

ET 304A
Electric Circuits Laboratory
Lab 2
Mesh Analysis-Solution of Equations Using Spreadsheets

PROPOSE This laboratory experimentally verifies mesh analysis. Comparing the theoretical values of mesh currents computed using an Excel spreadsheet program to the measurements shows how the errors in the instruments and the tolerances of the resistors affect the overall error. A method of computing the solutions to a set of linear simultaneous equations using the Excel spreadsheet program is introduced. A method of estimating the overall error in measured and calculated values is also covered.

Theoretical Background

Conducting a mesh analysis of an electrical network produces a system of linear simultaneous equations with the mesh currents as the unknowns. A system of linear equations in n unknowns has the following standard form:

$$\begin{array}{ccccccc} a_{11}x_1 + & a_{12}x_2 + & \dots & a_{1n}x_n = & b_1 \\ a_{21}x_1 + & a_{22}x_2 + & \dots & a_{2n}x_n = & b_2 \\ a_{31}x_1 + & a_{32}x_2 + & \dots & a_{3n}x_n = & b_3 \\ \vdots & \vdots & & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots \\ a_{n1}x_1 + & a_{n2}x_2 + & \dots & a_{nn}x_n = & b_n \end{array}$$

In standard form all variables must be represented in each equation. A variable that does not occur in an equation is represented with a zero coefficient. Writing linear equations in standard form leads to a matrix representation of the system of equations.

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \quad (1)$$

Where: **A** = an array of the equations' coefficients that has as many rows as columns. ($n \times n$ for a system with n unknowns)
b = an array of the constants on the right hand side (RHS) of the equations with one column and n rows for a system with n unknowns.
x = an array of variables with one column and n rows for a system of n unknowns.

A system of linear simultaneous equations in matrix form can be easily solved using a matrix inversion method. A computer program or spreadsheet function can easily compute the inverse of a matrix. The inverse of a matrix, A is written as

$$\mathbf{A}^{-1}$$

Only square matrices have an inverse. A matrix is square if it has the same number of rows and columns.

The inverse of a matrix has the following property:

$$\mathbf{A}^{-1} \cdot \mathbf{A} = \mathbf{I}$$

Where \mathbf{I} is the identity matrix. The identity matrix has ones along the principle diagonal and all other entries zero. An identity matrix has the following form.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

The matrix inverse and matrix multiplication can now solve the matrix equation given by (1). This equation represents the original system of equations. Matrix multiplication is not commutative-the order of the operands can not be reversed. Also, matrix multiplication is only defined when the number of columns in the first array equals the number of rows in the second array. With this in mind, the unknown values of the variables are found.

$$\begin{aligned} \mathbf{A} \cdot \mathbf{x} &= \mathbf{b} \\ \mathbf{A}^{-1} \mathbf{A} \cdot \mathbf{x} &= \mathbf{A}^{-1} \cdot \mathbf{b} \\ \mathbf{I} \cdot \mathbf{x} &= \mathbf{A}^{-1} \cdot \mathbf{b} \end{aligned}$$

so the variables are now given by

$$\mathbf{x} = \mathbf{A}^{-1} \cdot \mathbf{b} \quad (2)$$

The numerical calculations to solve (2) are easily performed using a spreadsheet or other computer program such as Mathcad or Matlab. The following procedure and example show how the Excel spreadsheet program can solve equation 2.

Solving a Linear System of Equations Using Excel

The following set of mesh equations must be solved for the currents I_1 , I_2 and I_3 .

$$\begin{aligned} 2I_1 + 3I_2 + I_3 &= 10 \\ 3I_1 + 2I_2 + 0I_3 &= 6 \\ 7I_1 + 0I_2 + 9I_3 &= 2 \end{aligned}$$

The Excel spreadsheet has build-in functions that compute the inverse and can perform matrix multiplication. To solve the system above open the Excel program and follow the steps given.

- 1.) Enter the coefficients of the equations, (**A** matrix), into spreadsheet cells. this should form an array of cell that have the same number of rows as columns. For the example, the matrix has the following values. Labels are added to help identify the rows and columns of the equations.

