

ET304A
Electric Circuits Laboratory
Lab 3
Nodal Analysis and Measurement Error Estimation

PURPOSE: In this lab, nodal analysis calculations find the theoretical circuit voltages with respect to the ground. Compare these values to the measured values of voltage. Current and power calculations made from the measured node voltages will illustrate the propagation of error and measurement uncertainty in laboratory work. Formulas for the calculation of measurement uncertainty are introduced and used to determine the limiting error in Ohm's law and power calculations.

Theoretical Background

Errors and uncertainty are a part of every experiment and engineering design testing. Gross errors and statistical errors comprise the two major groups of experimental error. Gross errors are due to blunders and mistakes made by the humans the conduct the experiments and tests. An example of a gross error is reading the incorrect scale on a meter. Statistical errors are due to randomness in the measurement process and equipment.

Statistical error can be further subdivided into fixed errors and random errors. Fixed errors, also called systematic or bias errors, occur in repeated readings. They produce approximately the same error in each measurement. Random errors are caused by personal fluctuations and random electronic fluctuations in equipment. These errors usually but not always follow a statistical probability distribution.

Experimental error is also sometimes called experimental uncertainty. This uncertainty or error propagates through calculations when measured values are used in formulas. Consider the following power calculation with measured values of voltage and current.

$$P = E \cdot I$$
$$E = 100 \text{ V} \pm 2\text{V} \quad 2\% \text{ of reading}$$
$$I = 10 \text{ A} \pm 0.2\text{A} \quad 2\% \text{ of reading}$$

The nominal value of power is computed from the readings as $P=(100\text{V})(10\text{A})=1000 \text{ W}$. Four possible values of power can occur considering the range of error in the voltage and current. Consider the cases where both readings are high and both are low simultaneously.

$$P_{\min} = (100\text{V}-2\text{V})(10\text{A}-0.2\text{A})=960.4 \text{ W}$$

$$P_{\max} = (100\text{V}+2\text{V})(10\text{A}+0.2\text{A})=1040.4 \text{ W}$$

The percentage error is computed from the nominal value and the values of P_{\max} and P_{\min} .

$$\left[\frac{P_{\min} - P_{\text{nominal}}}{P_{\text{nominal}}} \right] \cdot 100\% = \left[\frac{960.4 - 1000}{1000} \right] \cdot 100\% = -3.96\%$$

$$\left[\frac{P_{\max} - P_{\text{nominal}}}{P_{\text{nominal}}} \right] \cdot 100\% = \left[\frac{1040.4 - 1000}{1000} \right] \cdot 100\% = 4.04\%$$

This example shows that the uncertainty or error in the calculation can range from – 3.96% to 4.04% . It is unlikely that both of these measurements would be high or low simultaneously.

A better estimate of the error is use root sum of squares combination (RSS). Consider the general measurement result, R, that is a function of n measured variables: x_1, x_2, \dots, x_n as

$$R = f(x_1, x_2, \dots, x_n)$$

The overall uncertainty in the result is given by the root sum of squares formula. This formula is valid for any function that is a product or quotient of the measured variables.

$$\frac{\Delta R}{R} = \left(\left(\frac{\Delta x_1}{x_1} \right)^2 + \left(\frac{\Delta x_2}{x_2} \right)^2 + \dots + \left(\frac{\Delta x_n}{x_n} \right)^2 \right)^{1/2} \quad (1)$$

Where: $\Delta x_1, \Delta x_2, \dots, \Delta x_n$ are the numerical values of the uncertainty in the measured variables.

x_1, x_2, \dots, x_n are the nominal values of the measured values

ΔR = the uncertainty of the result in units of the result

R = nominal value of the result.

Note: The ratios $\left(\frac{\Delta x}{x} \right)$ are equal to the decimal value of the tolerance or instrument measurement error with $100\% \left(\frac{\Delta x}{x} \right) =$ the percentage error in the result.

Example

The current through a resistor is determined by measuring the voltage across the resistor and dividing by the nominal (marked) resistor value. The resistor has a marked value of 820 ohms and a tolerance of 5%. The voltage measured across the resistor is 4.93V with 4% error in the measured value. Determine the percent uncertainty in the current value and the magnitude of the current uncertainty.

