Packet 1

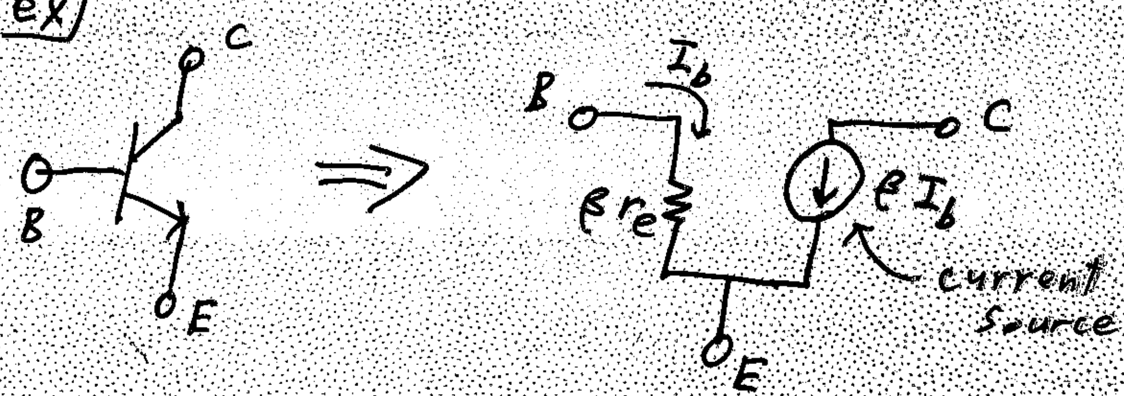
# Methods of Analysis

## Current Sources

A current source determines the current in the branch in which it is located.

The magnitude and polarity of the voltage across a current source are a function of the network to which it is applied.

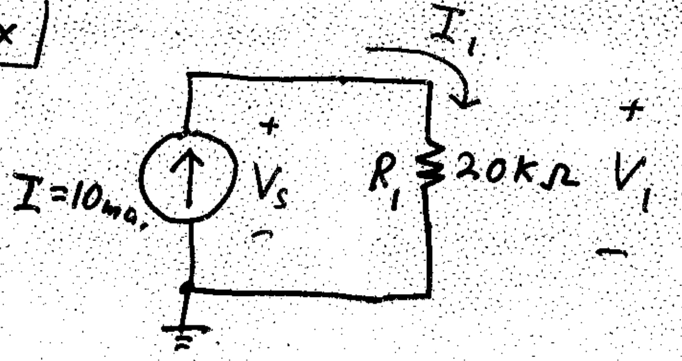
ex)




Transistor

Transistor's equivalent Circuit

ex)

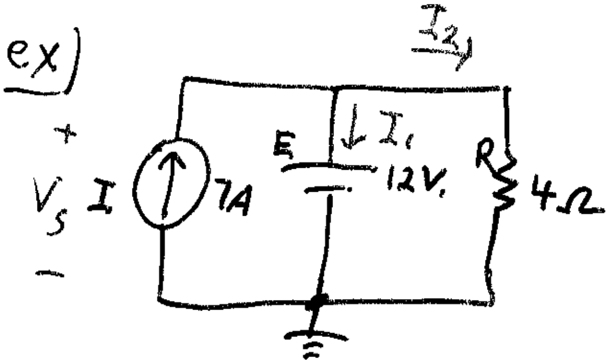



Find $V_s$ and $I_1$.

$I_1 = I = 10\text{ mA}$

$V_s = V_1 = I_1 R_1 = (10\text{ mA})(20\text{ k}\Omega) = 200\text{ V}$

ex)




Find $V_s$, $I_1$ and $I_2$

$V_s = E = 12V$

By Ohm's Law:

$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12V.}{4\Omega} = 3A.$$

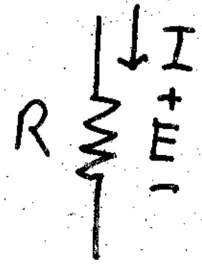
By Kirchhoff's Current Law (KCL):

$$I = I_1 + I_2$$

so $I_1 = I - I_2 = 7A - 3A = 4A.$

Ohm's Law:

Given the voltage polarity and current direction shown in the diagram:



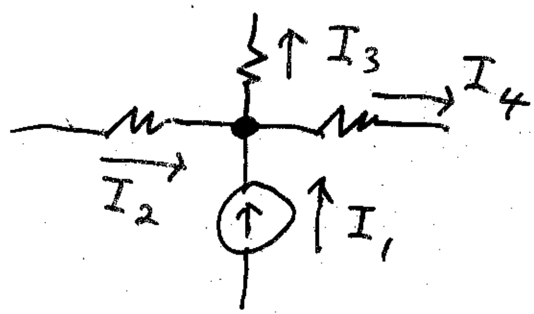

$$I = \frac{E}{R}, \quad E = IR$$

Note: The current is entering the positively labelled voltage terminal.

## Kirchhoff's Current Law (KCL):

The sum of the currents leaving a circuit node is equal to the sum of the currents entering the circuit node.

ex)



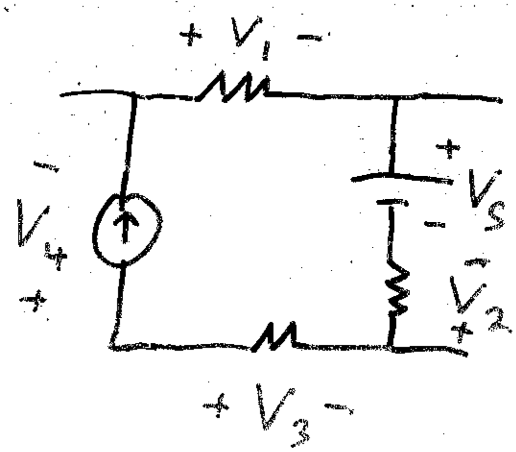

By KCL:

$$I_3 + I_4 = I_1 + I_2$$

## Kirchhoff's Voltage Law (KVL):

The algebraic sum of the voltages around a completed circuit path must equal zero.