	A	B	C	D
5	Coefficients of Mesh Equations (A -Matrix)			
6		I_1	I_2	I_3
7	Mesh1	2	3	1
8	Mesh2	3	5	0
9	Mesh3	7	0	9
10				

- 2.) Enter the constants from the right-hand side of the equation into an unused row on the spreadsheet. The first value on the right-hand side should be the first entry in the row.

	A	B	C	D
5	Coefficients of Mesh Equations (A -Matrix)			
6		I_1	I_2	I_3
7	Mesh1	2	3	1
8	Mesh2	3	5	0
9	Mesh3	7	0	9
10				
11	Right Hand Side Constants (b -Matrix)			
12		Mesh1	Mesh2	Mesh3
13		10	6	2
14				

- 3.) Select an unused area of the spreadsheet that is the same size as the **A**-Matrix array.
- 4.) Click the paste function command on the Excel toolbar and select the **Math&Trig** function category. From the function list, select MINVERSE. This function requires an array as an argument. Select the array of coefficients from step 1. End the command by simultaneously pressing CTRL-SHIFT-ENTER. This will enter the following formula in each cell of the array:

{=MINVERSE(C7:E9)}

After computing the inverse, the spreadsheet should look like this.

	A	B	C	D
5	Coefficients of Mesh Equations (A -Matrix)			
6		I_1	I_2	I_3
7	Mesh1	2	3	1
8	Mesh2	3	5	0
9	Mesh3	7	0	9
10				
11	Right Hand Side Constants (b -Matrix)			
12		Mesh1	Mesh2	Mesh3
13		10	6	2
14				
15	Inverse of A Matrix (A ⁻¹)			
16	-1.73077	1.038462	0.192308	
17	1.038462	-0.42308	-0.11538	
18	1.346154	-0.80769	-0.03846	
19				

- 5.) Select an empty cell in the spreadsheet with enough adjacent cells to make several copies. Click on the paste function toolbar selection. Select the **Math&Trig** function category. From the function name list select the SUMPRODUCT function. This function requires two arrays. Select the inverse for array 1. Select the **b**-matrix for array 2. Make the location of this array a constant by entering the \$ symbol in front of the row and column addresses. For the example given above the function should be

SUMPRODUCT(B16:D16,\$C\$13:\$E\$13)

The result in this cell is the value of I_1 .

- 6.) Copy this cell entry into the other two cells for the remaining variable values. The final spreadsheet follows on the next page. This sheet can be expanded to handle any size circuit that will be encountered in this course.

	A	B	C	D
5	Coefficients of Mesh Equations (A -Matrix)			
6		I_1	I_2	I_3
7	Mesh1	2	3	1
8	Mesh2	3	5	0
9	Mesh3	7	0	9
10				
11	Right Hand Side Constants (b -Matrix)			
12		Mesh1	Mesh2	Mesh3
13		10	6	2
14				
15	Inverse of A Matrix (A ⁻¹)			
16	-1.73077	1.038462	0.192308	
17	1.038462	-0.42308	-0.11538	
18	1.346154	-0.80769	-0.03846	
19				
20	Mesh Currents			
21		I_1	-10.6923	
22		I_2	7.615385	
23		I_3	8.538462	
24				

Final Spreadsheet for Circuit Example

PROCEDURE

- 1.) Construct the circuit shown in Figure 1.