The decimal values of resistor tolerance and meter error are:

$$e_R = 5\% / 100 = 0.05$$

$$e_M = 4\% / 100 = 0.04$$

The percent uncertainty is found from the following equation.

$$\frac{\Delta I}{I} = \left[\left(\frac{\Delta R}{R} \right)^2 + \left(\frac{\Delta V}{V} \right)^2 \right]^{1/2} \quad (2)$$

$$\frac{\Delta I}{I} = \left[(0.04)^2 + (0.05)^2 \right]^{1/2} = 0.064 \quad 6.4\%$$

The numerical value of uncertainty in I is found using the following formula.

$$\Delta I = \left[\left(\frac{\Delta V}{R} \right)^2 + \left(\frac{\Delta R}{R^2} V \right)^2 \right]^{1/2} \quad (3)$$

$$\Delta V = 0.04(4.93) = 0.1972 \text{ V} \quad \Delta R = 0.05(820) = 41 \text{ } \Omega$$

$$\Delta I = \left[\left(\frac{0.1972}{820} \right)^2 + \left(\frac{41}{820^2} 4.93 \right)^2 \right]^{1/2} = 3.8496 \times 10^{-4} \text{ A}$$

The nominal value of resistor current is given by

$$I = \frac{V}{R} = \frac{4.93 \text{ V}}{820 \text{ } \Omega} = 0.006012 \text{ A}$$

$$I = 0.006012 \text{ A} \pm 3.8496 \times 10^{-4} \text{ A}$$

Similar relationships can be found for other Ohm's law relationship. The table below summarizes these equations.

Uncertainty Formulas For Ohm's Law Relationships

Nominal Equation	Percent Uncertainty	Uncertainty Value
$I = \frac{V}{R}$	$\frac{\Delta I}{I} = \left[\left(\frac{\Delta R}{R} \right)^2 + \left(\frac{\Delta V}{V} \right)^2 \right]^{1/2}$	$\Delta I = \left[\left(\frac{\Delta V}{R} \right)^2 + \left(\frac{\Delta R}{R^2} V \right)^2 \right]^{1/2}$
$V = IR$	$\frac{\Delta V}{V} = \left[\left(\frac{\Delta R}{R} \right)^2 + \left(\frac{\Delta I}{I} \right)^2 \right]^{1/2}$	$\Delta V = \left[\left(\frac{\Delta R}{R} \right)^2 V^2 + \left(\frac{\Delta I}{I} \right)^2 V^2 \right]^{1/2}$
$R = \frac{V}{I}$	$\frac{\Delta R}{R} = \left[\left(\frac{\Delta V}{V} \right)^2 + \left(\frac{\Delta I}{I} \right)^2 \right]^{1/2}$	$\Delta R = \left[\left(\frac{\Delta V}{V} \right)^2 R^2 + \left(\frac{\Delta I}{I} \right)^2 R^2 \right]^{1/2}$
$P = \frac{V^2}{R}$	$\frac{\Delta P}{P} = \left[4 \left(\frac{\Delta V}{V} \right)^2 + \left(\frac{\Delta R}{R} \right)^2 \right]^{1/2}$	$\Delta P = \left[4 \left(\frac{\Delta V}{V} \right)^2 P^2 + \left(\frac{\Delta R}{R} \right)^2 P^2 \right]^{1/2}$
$P = I^2 R$	$\frac{\Delta P}{P} = \left[4 \left(\frac{\Delta I}{I} \right)^2 + \left(\frac{\Delta R}{R} \right)^2 \right]^{1/2}$	$\Delta P = \left[4 \left(\frac{\Delta I}{I} \right)^2 P^2 + \left(\frac{\Delta R}{R} \right)^2 P^2 \right]^{1/2}$

PROCEDURE

- 1.) Construct the circuit in Figure 1 and measure the voltages V1, V2, V3, V4. Record the values in Table 1.

