Nodal Analysis (General Approach)

This method employs KCL's at network nodes and works best when network has mostly current sources.

1.) Determine the number of nodes in the network.

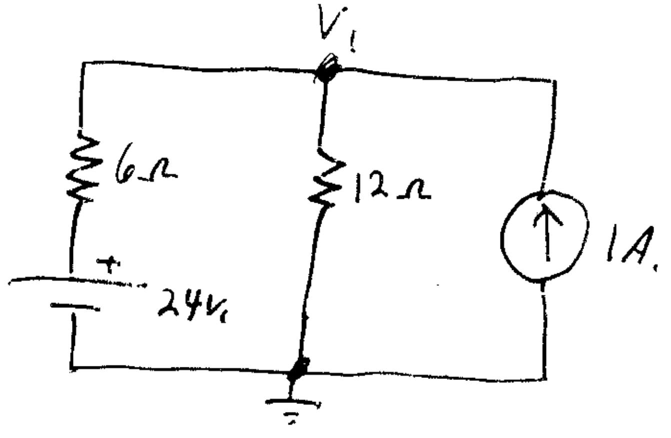
2.) Pick a reference node and assign voltages ($V_1, V_2 \ldots$) to all other nodes.

3.) Apply KCL at each node except the reference node.

4.) Solve resulting equations for node voltages.

> Warning:
>
> Do not write a KCL at a node connected to a voltage souce. (Its current can't be found with ohms Law)
>
> Do not write a KVL through a current source. (Its voltage can't be found with ohms Law)

ex/



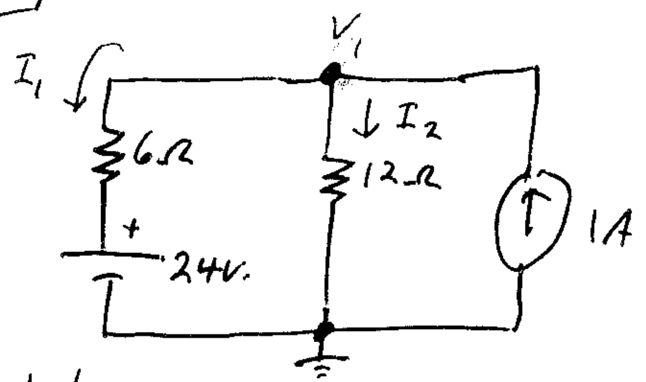
Apply nodal analysis:

1.) Circuit has 2 nodes

2.) Label nodes : V_1

(Continued)

ex/ continued



3.) Write KCL at V_1:

$$1A = I_1 + I_2$$

Apply ohms law:

$$I_1 = \frac{V_1 - 24}{6}, \quad I_2 = \frac{V_1}{12}$$

Subst:

$$1 = \frac{V_1 - 24}{6} + \frac{V_1}{12}$$

solve for V_1:

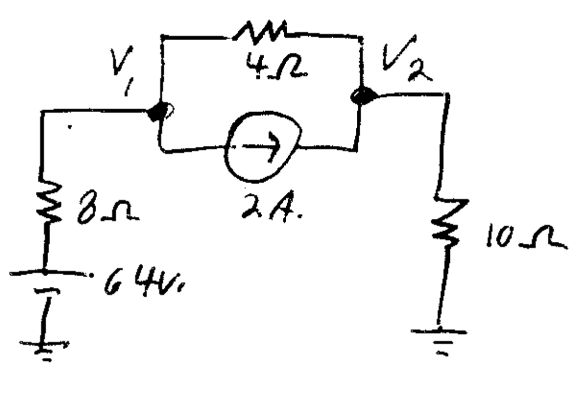
$$12 = 2(V_1 - 24) + V_1$$
$$12 = 3V_1 - 48$$
$$V_1 = \frac{60}{3} = \boxed{20\,V_1}$$

Substituting into I_1 and I_2 eqns. gives:

$$I_1 = \frac{V_1 - 24}{6} = \frac{20-24}{6} = \frac{-4}{6} = \boxed{-0.667\,A.}$$

$$I_2 = \frac{V_1}{12} = \frac{20}{12} = \boxed{1.667\,A.}$$

ex/



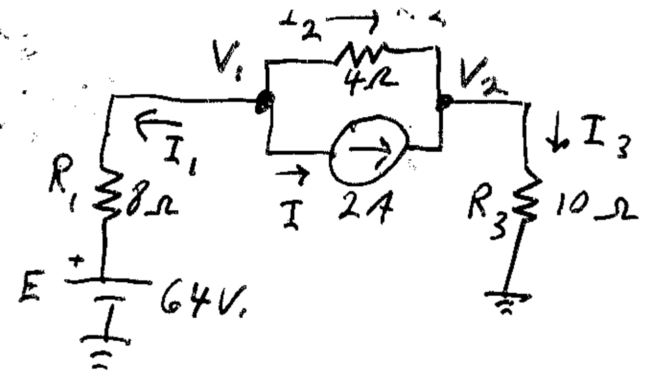
Find all I and V for ckt.