ex)

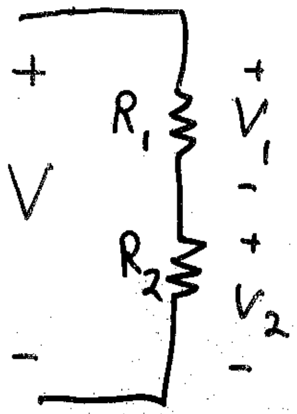



By KVL:

$$V_1 + V_s - V_2 - V_3 + V_4 = 0$$

## Voltage Divider Rule:

Given resistors in series (same current through them) the voltage will divide as follows:

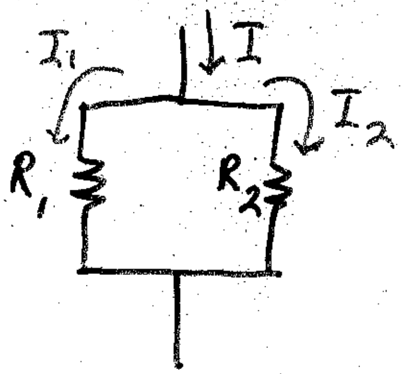



$$V_1 = V \frac{R_1}{R_1 + R_2}$$

$$V_2 = V \frac{R_2}{R_1 + R_2}$$

## Current Divider Rule:

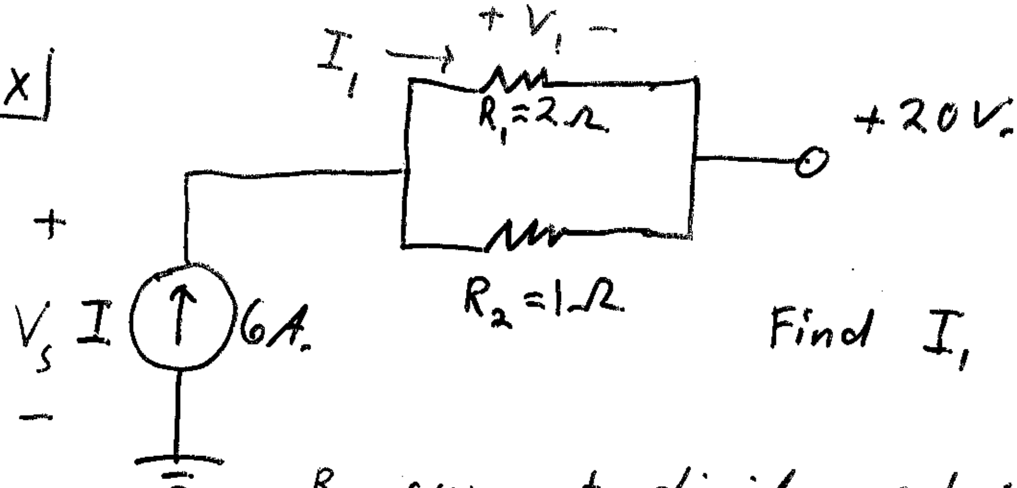
Given resistors in parallel (same voltage across them) the current will divide as follows:




$$I_1 = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \frac{R_1}{R_1 + R_2}$$

ex)




Find $I_1$ and $V_s$

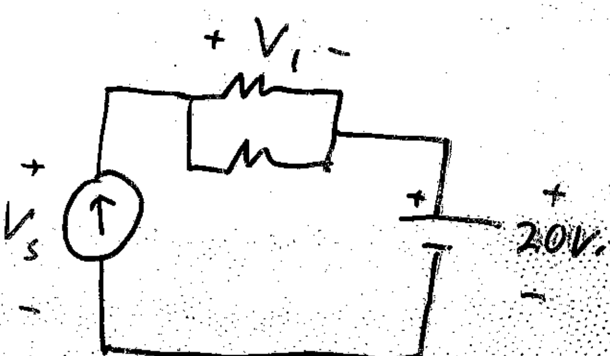
By current divider rule:

$$I_1 = I \frac{R_2}{R_1 + R_2} = \frac{(1\Omega)(6A)}{1\Omega + 2\Omega} = \boxed{2 A.}$$

By Ohm's Law: $V_1 = I_1 R_1 = (2A)(2\Omega) = 4V.$

By KVL:

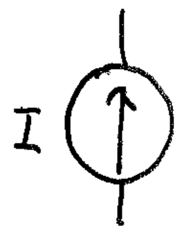
equiv. ckt. $\Rightarrow$




$$V_1 + 20 - V_s = 0$$

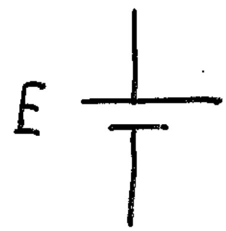
$$V_s = V_1 + 20 = 4 + 20 = \boxed{24 V.}$$