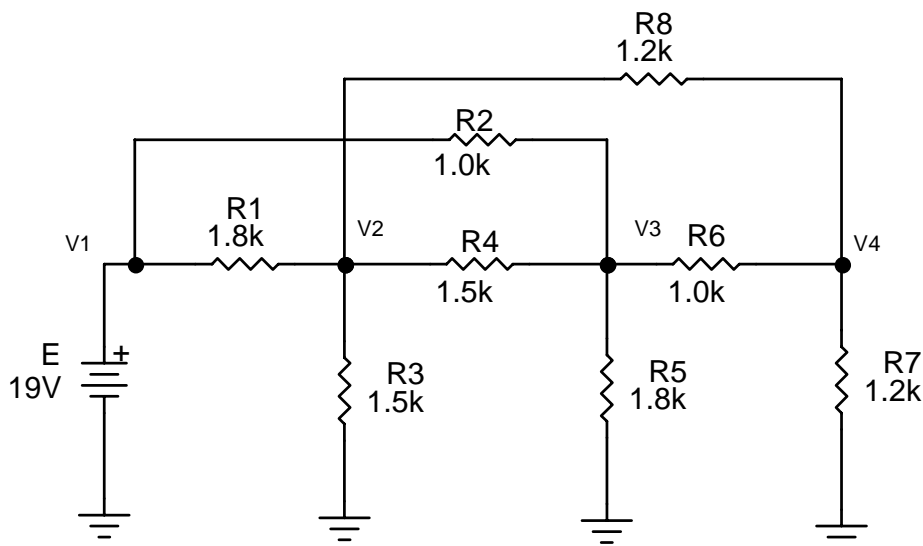


Figure 1. Nodal Analysis Circuit 1.

- 2.) Use nodal analysis to find the theoretical values of the voltages V1 through V4. Record these values in Table 1 also. Use the following formula to compute the percent error between the measured and calculated values of voltage.

$$\left[\frac{V_{\text{measured}} - V_{\text{theoretical}}}{V_{\text{theoretical}}} \right] \cdot 100\% = \text{error}$$

Where: V_{measured} = the measured value of voltage,
 $V_{\text{theoretical}}$ = the calculated value of voltage.

Record the error values in Table 1 also.

- 3.) Compute the current through the resistors R3, R5 and R6 from the voltage measurement and nominal resistance value using Ohm's law. Record these values in Table 2. Determine the DVM's percentage error and the resistors' tolerance and use these values to compute the percent uncertainty of this calculation. Record these values in Table 2. Compute the currents using the values found from nodal analysis and record them in the table. Also compute the percent error using the following equation

$$\left[\frac{I_{\text{calculated}} - I_{\text{theoretical}}}{I_{\text{theoretical}}} \right] \cdot 100\% = \text{error}$$

Where: $I_{\text{calculated}}$ = the calculated value of current from voltage measurement,
 $I_{\text{theoretical}}$ = the calculated value of current

- 4.) Use the measured values of node voltages to compute the power dissipated in resistors R3, R5, R7. Enter these values into Table 3. Use the percent error of the voltage measurement and the resistor tolerance to compute the percent uncertainty of the power calculations and enter them into Table 3. Compute the power dissipated from the resistors using the voltages from the nodal analysis and enter them into the theoretical column in Table 3. Compute the error in the power values by using the formula below and enter them into Table 3.

$$\left[\frac{P_{\text{calculated}} - P_{\text{theoretical}}}{P_{\text{theoretical}}} \right] \cdot 100\% = \text{error}$$

Where: $P_{\text{calculated}}$ = the calculated value of power from voltage measurement,
 $P_{\text{theoretical}}$ = the calculated value of power from analysis.

- 5.) Connect the circuit shown in Figure 2 and measure the voltages V1, V2 and V3. Record these values in Table 4.