Solution:

1.) Determine the number of nodes requiring KCL's. (2 nodes)

2.) Label nodes : V_1 & V_2

3.) Apply KCL's at V_1 & V_2



KCL at node V_1:

$$I + I_1 + I_2 = 0$$

Apply Ohm's Law:

$$I_1 = \frac{V_1 - E}{R_1}, \quad I_2 = \frac{V_1 - V_2}{R_2}$$

note: voltage on side of R that I enters

minus voltage on side of R that I leaves

Now substitute into KCL eqn:

$$I + \frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$2 + \frac{V_1 - 64}{8} + \frac{V_1 - V_2}{4} = 0$$ ← multiply both sides by 8

$$16 + V_1 - 64 + 2V_1 - 2V_2 = 0$$

$$3V_1 - 2V_2 = 48$$ ← eqn. 1

KCL at node V_2:

$$I_2 + I = I_3$$

Apply Ohm's Law:

$$I_2 = \frac{V_1 - V_2}{R_2}, \quad I_3 = \frac{V_2 - 0}{R_3}$$

Substitute into KCL:

$$\frac{V_1 - V_2}{R_2} + I = \frac{V_2}{R_3}$$

$$\frac{V_1 - V_2}{4} + 2 = \frac{V_2}{10}$$ ← multiply both sides by 20

$$5V_1 - 5V_2 + 40 = 2V_2$$

$5V_1 - 7V_2 = -40$ eqn. 2

Now solve eqns 1 and 2 simultaneously for V_1 and V_2:

$$3V_1 - 2V_2 = 48 \quad \text{eqn 1}$$
$$5V_1 - 7V_2 = -40 \quad \text{eqn 2}$$

$$V_1 = \frac{\begin{vmatrix} 48 & -2 \\ -40 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 5 & -7 \end{vmatrix}} = \frac{-336-(80)}{-21-(-10)} = \frac{-416}{-11} = \boxed{37.82\ V.}$$

$$V_2 = \frac{\begin{vmatrix} 3 & 48 \\ 5 & -40 \end{vmatrix}}{-11} = \frac{-120-240}{-11} = \frac{-360}{-11} = \boxed{32.73\ V.}$$

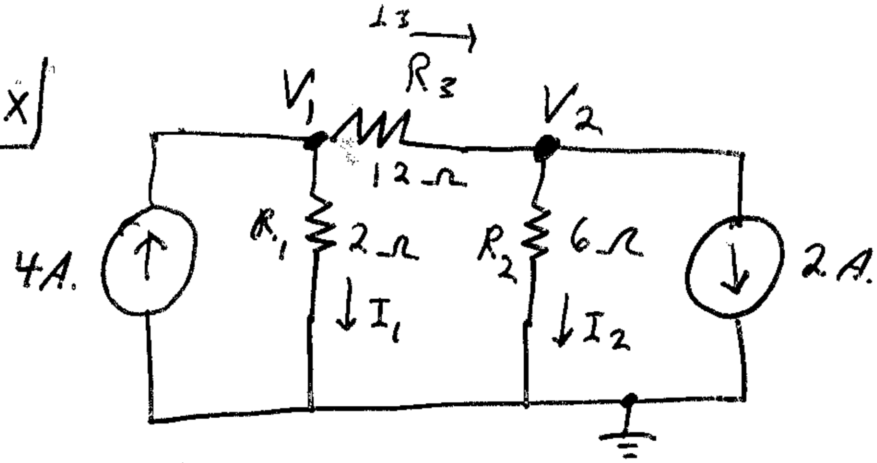
Can now use ohm's law to find currents if desired:

$$I_1 = \frac{V_1 - E}{R_1} = \frac{37.82-64}{8} = \boxed{-3.273\ A.}$$

$$I_2 = \frac{V_1 - V_2}{R_2} = \frac{37.82-32.73}{4} = \boxed{1.273A.}$$

$$I_3 = \frac{V_2 - 0}{R_3} = \frac{V_2}{R_3} = \frac{32.73}{10} = \boxed{3.273\ A.}$$

ex



Find node voltages:

1.) Determine the number nodes requiring KCLs.
There are 2 non-reference nodes

2.) Label nodes: V_1 and V_2

3.) Apply KCL's at V_1 & V_2 nodes:

KCL at node V_1:

$$4 = I_1 + I_3$$

Apply ohm's law: $I_1 = \frac{V_1 - 0}{2}$, $I_3 = \frac{V_1 - V_2}{12}$

Now substitute into KCL eqn:

$$4 = \frac{V_1}{2} + \frac{V_1 - V_2}{12}$$

$$48 = 6V_1 + V_1 - V_2$$

$$-7V_1 + V_2 = -48$$

$$7V_1 - V_2 = 48 \quad \leftarrow \text{eqn } ①$$

KCL at node V_2:

$$I_3 = I_2 + 2$$

Apply Ohm's Law: $I_3 = \frac{V_1 - V_2}{12}$, $I_2 = \frac{V_2 - 0}{6}$

Substitute into KCL eqn:

$$\frac{V_1 - V_2}{12} = \frac{V_2}{6} + 2$$

$$V_1 - V_2 = 2V_2 + 24$$

$$V_1 - 3V_2 = 24 \quad \leftarrow \text{eqn } (2)$$

4.) Now solve eqn. (1) and (2) simultaneously:

$$7V_1 - V_2 = 48 \quad \leftarrow \text{eqn } (1)$$

$$V_1 - 3V_2 = 24 \quad \leftarrow \text{eqn } (2)$$

$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ 24 & -3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ 1 & -3 \end{vmatrix}} = \frac{-144 - (-24)}{-21 - (-1)} = \frac{-120}{-20} = \boxed{6\text{ V.}}$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ 1 & 24 \end{vmatrix}}{-20} = \frac{168 - 48}{-20} = \frac{120}{-20} = \boxed{-6\text{ V.}}$$

If desired, I_1, I_2 & I_3 can be found:

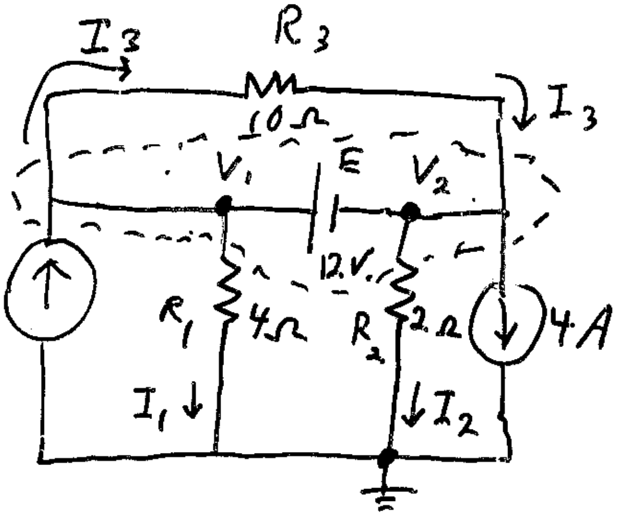
$$I_1 = \frac{V_1}{2} = \frac{6}{2} = \boxed{3\text{ A}} \qquad I_2 = \frac{V_2}{6} = \frac{-6}{6} = \boxed{-1\text{ A}}$$

$$I_3 = \frac{V_1 - V_2}{12} = \frac{6 - (-6)}{12} = \frac{12}{12} = \boxed{1\text{ A}}$$

Supernode

If there are voltage sources in the network, do not write a KCL equation at any node adjacent to a voltage source. If necessary combine more than one node into a supernode or region and then write a KCL for the currents entering and leaving the supernode or region. The current passing thru the voltage source should not be included in the KCL.

ex)



Find V_1 and V_2 by nodal analysis.

Since a voltage source is present, it is necessary to form a Supernode.