# Source Conversions



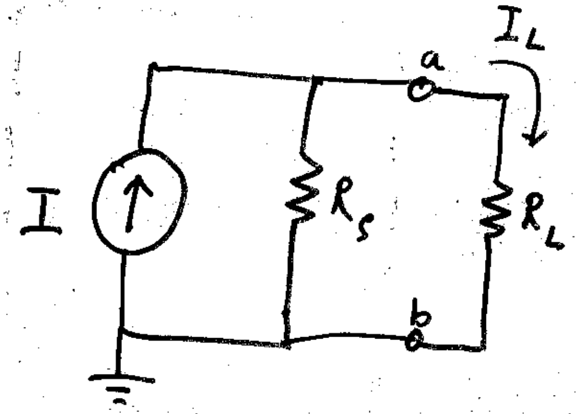
Ideal Current Source

$R_s = \infty$



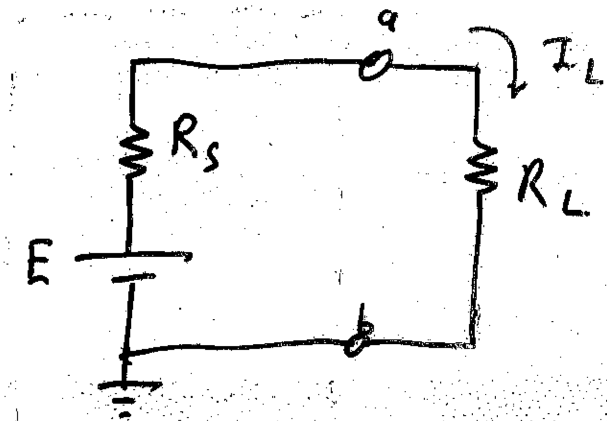
Ideal Voltage Source

$R_s = 0$



Practical Current Source

$$I_L = \frac{I\,R_s}{R_s + R_L}$$



Practical Voltage Source

$$I_L = \frac{E}{R_s + R_L}$$

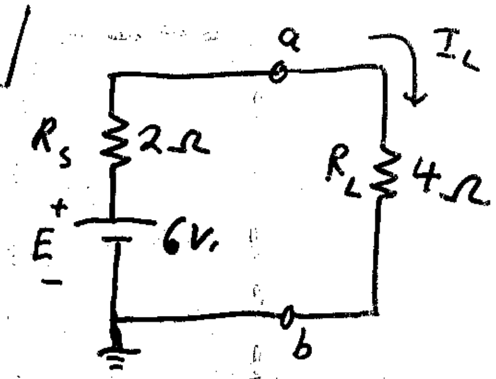
The practical current and voltage sources are <u>equivalent</u> if

$$E = I\,R_s \quad \text{and} \quad I = \frac{E}{R_s}$$

and measurements taken at terminals a and b can not tell them apart.

Equivalent circuits may be substituted for one another to simplify the circuit analysis.

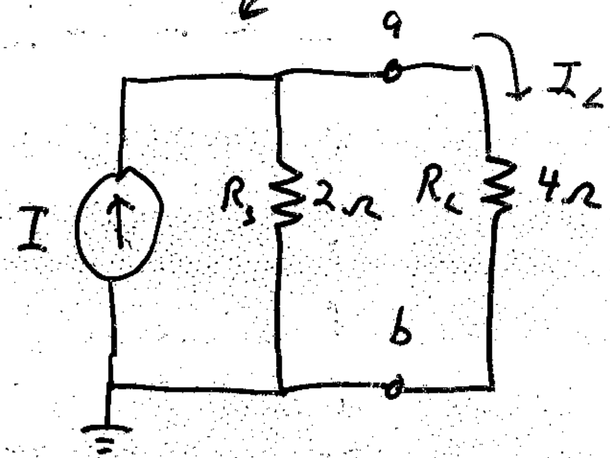
ex/




Find $I_L$

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+4} = \boxed{1A}$$

The voltage source could be replaced by an equivalent current source to solve the problem.




$$I = \frac{E}{R_s} = \frac{6}{2} = 3A.$$

Then by CDR:

$$I_L = \frac{I\ R_s}{R_s + R_L} = \frac{3(2)}{2+4} = \boxed{1A.}$$

If $R_L >> R_s$, then the practical voltage source model will act like an ideal Voltage source ($R_s = 0$).

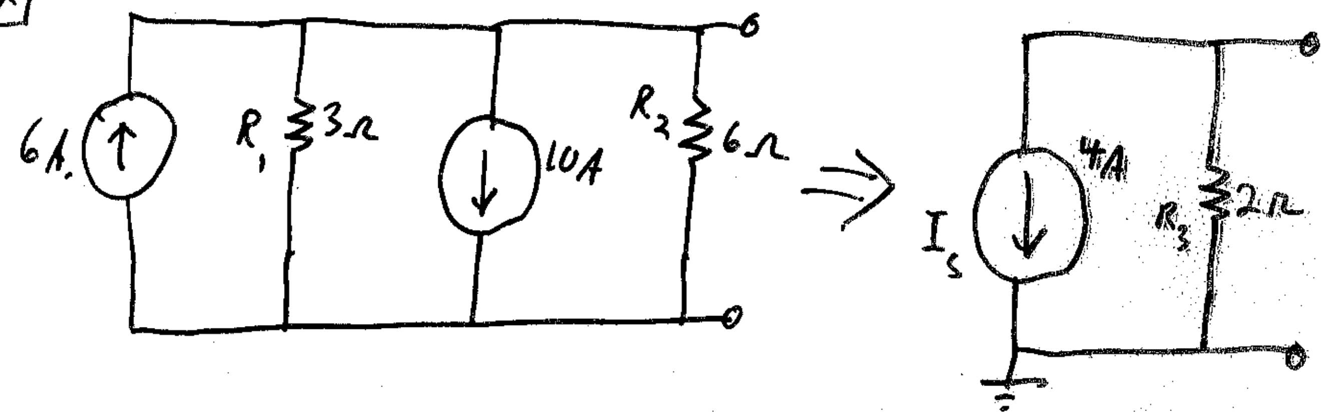
ex/ Given $R_s = 2\Omega$ and $R_L = 1k\Omega$

$$I_L = \frac{E}{R_s + R_L} = \frac{6}{2+1K} = 5.99\ mA$$

Assuming $R_s = 0$ gives: $I_L = \frac{E}{R_L} = \frac{6}{1K} = 6\ mA \approx 5.99 mA$

Two current sources connected in parallel may be replaced by a single current source whose value is their algebraic sum.