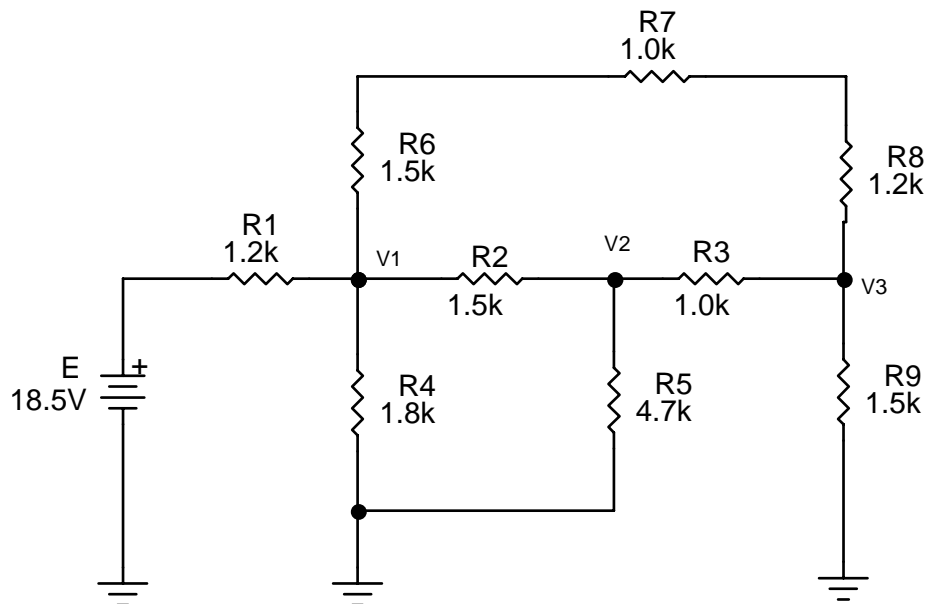


Figure 2. Nodal Analysis Circuit 2.

- 6.) Use nodal analysis to find the theoretical values of the voltages V1 through V3. Record these values in Table 4 also. Use the formula from step 2 to compute the error between the measured and calculated values.
- 7.) Compute the current through the resistors R4, R5 and R9 from the voltage measurement and nominal resistance value using Ohm's law. Record these values in Table 5. Determine the DVM's percentage error and the resistors'

tolerance and use these values to compute the percent uncertainty of this calculation. Record these values in Table 5. Compute the currents using the values found from nodal analysis and record them in the table. Also compute the percent error using the percent error equation from step 3.

- 8.) Use the measured values of node voltages to compute the power dissipated in resistors R4, R5, R9. Enter these values into Table 5. Use the percent error of the voltage measurement and the resistor tolerance to compute the percent uncertainty of the power calculations and enter them into Table 5. Compute the power dissipated from the resistors using the voltages from the nodal analysis and enter them into the theoretical column in Table 5. Compute the error in the power values by using the formula from step 4 and enter them into Table 5.

What to Include in the Report

- 1.) Do the nodal voltage calculations on engineering paper or graph paper. Include the calculations in the report appendix.
- 2.) Reproduce the data tables for the circuit in the data and results section of the report. Give each table an appropriate name and table number.
- 3.) Discuss procedure used in nodal analysis in the theory section of the report.
- 4.) Compare the percent uncertainty to the error percentages for the current and power calculations. Are the error percentages less than the uncertainty percentages? Discuss how the uncertainty percentage can be reduced.

Table 1-Circuit 1 Nodal Voltage Values

	Measured V	Calculated V	% Error
V1			
V2			
V3			
V4			

Table 2-Circuit 1 Resistor Currents (mA)

	Calculated (V/R)	Theoretical	% Uncertainty	% error
I_{R3}				
I_{R5}				
I_{R7}				

Table 3-Circuit 1 Resistor Powers (mW)

	Calculated (V^2/R)	Theoretical	% Uncertainty	% error
P_{R3}				
P_{R5}				
P_{R7}				

Table 4-Circuit 2 Nodal Voltage Values

	Measured V	Calculated V	% Error
V1			
V2			
V3			

Table 5-Circuit 2 Resistor Currents (mA)

	Calculated (V/R)	Theoretical	% Uncertainty	% error
I_{R4}				
I_{R5}				
I_{R9}				

Table 6-Circuit 2 Resistor Powers (mW)

	Calculated (V^2/R)	Theoretical	% Uncertainty	% error
P_{R4}				
P_{R5}				
P_{R9}				