(Note: all of the currents crossing the boundry of the supernode can be found with Ω law or are given)

KCL: for supernode $\qquad \sum I_{entering} = \sum I_{leaving}$

$$6A + I_3 = I_3 + I_1 + I_2 + 4A$$

$I_1 + I_2 = 6 - 4 = 2$

Apply ohm's law to get a node voltage eqn:

$$I_1 = \frac{V_1 - 0}{4} \quad , \quad I_2 = \frac{V_2 - 0}{2}$$

Substituting:

$$\frac{V_1}{4} + \frac{V_2}{2} = 2$$

$$V_1 + 2V_2 = 8 \qquad \text{eqn } 1$$

Get another equation by relating V_1 and V_2 to the value of the voltage source.

$$V_1 - V_2 = E$$

$$V_1 - V_2 = 12 \qquad \text{eqn } 2$$

Now solve eqns 1 and 2 simultaneously:

$$V_1 + 2V_2 = 8 \leftarrow \text{eqn. } 1$$

$$V_1 - V_2 = 12 \leftarrow \text{eqn. } 2$$

$$V_1 = \frac{\begin{vmatrix} 8 & 2 \\ 12 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 1 & -1 \end{vmatrix}} = \frac{-8-24}{-1-2} = \frac{-32}{-3} = \boxed{10.67\text{V.}}$$

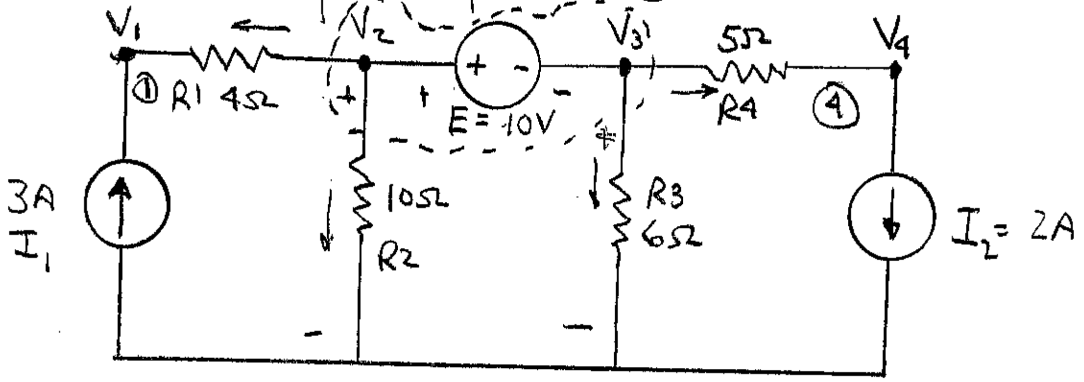
$$V_2 = \frac{\begin{vmatrix} 1 & 8 \\ 1 & 12 \end{vmatrix}}{-3} = \frac{12-8}{-3} = \frac{4}{-3} = \boxed{-1.333\text{V.}}$$

$$I_1 = \frac{V_1}{4} = \frac{10.67}{4} = \boxed{2.667\text{A.}} \qquad I_2 = \frac{V_2}{2} = \frac{-1.333\text{V.}}{2} = \boxed{-0.667\text{A.}}$$

$$I_3 = \frac{V_1 - V_2}{10} = \frac{10.67 - (-1.333)}{10} = \boxed{1.2\text{A}}$$

Nodal Analysis with voltage sources

Short voltage source and write KCL around resulting super-node



KCL at node 1

$$I_1 = \frac{V_1 - V_2}{R_1} \quad ①$$

KCL at super node (assume all currents leave)

$$-I_{R1} - I_{R2} - I_{R3} - I_{R4} = 0 \rightarrow I_{R1} + I_{R2} + I_{R3} + I_{R4} = 0$$

write in terms of node voltages

$$\frac{V_2 - V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} + \frac{V_3 - V_4}{R_4} = 0 \quad ②$$

Relate E to node voltages, KVL $V_2 - E - V_3 = 0$ $E = 10V$

Assume all R current leave $V_2 - V_3 = E \Rightarrow V_2 - V_3 = 10$ ③

KCL AT NODE 4

$$-I_{R4} - I_2 = 0, \quad I_{R4} = -I_2, \quad \frac{V_4 - V_3}{R_4} = -I_2 \quad ④$$

$$3A = \frac{V_1 - V_2}{4} \quad ①$$

$$\frac{V_2 - V_1}{4} + \frac{V_2}{10} + \frac{V_3}{6} + \frac{V_3 - V_4}{5} = 0 \quad ②$$