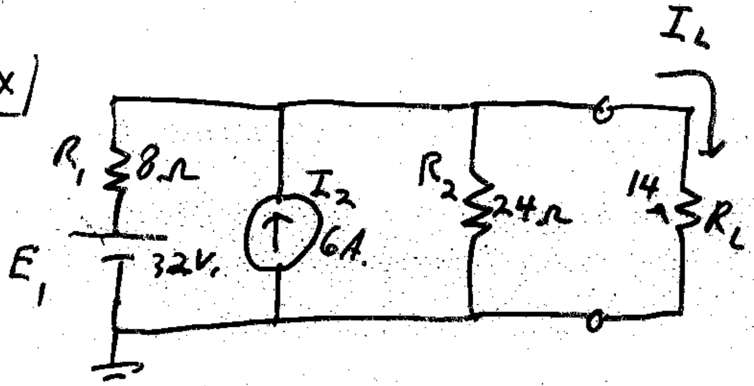
ex/




$I_s = 10A. - 6A. = 4A.$

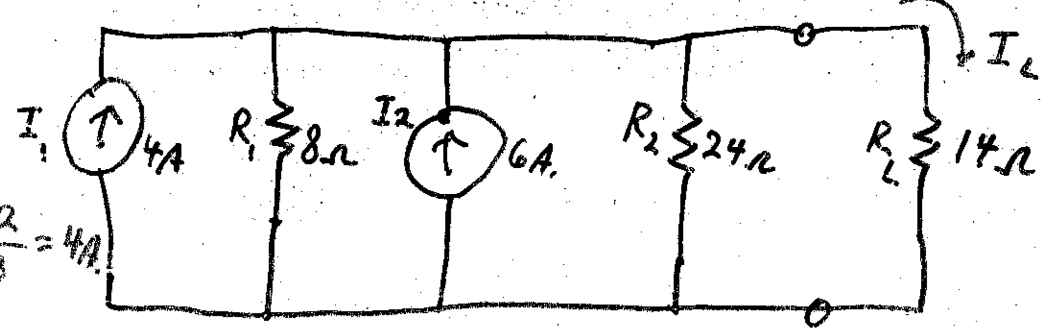
$R_s = 3\Omega \| 6\Omega = 2\Omega$

ex/



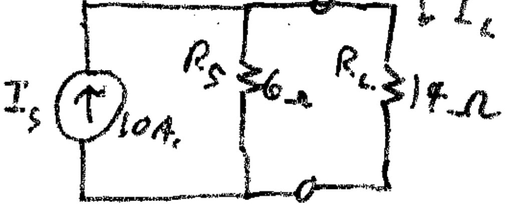

Reduce ckt. to a single current source and find $I_L$

First convert the voltage source to a current source




$I_1 = \frac{32}{8} = 4A.$

Then combine current sources.



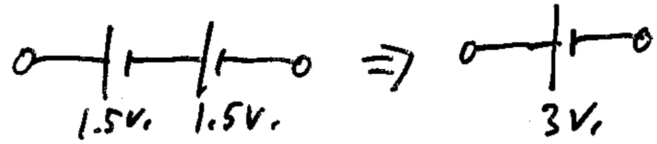

$I_s = 6 + 4 = 10A$ , $R_s = 8 \| 24 = 6\,\Omega$

By CDR:

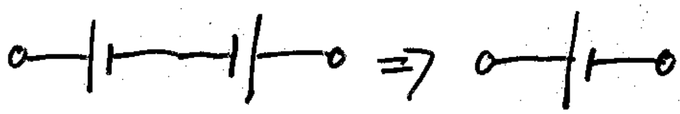
$I_L = \frac{10(6)}{6+14} = \boxed{3\,A.}$

Two voltage sources connected in series may be replaced by a single voltage source whose value is their algebraic sum.

ex/

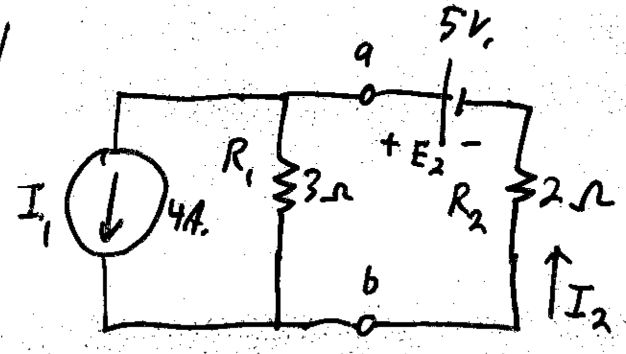



ex/



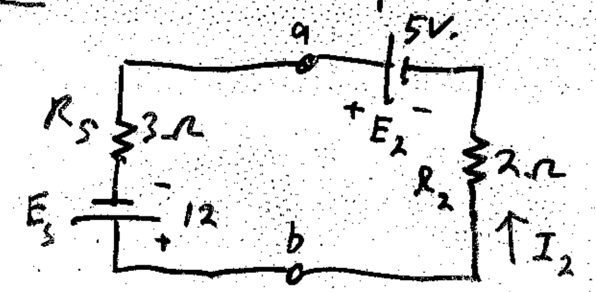

$V_T = 12 - 6 = 6V.$

ex/




Find $I_2$

Sol: Convert $I_1$ into a voltage source

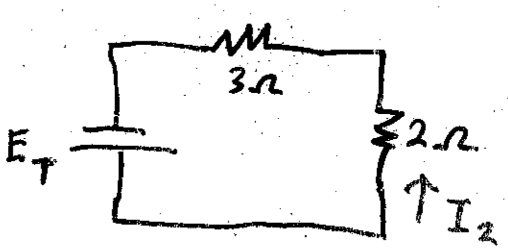



$E_S = I_1 R_1 = (4)(3) = 12V.$

$R_S = R_1 = 3\Omega$

Combine voltage sources:

$E_T = 12 + 5 = 17V.$




$$I_2 = \frac{17}{3+2} = \boxed{3.4A.}$$

Pocket 2

# Branch - Current Analysis

1. Assign a current of arbitrary direction to each branch of the network.

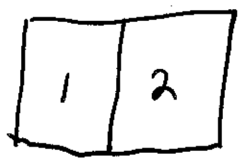
2. Label the voltage polarities across each resistor such that the positive side is where the assigned current enters.

