Lesson 7: Anti-Aliasing Filtering

ET 438b Sequential Control and Data Acquisition
Department of Technology

Learning Objectives

After this presentation you will be able to:

- Explain why anti-aliasing filters are used.
- Design a first order anti-aliasing filter using an OP AMP
- Design a 2nd order anti-aliasing filter using an OP AMP
- Verify the performance of an anti-aliasing filter using simulation software.
Anti-Aliasing Filters

Dealing with Aliasing in practical systems

- Exact frequency components of a sampled signal are unknown.
- Can not determine if signal component is alias or real.
- Sampling rate limited by hardware selection.
- Lab DAQ cards rate is 200 kHz - 250 kHz.
- \( f_{\text{nyquist}} = 100 \text{ kHz} - 125 \text{ kHz} \) sets input frequency limits.
- Bandwidth limit input signals using anti-aliasing filters to eliminate frequencies above \( f_{\text{nyquist}} \).

Ideal low pass filter Bode plot:
- **Goal**: reduce amplitude of all frequencies above \( f_{\text{nyquist}} \) to zero level.

Practical (Butterworth) filters have sloping characteristics:
- 1st Order: -20 dB/decade
- 2nd Order: -40 dB/decade
- 3rd Order: -60 dB/decade

Set \( f_c = f_{\text{nyquist}} \) for perfect signal elimination.
Anti-Aliasing Filters: OP AMP Circuits

First order Butterworth filter

Design formulas

\[ A_v = \frac{-R_2}{R_1} \quad \text{Voltage gain in the pass band} \]
\[ f_c = \frac{1}{2\pi \cdot R_2 \cdot C_1} \quad \text{Cut-off frequency (Hz)} \]

Design Procedure

1.) Determine the acceptable level of signal gain, \( A_v \), at sampling frequency \( f_s \)
2.) Use the formula below to determine the value of \( f_c \) based on the designed signal gain

\[ f_c = \frac{f_s}{2} \cdot \sqrt{1 - \frac{A_v^2}{A_v'^2}} \quad \text{for } A_v < 1 \]

3.) Select a value of \( C_1 \) from standard values and compute value of \( R_2 \)

Design Procedure (continued)

4.) Set \( R_1 = R_2 \) to give pass band gain of -1 (0 dB). Amplify signal after filtering to reduce noise and unwanted signal components. (e.g. 60 Hz)

Design Example: A data acquisition system samples an analog signal at \( T_s = 0.0004 \) seconds. Design a 1st order anti-aliasing filter that will reduce the voltage level of all signal above the Nyquist frequency to 0.4

Solution: Determine the sampling frequency from \( T_s \) then find the \( f_{\text{nyquist}} \)

\[ f_s = \frac{1}{T_s} = \frac{1}{0.0004} = 2500 \text{ Hz} \]
\[ f_{\text{nyquist}} = \frac{f_s}{2} = \frac{2500}{2} = 1250 \text{ Hz} \]

Find the value of \( f_c \) for the given level of \( A_v \) above \( f_{\text{nyquist}} \)

\[ f_c = \frac{f_s}{2} \cdot \sqrt{1 - \frac{A_v^2}{A_v'^2}} \]

\[ = \frac{2500}{2} \cdot \sqrt{1 - \frac{0.4^2}{0.4^2}} = \frac{2500}{2} \cdot \sqrt{1 - 0.4^2} = \frac{2500}{2} \cdot \sqrt{0.525} = 546 \text{ Hz} \]
Design Example (continued)

Select a capacitor value of 0.1 μF. Use larger values for lower f’s to keep values of resistors in range of 1k to 820k.

\[ f_c = \frac{1}{2\pi R2 \cdot C1} \]
\[ R2 = \frac{1}{2\pi \cdot 0.1 \times 10^{-6} \cdot 0.546} = 2916 \Omega \]

Set \( R1 = R2 \) to give gain of -1.

Design complete. Check result with circuit simulation.

Anti-Aliasing Filters - 2\(^{nd}\) Order Filters

Second order Butterworth filter

Unity gain is fixed by negative feedback loop in this design.

Use same design procedure as 1\(^{st}\) order filter but use following formula to find cut-off frequency.

\[ f_c = \frac{f_s}{2 \sqrt{1 - A_c^2}} \text{ for } A_c < 1 \]

2\(^{nd}\) order filter produces -40 dB/decade rolloff in stop band.
Anti-Aliasing 2\textsuperscript{nd} Order Filter Design

Design Example 2: Repeat the design of the 1\textsuperscript{st} order filter example using a 2\textsuperscript{nd} order filter

\( T_s = 0.0004 \text{ seconds sampling} \). Design a 2\textsuperscript{nd} order anti-aliasing filter that will reduce the voltage level of all signal above the Nyquist frequency to 0.4 Hz

Same \( f_s \) and \( f_{nyquist} \) as previous case

\[
f_s = \frac{1}{T_s} = \frac{1}{0.0004} = 2500 \text{ Hz}
\]
\[
f_{nyquist} = \frac{f_s}{2} = \frac{2500 \text{ Hz}}{2} = 1250 \text{ Hz}
\]

Find the value of \( f_c \) for the given level of \( A_v \) using 2\textsuperscript{nd} order filter formula

\[
f_c = \frac{f_s}{2 \cdot \sqrt{1 - \frac{A_v^2}{A_c^2}}} = \frac{2500 \text{ Hz}}{2 \cdot \sqrt{1 - 0.4^2}} = \frac{2500 \text{ Hz}}{2 \cdot \sqrt{0.75}} = 826 \text{ Hz}
\]

Design Example (continued)

Select value of \( C_2 \) and compute \( C_3 \).

Let \( C_2 = 0.1 \mu F \) so.....

\[
C_3 = 2 \cdot C_2 = 2 \cdot (0.1 \mu F) = 0.2 \mu F
\]

Use design formulas to find value of \( R_4 = R_5 \)

\[
f_c = \frac{1}{2 \pi \cdot \sqrt{2 \cdot R_4 \cdot R_5 \cdot C_3 \cdot C_2}} \quad \text{With } R_4 = R_5 \quad \text{and } C_3 = 2 \cdot C_2
\]

\[
f_c = \frac{1}{2 \pi \cdot \sqrt{2} \cdot R_4 \cdot C_2^2} = \frac{1}{2 \pi \cdot \sqrt{2} \cdot R_4 \cdot C_2}
\]

Solve for \( R_4 \) given \( C_2 \) and \( f_c \)

\[
f_c = \frac{1}{2 \pi \cdot \sqrt{2} \cdot R_4 \cdot C_2}
\]

\[
R_4 = \frac{1}{2 \pi \cdot \sqrt{2} \cdot C_2 \cdot f_c}
\]

Use standard value of 1.5k.

Response very sensitive to \( R \) values

\[
R_4 = \frac{1}{2 \pi \cdot \sqrt{2} \cdot 0.1 \times 10^{-6} \text{ F} \cdot 826 \text{ Hz}} = \frac{1}{2 \pi \cdot 1363 \Omega} \approx R_5
\]
Design Example (continued)

Design complete. Check result with circuit simulation.

\[ v = -9.4 \text{ dB} \]

\[ \frac{\text{dB}}{10} = \log(A_v) \]

\[ A_v = 10^{\frac{-9.4}{20}} = 0.34 \]

Flatter response in pass band sharper roll off.

End Lesson 7: Anti-Aliasing Filtering

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