$$3A = 0.25V_1 - 0.25V_2 \quad ①$$

$$-0.25V_1 + 0.35V_2 + 0.3667V_3 - 0.2V_4 = 0 \quad ②$$

④ $$\frac{V_4 - V_3}{5} = -2A$$

$$-0.2V_3 + 0.2V_4 = -2A$$

Use Relationship $V_2 = 10 + V_3$ to eliminate V_2 from ① & ②

$$3 = 0.25V_1 - 0.25(10 + V_3)$$

$$3 = 0.25V_1 - 2.5 - 0.25V_3$$

$$0.25V_1 - 0.25V_3 = 5.5$$

Remove V_2 from equ. 2

$$-0.25V_1 + 0.35(10+V_3) + 0.3667V_3 - 0.2V_4 = 0$$

$$-0.25V_1 + 3.5 + 0.35V_3 + 0.3667V_3 - 0.2V_4 = 0$$

$$-0.25V_1 + 0.7167V_3 - 0.2V_4 = -3.5$$

$$-0.2V_3 + 0.2V_4 = -2$$

Collect equations

$$0.25V_1 - 0.25V_3 + 0V_4 = 5.5$$
$$-0.25V_1 + 0.7167V_3 - 0.2V_4 = -3.5$$
$$0V_1 - 0.2V_3 + 0.2V_4 = -2$$

Note symmentry of coefficients, diagonals are
sum of conductances at nodes
off-diagonals neg of conductances between nodes

solve using Cramer's rule or TI-85

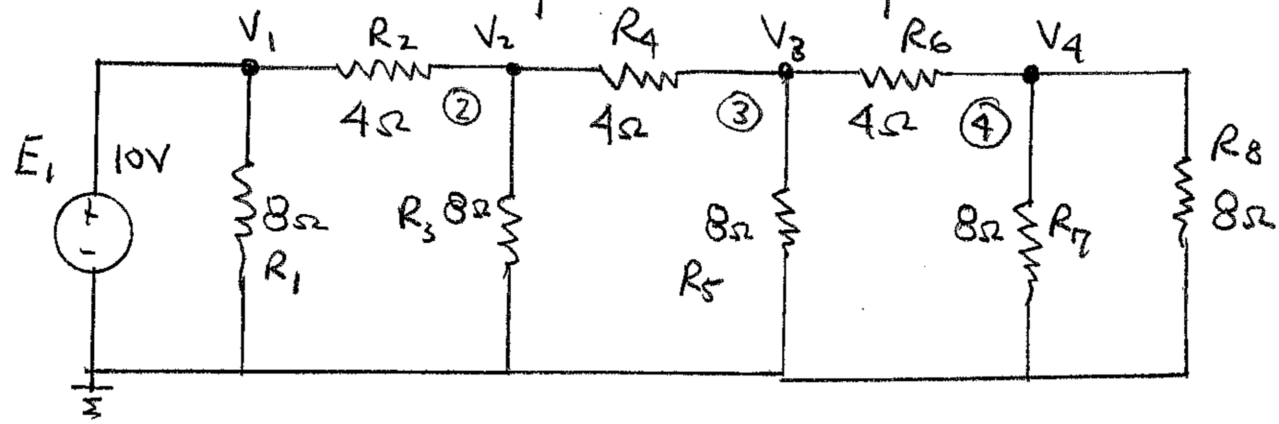
TI-85 solution

$V_1 = \underline{\underline{22\text{ V}}}$ $V_3 = \underline{\underline{0}}$ $V_4 = \underline{\underline{-10}}$

To find V_2 use $V_2 = 10 + V_3 \Rightarrow V_2 = 10 + 0 = \underline{\underline{10\text{ V}}}$

<u>Ex</u>

Ladder network by nodal analysis



Voltage known at V_1 $V_1 = E_1$

WRITE KCL AT OTHER NODES

node 2 Assume all currents leave nodes

$$\frac{V_2-V_1}{R_2}+\frac{V_2}{R_3}+\frac{V_2-V_3}{R_4}=0 \qquad \frac{V_2-V_1}{4}+\frac{V_2}{8}+\frac{V_2-V_3}{4}=0 \quad (1)$$

node 3

$$\frac{V_3-V_2}{R_4}+\frac{V_3}{R_5}+\frac{V_3-V_4}{R_6}=0 \qquad \frac{V_3-V_2}{4}+\frac{V_3}{8}+\frac{V_3-V_4}{4}=0 \quad (2)$$

node 4

$$\frac{V_4-V_3}{R_6}+\frac{V_4}{R_7}+\frac{V_4}{R_8}=0 \qquad \frac{V_4-V_3}{4\Omega}+\frac{2V_4}{8} \quad (3)$$