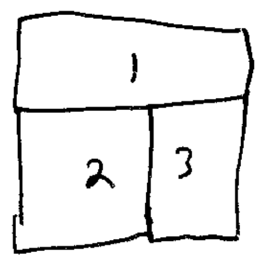
3. Apply KVL around each closed circuit path. (loop)

4. Apply KCL at the minimum number of nodes that will include all the branch currents of the network. (# KCL's = # nodes − 1)

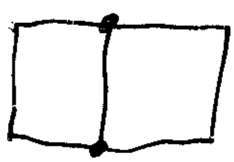
5. Solve the resulting simultaneous equations for the assumed branch currents.

ex/ closed loops:



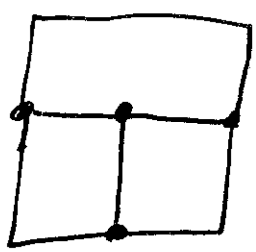




ex/ required node eqns:



2 nodes

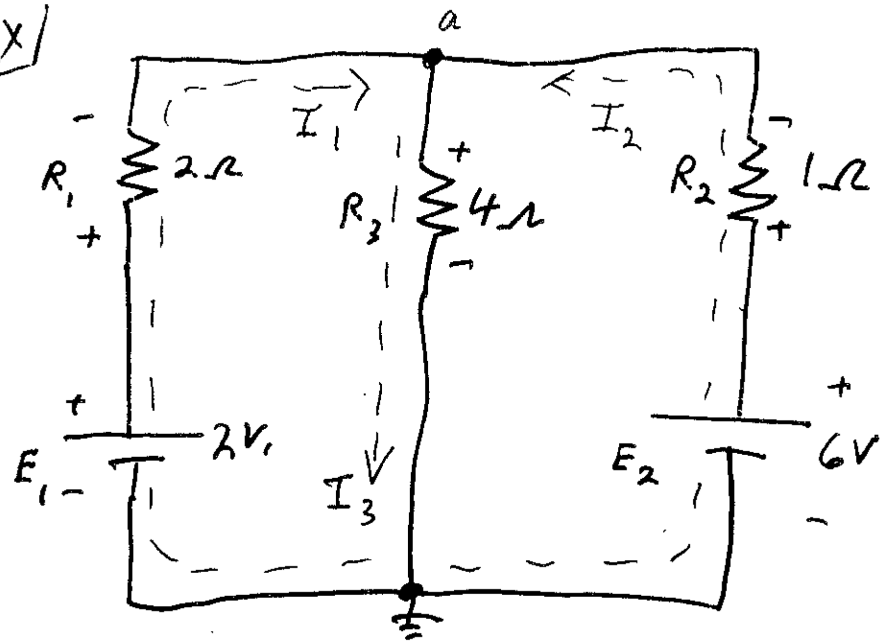
2 - 1 = 1 eqn.



4 nodes

4 - 1 = 3 eqns.

ex/




Solve by branch-current method

Solution: 1.) Assign currents $I_1$, $I_2$, $I_3$

2.) Label Voltage polarities across resistors

3.) Apply KVL to 2 closed loops.

Loop 1: $+E_1 - V_{R_3} - V_{R_1} = 0$

$$2 - V_{R_3} - V_{R_1} = 0$$

Apply Ω Law: $V_{R_3} = (4\Omega)(I_3)$ and $V_{R_1} = (2\Omega)(I_1)$

substitute into KVL: $2 - 4I_3 - 2I_1 = 0 \leftarrow$ eqn. 1

Loop 2: $+E_2 - V_{R_3} - V_{R_2} = 0$

$$6 - V_{R_3} - V_{R_2} = 0$$

Apply Ω Law: $V_{R_3} = (4\Omega)(I_3)$ and $V_{R_2} = (1\Omega)(I_2)$

substitute into KVL: $6 - 4I_3 - I_2 = 0 \leftarrow$ eqn. 2

4.) Apply KCL at node a.

$$I_1 + I_2 = I_3 \quad \leftarrow \text{eqn. } (3)$$

5.) Solve equations (1), (2) and (3) simultaneously

Rewriten:

$$2 - 4I_3 - 2I_1 = 0 \quad \longrightarrow \quad 2I_1 + 0 + 4I_3 = 2$$

$$6 - 4I_3 - I_2 = 0 \quad \longrightarrow \quad 0 + I_2 + 4I_3 = 6$$

$$I_1 + I_2 = I_3 \quad \longrightarrow \quad I_1 + I_2 - I_3 = 0$$

A system of 3 eqns. & 3 unknowns can be solved using Cramer's Rule.

$$I_1 = \frac{\begin{vmatrix} 2 & 0 & 4 \\ 6 & 1 & 4 \\ 0 & 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = \frac{(2)(1)(-1) + (6)(1)(4) + (0)(4)(0) - [0+0+8]}{-2+0+0-[4+8+0].} = \boxed{-1A.}$$

a minus sign indicates the actual current flows in the opposite direction to the arrow on the schematic.

$$I_2 = \frac{\begin{vmatrix} 2 & 2 & 4 \\ 0 & 6 & 4 \\ 1 & 0 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = \boxed{2A}$$

$$I_3 = \frac{\begin{vmatrix} 2 & 0 & 2 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} 2 & 0 & 4 \\ 0 & 1 & 4 \\ 1 & 1 & -1 \end{vmatrix}} = \boxed{1A}$$

## Cramer's Rule

Given a set of simultaneous, linear equations written in matrix form as $AX = B$, let $\Delta = \det A$. Define $\Delta_i$ to be the determinant of the matrix that is obtained by replacing the $i$th column of $A$ with the column comprising $B$.