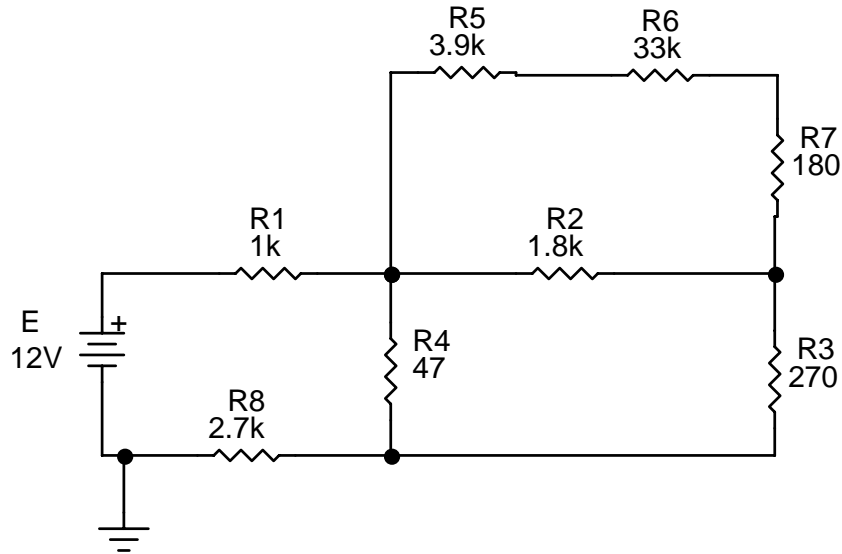


Figure 1. Mesh Circuit 1. All values in Ohm's

- 2.) Measure the voltage across each resistor using the DVM's found in lab and compute the current flowing in each resistor using Ohm's law. Record this values in Table 1. Also record the tolerance of the resistors used in the circuit.
- 3.) Calculate the theoretical resistor values by using mesh analysis.
- 4.) Compare the measured values of current to the theoretical values by computing the percentage error between the theoretical and measured values of resistor currents using the following formula.

$$\left[\frac{V_{\text{measured}} - V_{\text{theoretical}}}{V_{\text{theoretical}}} \right] \cdot 100\% = \text{error}$$

Where: V_{measured} = the measured value of circuit voltage,
 $V_{\text{theoretical}}$ = the calculated value of circuit voltage.

Record these values for use in the lab report.

- 5.) Construct the circuit shown in Figure 2.
- 6.) Measure the voltage across each resistor using the DVM's found in lab and compute the current flowing in each resistor using Ohm's law. Record this values in Table 2. Also record the tolerance of the resistors used in the circuit.
- 7.) Calculate the theoretical resistor values by using mesh analysis.

- 8.) Compare the measured values of current to the theoretical values by computing the percentage error between the theoretical and measured values of resistor currents using the formula from step 4.

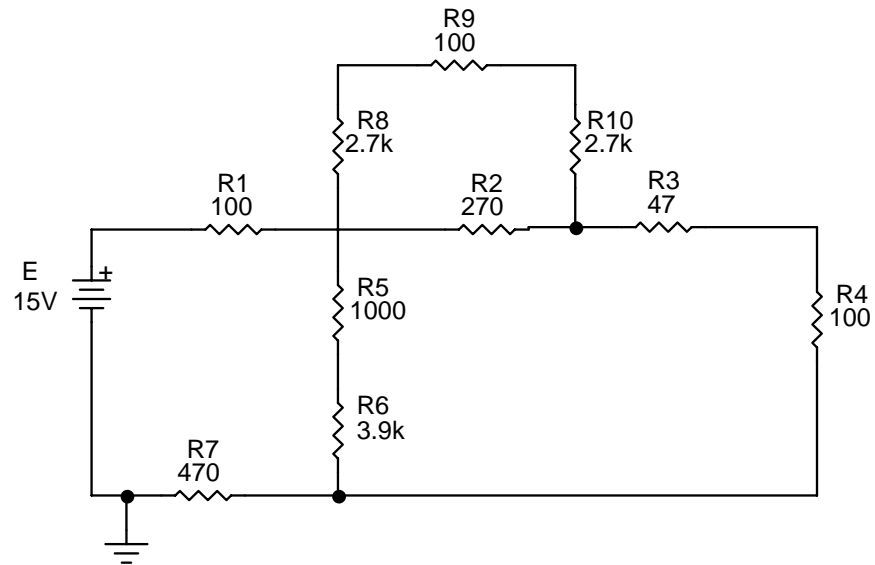


Figure 2. Mesh Circuit 2. All values in Ohms