Simplify (1) (2) (3) and substitute $V_1 = E_1 = 10$

(1) $0.25V_2 - 0.25V_1 + 0.125V_2 + 0.25V_2 - 0.25V_3 = 0$

$-0.25(10) + 0.625V_2 - 0.25V_3 = 0$

$0.625V_2 - 0.25V_3 = 2.5$

(2) $0.25V_3 - 0.25V_2 + 0.125V_3 + 0.25V_3 - 0.25V_4 = 0$

$-0.25V_2 + 0.625V_3 - 0.25V_4 = 0$

(3)

$0.25V_4 - 0.25V_3 + 0.25V_4 = 0$

$-0.25V_3 + 0.5V_4 = 0$ Collect equations (1) (2) (3)
Place in standard form

$0.625V_2 - 0.25V_3 + 0V_4 = 2.5$

$-0.25V_1 + 0.625V_3 - 0.25V_4 = 0$

$0V_1 + 0.25V_3 + 0.5V_4 = 0$

note symmetry of equations check signs of diagonal and off-diagonal

solve using Cramer's rule of other numerical method.

4- Using "SOLVE" Function of TI-85

$V_2 = \underline{5\,V}$ $\underline{V_1 = 10}$ from inspection

$V_3 = \underline{2.5}$

$V_4 = \underline{1.25}$

Calcute Currents in R_1, R_3, R_5, R_7 & R_8

$I_{R8} = \frac{V_4}{R_8} = \frac{1.25V}{8} = \underline{0.1563\,A}$ $I_{R3} = \frac{V_2}{R_3} = \frac{5}{8} = \underline{0.625\,A}$

$I_{R7} = I_{R8} = \underline{0.1563\,A}$ $I_{R1} = \frac{V_1}{R_1} = \frac{10}{8} = \underline{1.25A}$

$I_{R5} = \frac{V_3}{R_5} = \frac{2.5}{8} = \underline{0.3125\,A}$

CURRENTS DIVIDE BY 2 with each step through Network

BASIC CIRCUIT USED IN DIGITAL-TO-ANALOG CONVERTERS CALLED R-2R Ladder

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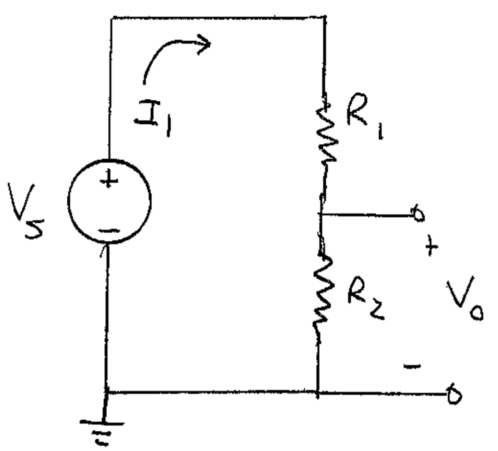
Resistive Bridges

Consider a basic dc voltage divider.



$$V_o = \frac{R_2}{R_1 + R_2} V_s$$

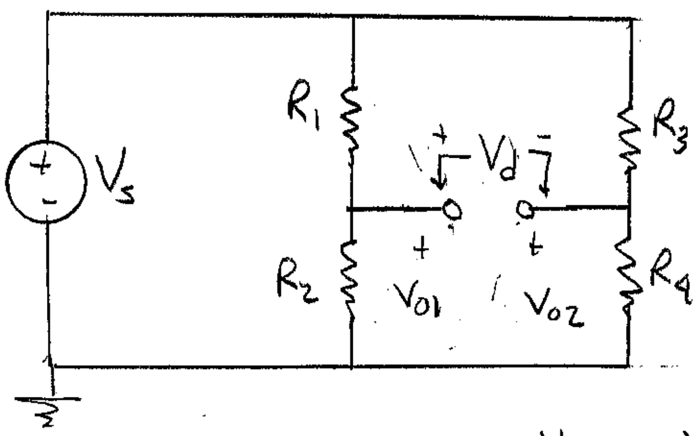
Increasing R_2 increases V_o

Decreasing R_2 decreases V_o

for Fixed V_s & R_1

-5-

Consider 2 voltage dividers supplied from same source.



find V_{o1} and V_{o2}

$$V_{o1} = \frac{R_2}{R_1 + R_2} V_s$$

$$V_{o2} = \frac{R_4}{R_3 + R_4} V_s$$

Difference in voltage $V_d = V_{o1} - V_{o2}$

When $V_{o1} = V_{o2}$ Then

$$\frac{R_2}{R_1 + R_2} V_s = \frac{R_4}{R_3 + R_4} V_s \qquad \frac{R_2}{R_1 + R_2} = \frac{R_4}{R_3 + R_4}$$