Then $\boxed{X_i = \dfrac{\Delta_i}{\Delta}}$

ex| Given equations: $X_1 - 2X_2 = 28$ Find $X_1$ and $X_2$

$4X_1 + 6X_2 = 21$

This can be expressed as a matrix equation: $AX = B$

$$\begin{bmatrix} 1 & -2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 28 \\ 21 \end{bmatrix}$$

First find $\Delta = \det A = \begin{vmatrix} 1 & -2 \\ 4 & 6 \end{vmatrix} = 6 - (-8) = 14$

Replacing the first column of $A$ by the column comprising $B$, we get:

$$\Delta_1 = \begin{vmatrix} 28 & -2 \\ 21 & 6 \end{vmatrix} = 168 - (-42) = 210$$

Thus $X_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{210}{14} = \boxed{15}$

Similarly: $\Delta_2 = \begin{vmatrix} 1 & 28 \\ 4 & 21 \end{vmatrix} = 21 - 112 = -91$

Then $X_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{-91}{14} = \boxed{-\dfrac{13}{2}}$

# EXPANDING DETERMINANTS

For 2x2 MATRIX

$$A = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \qquad |A| = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc$$

EX.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad |A| = (1)(4) - (3)(2) = 4 - 6 = -2$$

$$B = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} \qquad |B| = -1(1) - 2(0) = -1$$

FOR 3x3 matrix     EXPAND ALONG TOP ROW
3 2x2 det.

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

1st. strike 1st row & Column mult. by a

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ \cancel{d} & e & f \\ \cancel{g} & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix}$$

2nd strike 1st row & 2nd Column mult. by -b

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & \cancel{e} & f \\ g & \cancel{h} & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix}$$

# EXPANDING DETERMINANTS

3rd step 3x3 matrix

strike 1st row & 3rd column mult by $c$

$$\begin{vmatrix} \cancel{a} & \cancel{b} & \cancel{c} \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

expand 2x2 det. and simplify

Ex.

$$\begin{vmatrix} 3 & 0 & -1 \\ 2 & 4 & 3 \\ -1 & 2 & 2 \end{vmatrix} = 3\begin{vmatrix} 4 & 3 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$$

$$= 3(4(2)-3(2)) - 0(2(2)-(-1)(3)) - 1(2(2)-(4)(-1))$$

$$= 3(8-6) - 0 - 1(4+4)$$

$$= 3(2) - 8$$

$$= \underline{\underline{-2}}$$

Ex

$$\begin{vmatrix} 3 & -2 & 1 \\ 0 & 2 & 1 \\ -1 & 0 & 1 \end{vmatrix} = 3\begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} - (-2)\begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix}$$

$$= 3[2(1)-(0)(1)] + 2[0(1)-(-1)(1)] + 1\cdot[0(0)-(2)(-1)]$$

$$= 3[2] + 2[1] + [2](1)$$

$$= 6 + 2 + 2 = \underline{\underline{10}}$$

ex| Given equations: $2x_1 - 3x_2 - 7x_3 = 5$

Find $x_1, x_2, x_3$ $\quad 5x_1 + x_2 = -2$

by Cramer's Rule. $\quad -8x_1 + 2x_2 + 7x_3 = 1$

matrix form: $\rightarrow AX = B$

First find $\Delta = \det A$

$$\begin{bmatrix} 2 & -3 & -7 \\ 5 & 1 & 0 \\ -8 & 2 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 2 & -3 & -7 \\ 5 & 1 & 0 \\ -8 & 2 & 7 \end{vmatrix} = (2)(1)(7) + (-3)(-8)(0) + (-7)(5)(2)$$

$$- \left[ (2)(0)(2) + (7)(-3)(5) + (-7)(1)(-8) \right]$$

$$= 14 + 0 - 70 - [0 - 105 + 56]$$

$$= -56 + 49 = -7$$

Then $\Delta_1 = \begin{vmatrix} 5 & -3 & -7 \\ -2 & 1 & 0 \\ 1 & 2 & 7 \end{vmatrix} = 35 + 0 + 28 - [-7 + 0 + 42] = 28$

Thus $x_1 = \dfrac{\Delta_1}{\Delta} = \dfrac{28}{-7} = \boxed{-4}$

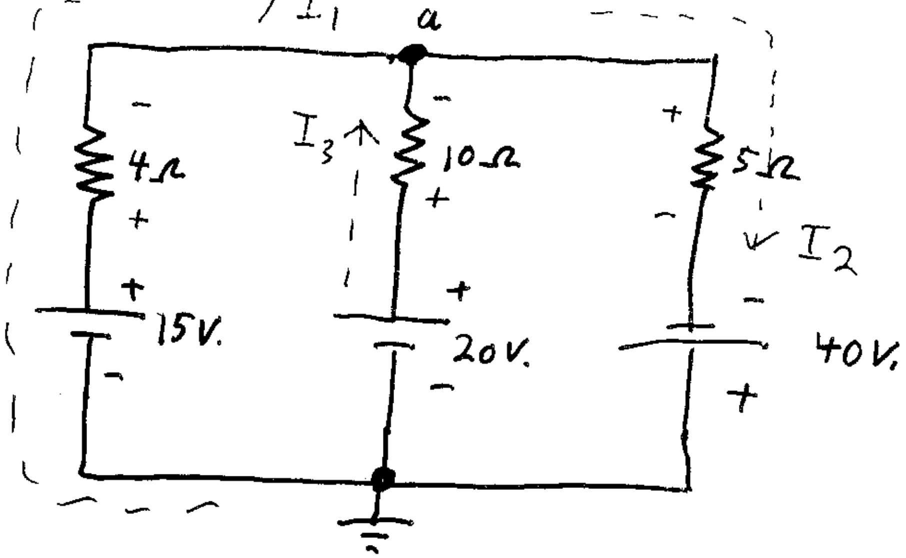
Then $\Delta_2 = \begin{vmatrix} 2 & 5 & -7 \\ 5 & -2 & 0 \\ -8 & 1 & 7 \end{vmatrix} = -28 + 0 - 35 - [-112 + 0 + 175] = -126$

Thus $x_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{-126}{-7} = \boxed{18}$

Then $\Delta_3 = \begin{vmatrix} 2 & -3 & 5 \\ 5 & 1 & -2 \\ -8 & 2 & 1 \end{vmatrix} = 2 - 48 + 50 - (-40 - 8 - 15) = 67$

Finally $x_3 = \dfrac{\Delta_3}{\Delta} = \boxed{\dfrac{67}{-7}}$

ex




Solve by Branch - current Method

Solution: 1.) Assign currents $I_1$, $I_2$, $I_3$ (arbitrary →)

2.) Label voltage polarities across resistors (+ on side I enters)

3.) Apply KVL to 2 closed loops.

