constant for this environment is 20 mW/°C. The air temperature is 50°C, and the RTD has a resistance of 200  $\Omega$  at 50°C. Determine the self-heating error for each of the following sensor currents: 1 mA, 10 mA, and 20 mA.

*Solution* For  $i_s = 1$  mA:

$$\text{Self-heating error} = (0.001)^2 \left( \frac{200}{0.02} \right) = 0.01^\circ\text{C}$$

For  $i_s = 10$  mA:

$$\text{Self-heating error} = (0.01)^2 \left( \frac{200}{0.02} \right) = 1.0^\circ\text{C}$$

For  $i_s = 20$  mA:

$$\text{Self-heating error} = (0.02)^2 \left( \frac{200}{0.02} \right) = 4.0^\circ\text{C}$$

◆

## Thermistors

A *thermistor* is a semiconductor temperature sensor whose resistance changes inversely with temperature. The resistance versus temperature response of a thermistor is nonlinear and very sensitive. Small changes in temperature produce large decreases in resistance. Consequently, thermistors are often used where high sensitivity is required and accurate temperature indication is not, for example, sensing for high or low temperature limits.

Desirable features of thermistors include small size, fast response, narrow span, and high sensitivity. Less desirable features include a very nonlinear calibration graph, poor high-temperature stability, unsuitability for large spans, and high impedance. Most thermistor applications are in the range from  $-80$  to  $+150^\circ\text{C}$ . Table 8.2 lists resistance versus temperature values for a typical thermistor.

Signal conditioning of thermistors involves the measurement of relatively large changes in resistance. The direct methods and bridge methods in Figure 8.5 can be used, but close attention must be given to the self-heating error. Typical thermistors have dissipation constants on the order of 1 to 10 mW/°C in still air and 25 to 250 mW/°C in a well-stirred oil bath. The relatively high resistance of thermistors, especially in the low end of the temperature range, exacerbates the potential for self-heating errors. The voltage divider circuit shown in Figure 8.7 can be used as a signal conditioner for thermistors. The nonlinearity of the voltage divider compensates for the nonlinearity of the thermistor, resulting in an output voltage that is rea-

The magnitude of the thermocouple emf (voltage) depends on the temperature difference between the two junctions and the material used in the two dissimilar wires. There are many possible materials for use in thermocouples, but only a limited number are used for thermocouples, and most of those are alloys that were developed specifically for such use. The most commonly used and available thermocouples have been standardized. The American National Standard (ANSI) MC 96.1 established type designations for thermocouples (see Table 8.3), thermocouple wire, and thermocouple extension wire. The platinum/platinum-10% rhodium (ANSI Type S) thermocouple is especially important because it is used to define the International Temperature Scale between 630.5 and 1063°C.

The National Bureau of Standards has published reference tables that show the emf (in millivolts) generated by the standard thermocouples in small increments of temperature over their useful range. Tables in degrees Fahrenheit and in degrees Celsius are contained in the National Bureau of Standards (NBS) Monograph 125 (U.S. Government Printing Office, Washington, DC). Figure 8.8 shows the graphs of four standard thermocouples over their useful range. Observe the different sensitivities, useful ranges, and nonlinearities displayed in the four graphs. Table 8.4 shows the millivolts generated by six standard thermocouple types in 100°C increments over their useful ranges. The table clearly shows the different sensitivities and useful ranges of the six types, but their nonlinearities are more apparent on a graph.

◆ **TABLE 8.4** Output Voltages of Six Thermocouples Over Their Useful Range of Temperatures in 100°C Increments (The Thermocouple Reference Temperature is 0°C)

| Temperature<br>(°C) | Output Voltage (mV) |       |                    |        |        |        |
|---------------------|---------------------|-------|--------------------|--------|--------|--------|
|                     | E                   | J     | K                  | R      | S      | T      |
| -190                | -8.45               | -7.66 | -5.60              |        |        | -5.379 |
| -100                | -5.18               | -4.63 | -3.49              |        |        | -3.348 |
| 0                   | 0.00                | 0.00  | 0.00               | 0.000  | 0.000  | 0.000  |
| 100                 | 6.32                | 5.27  | 4.10               | 0.645  | 0.643  | 4.277  |
| 200                 | 13.42               | 10.78 | 8.13               | 1.465  | 1.436  | 9.288  |
| 300                 | 21.04               | 16.32 | 12.21              | 2.395  | 2.316  | 14.864 |
| 400                 | 28.95               | 21.85 | 16.40              | 3.399  | 3.251  | 20.873 |
| 500                 | 37.01               | 27.39 | 20.65              | 4.455  | 4.221  |        |
| 600                 | 45.10               | 33.11 | 24.91              | 5.563  | 5.224  |        |
| 700                 | 53.14               | 39.15 | 29.14              | 6.720  | 6.260  |        |
| 800                 | 61.08               | 45.53 | 33.30              | 7.925  | 7.329  |        |
| 900                 | 68.85               |       | 37.36              | 9.175  | 8.432  |        |
| 1000                | 76.54               |       | 41.31              | 10.471 | 9.570  |        |
| 1100                |                     |       | 45.16              | 11.817 | 10.741 |        |
| 1200                |                     |       | 48.89              | 13.193 | 11.935 |        |
| 1300                |                     |       | 52.46              | 14.583 | 13.138 |        |
| 1400                |                     |       | 54.88 <sup>a</sup> | 15.969 | 14.337 |        |
| 1500                |                     |       |                    | 17.355 | 15.530 |        |
| 1600                |                     |       |                    | 18.727 | 16.715 |        |
| 1700                |                     |       |                    | 20.090 | 17.891 |        |
| 1765                |                     |       |                    |        | 18.648 |        |

<sup>a</sup> The value 54.88 mV is at a temperature of 1370°C.