Bridge is considered Balanced when $V_{o1} = V_{o2}$

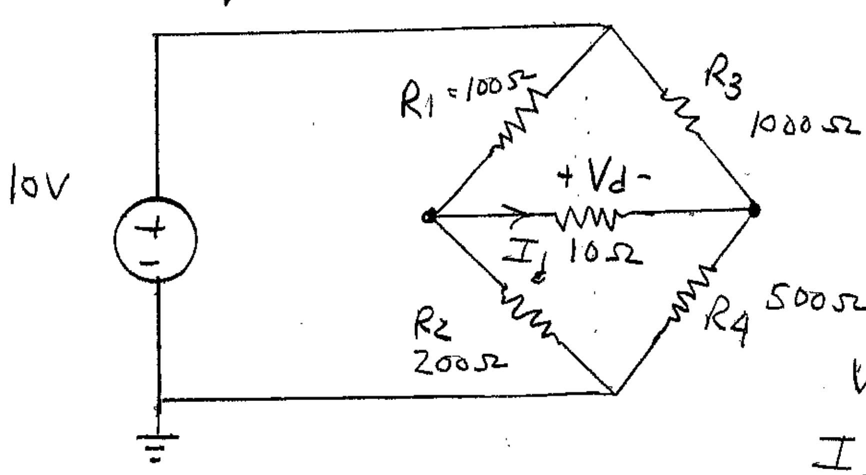
$$\frac{R_2}{R_1} + 1 = \frac{R_4}{R_3} + 1 \Rightarrow \frac{R_2}{R_1} = \frac{R_4}{R_3}$$

---

when Ratio of divider resistors is equal

$V_d = 0$ $\quad V_d = 0$ means no current will flow

When $\frac{R_2}{R_1} \neq \frac{R_4}{R_3}$ must solve circuit using mesh or node equations



$$\frac{R_1}{R_2} = \frac{100\Omega}{200\Omega} = 1/2$$

$$\frac{R_3}{R_4} = \frac{1000\Omega}{500\Omega} = 2$$

$$\frac{1}{2} \neq 2$$

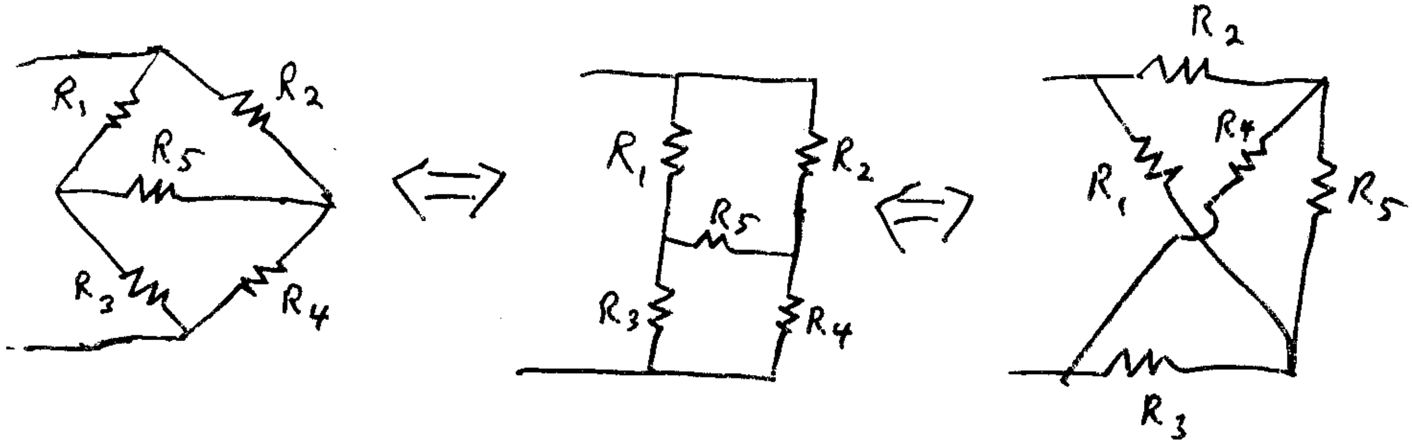
$V_d \neq 0$

$I_d \neq 0$

29E

Bridge Networks

General Form of a Bridge Circuit:



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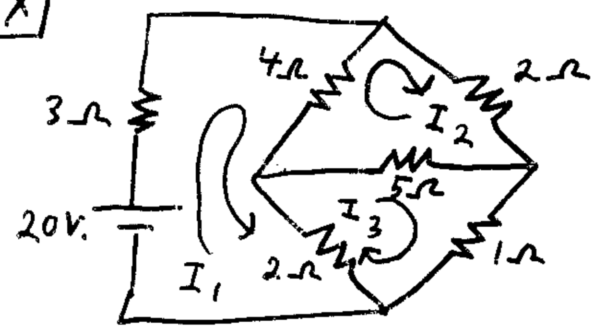


find current through each resistor.