Loop 1: $-10 I_3 + 20 - 15 + 4 I_1 = 0$

$$4 I_1 + 0 - 10 I_3 = -5 \quad \text{eqn. } 1$$

Loop 2: $5 I_2 - 40 - 20 + 10 I_3 = 0$

$$0 + 5 I_2 + 10 I_3 = 60 \quad \text{eqn. } 2$$

4.) Apply KCL at node a

$$I_1 + I_3 = I_2 \quad \text{eqn. } 3$$

5.) Solve equations 1, 2 and 3 simultaneously

Cramers rule could be used or we can first solve eqn 3 for $I_2$ and then substitute $I_2$ into eqn. 2 to get 2 eqns. and 2 unknowns.

$$I_2 = I_1 + I_3$$

substituting:

$$5(I_1 + I_3) + 10 I_3 = 60 \qquad \text{eqn. } (2)$$

$$5 I_1 + 15 I_3 = 60$$

Solve these 2 eqns. simultaneously

$$\begin{cases} I_1 + 3 I_3 = 12 \\ 4 I_1 - 10 I_3 = -5 \quad \leftarrow \text{eqn } (1) \end{cases}$$

$$I_1 = \frac{\begin{vmatrix} 12 & 3 \\ -5 & -10 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 4 & -10 \end{vmatrix}} = \frac{-120 - [-15]}{-10 - 12} = \frac{-105}{-22} = \boxed{4.773 \text{ A.}}$$

$$I_3 = \frac{\begin{vmatrix} 1 & 12 \\ 4 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & 3 \\ 4 & -10 \end{vmatrix}} = \frac{-5 - 48}{-22} = \frac{-53}{-22} = \boxed{2.409 \text{ A.}}$$

Then from eqn. (3)

$$I_2 = I_1 + I_3 = 4.773 + 2.409 = \boxed{7.182 \text{ A.}}$$

# Mesh Analysis (General Approach)

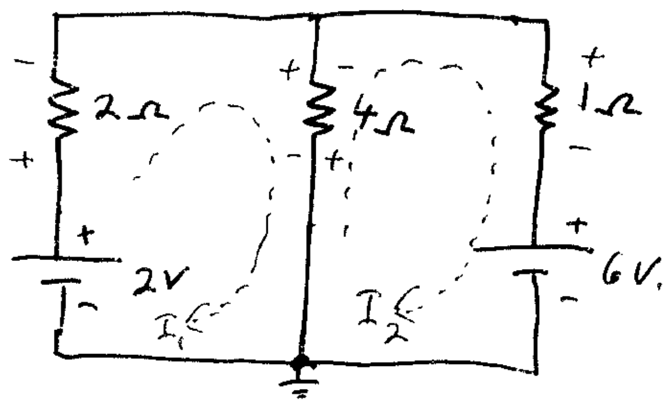
1. Assign a clockwise current to each mesh. (closed loop)

2. Label voltage polarities across each resistor due to each mesh current.
   (+ on side I enters R)

3. Apply KVL in clockwise direction around each mesh.
   a.) For resistors shared by 2 meshes, the Total of both mesh currents must be used in ohms law calculation.
   b.) Polarity of voltage sources is unaffected by mesh current directions

4. Solve the resulting set of equations for the assumed mesh currents.

ex)




(note sum of I thru 4Ω R)

Solve:

For mesh 1:

$$-2 + 2I_1 + 4(I_1 - I_2) = 0$$

eqn.1 $\rightarrow$ $6I_1 - 4I_2 = 2$

For mesh 2:

$$6 + 4(I_2 - I_1) + I_2 = 0$$

eqn.2 $\rightarrow$ $-4I_1 + 6I_2 = -6$

Packet 3

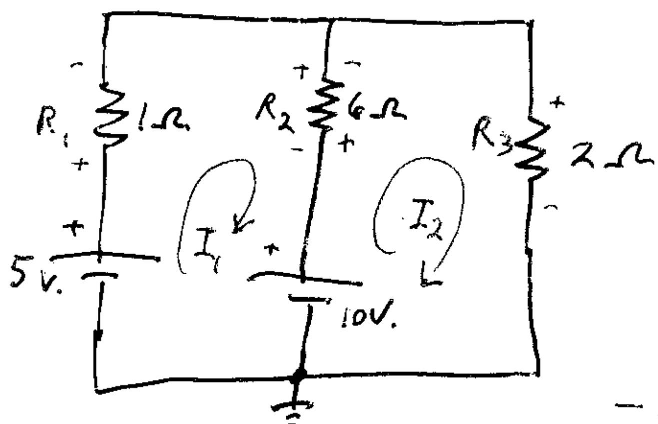
Solving with Cramer's Rule:

$$I_1 = \frac{\begin{vmatrix} 2 & -4 \\ -6 & 5 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{10-24}{30-16} = \frac{-14}{14} = \boxed{-1A.}$$

$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix}}{\begin{vmatrix} 6 & -4 \\ -4 & 5 \end{vmatrix}} = \frac{-36+8}{14} = \frac{-28}{14} = \boxed{-2A}$$

$$I_{4\Omega_R} = I_1 - I_2 = -1-(-2) = 1A$$

ex




Solve for branch currents:

Solution:

For mesh 1:

$$-5 + I_1 + 6(I_1 - I_2) + 10 = 0$$

$$7I_1 - 6I_2 = -5 \leftarrow \text{eqn. } 1$$

For mesh 2:

$$2I_2 - 10 + 6(I_2 - I_1) = 0$$

$$-6I_1 + 8I_2 = 10$$

$$-3I_1 + 4I_2 = 5 \leftarrow \text{eqn. } 2$$

$$I_1 = \frac{\begin{vmatrix} -5 & -6 \\ 5 & 4 \end{vmatrix}}{\begin{vmatrix} 7 & -6 \\ -3 & 4 \end{vmatrix}} = \frac{-20+30}{28-18} = \frac{10}{10} = \boxed{1A.} = I_{R_1}$$

$$I_2 = \frac{\begin{vmatrix} 7 & -5 \\ -3 & 5 \end{vmatrix}}{10} = \frac{35-15}{10} = \boxed{2A} = I_{R_3}$$

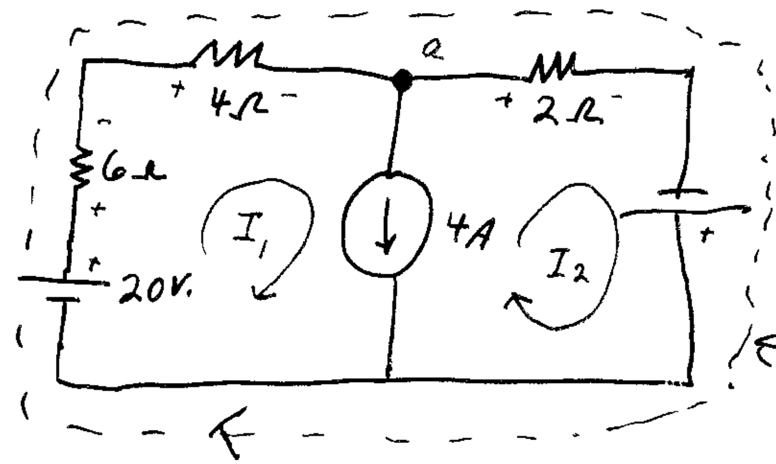
$$I_{R_2} = I_2 - I_1 = 2-1 = \boxed{1A.}$$

(in the $I_2$ direction)

# Supermesh Currents

If there are current sources in the network, do not construct any KVL loops which include a current source. If necessary combine more than one mesh together to form a supermesh which avoids the current source in its path.

ex|




Find the currents:

← consider supermesh to avoid current source

solution: 1.) create supermesh

2.) draw mesh currents $I_1$ & $I_2$

3.) write KVL around supermesh path:

$$-20 + 6I_1 + 4I_1 + 2I_2 - 12 = 0$$

$$10I_1 + 2I_2 = 32 \quad \leftarrow \text{eqn. } 1$$

4.) Get needed 2nd equation by writing KCL at current source's node a.

$$I_1 = I_2 + 4$$

$$I_1 - I_2 = 4 \quad \leftarrow \text{eqn. } 2$$

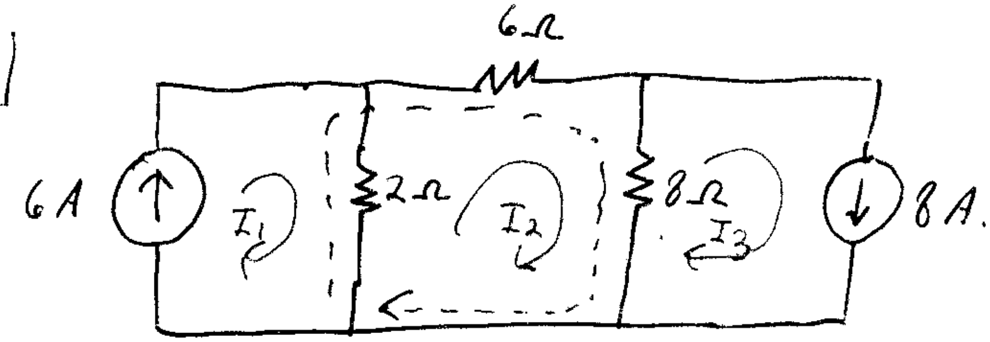
Can now solve 2 eqn & 2 unknowns.

$$\left.\begin{aligned} 10 I_1 + 2 I_2 &= 32 \\ I_1 - I_2 &= 4 \end{aligned}\right\}$$

$$I_1 = \frac{\begin{vmatrix} 32 & 2 \\ 4 & -1 \end{vmatrix}}{\begin{vmatrix} 10 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{-32-8}{-10-2} = \frac{-40}{-12} = \boxed{3.33A.}$$

$$I_2 = \frac{\begin{vmatrix} 10 & 32 \\ 1 & 4 \end{vmatrix}}{-12} = \frac{40-32}{-12} = \frac{8}{-12} = \boxed{-0.67A}$$

ex)




Find network currents:

1.) Draw mesh currents

2.) Construct supermesh to avoid current sources.

3.) Write KVL around supermesh path

$$6 I_2 + 8(I_2 - I_3) + 2(I_2 - I_1) = 0$$

$$-2 I_1 + 16 I_2 - 8 I_3 = 0 \leftarrow \text{eqn } 1$$

4.) Since each current source only has one mesh current passing through it:

$$\boxed{I_1 = 6A} \qquad \boxed{I_3 = 8A}$$

Solving eqn 1 for $I_2$:

$$16 I_2 = 2 I_1 + 8 I_3$$

$$I_2 = \frac{I_1}{8} + \frac{I_3}{2}$$

$$I_2 = \frac{6}{8} + \frac{8}{2} = \boxed{4.75A}$$

Note: $I_{6\Omega} = I_2 = 4.75A \rightarrow$

$I_{2\Omega} = I_1 - I_2 = 1.25A. \downarrow$

$I_{8\Omega} = I_3 - I_2 = 3.25A. \uparrow$