Apply mesh analysis for 3 meshes.

KVL mesh 1:

$$-20 + 3I_1 + 4(I_1 - I_2) + 2(I_1 - I_3) = 0$$

$$9I_1 - 4I_2 - 2I_3 = 20 \quad \leftarrow \text{eqn. } 1$$

KVL mesh 2:

$$4(I_2 - I_1) + 2I_2 + 5(I_2 - I_3) = 0$$

$$-4I_1 + 11I_2 - 5I_3 = 0 \quad \leftarrow \text{eqn. } 2$$

KVL mesh 3:

$$2(I_3 - I_1) + 5(I_3 - I_2) + 1(I_3) = 0$$

$$-2I_1 - 5I_2 + 8I_3 = 0 \quad \leftarrow \text{eqn. } 3$$

Now solve eqns. 1, 2 and 3 simultaneously

$9 I_1 - 4 I_2 - 2 I_3 = 20$ ← eqn ①

$4 I_1 - 11 I_2 + 5 I_3 = 0$ ← eqn ② (multiplied by −1)

$2 I_1 + 5 I_2 - 8 I_3 = 0$ ← eqn ③ (multiplied by −1)

$$I_1 = \frac{\begin{vmatrix} 20 & -4 & -2 \\ 0 & -11 & 5 \\ 0 & 5 & -8 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -2 \\ 4 & -11 & 5 \\ 2 & 5 & -8 \end{vmatrix}} = \boxed{4A}$$

$$I_2 = \frac{\begin{vmatrix} 9 & 20 & -2 \\ 4 & 0 & 5 \\ 2 & 0 & -8 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -2 \\ 4 & -11 & 5 \\ 2 & 5 & -8 \end{vmatrix}} = \boxed{2.667\,A.}$$

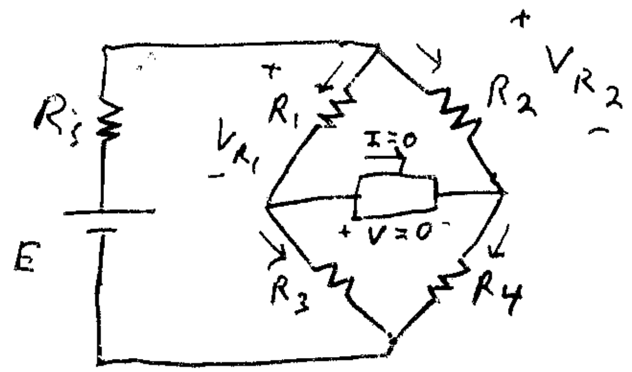
$$I_3 = \frac{\begin{vmatrix} 9 & -4 & 20 \\ 4 & -11 & 0 \\ 2 & 5 & 0 \end{vmatrix}}{\begin{vmatrix} 9 & -4 & -2 \\ 4 & -11 & 5 \\ 2 & 5 & -8 \end{vmatrix}} = \boxed{2.667\,A.}$$

The net current thru the 5Ω resistor is:

$$I_{5\Omega} = I_2 - I_3 = 2.667 - 2.667 = 0A.$$

The voltage $V_{5\Omega} = I_{5\Omega}(5\Omega) = 0\,(5\Omega) = 0$

> A bridge network is said to be <u>balanced</u> when $I=0$ and $V=0$ for the center element.



For the balanced condition:

$I=0,\ V=0$

Then $V = V_{R_2} - V_{R_1}$ or

$V_{R_1} = V_{R_2}$

Then $I_{R_1} R_1 = I_{R_2} R_2$

so $I_{R_1} = I_{R_2} \dfrac{R_2}{R_1}$ ← eqn 1

Similarly, $V_{R_3} = V_{R_4}$

$I_{R_3} R_3 = I_{R_4} R_4$ ← eqn. 2

Since $I=0$ at balance, then $I_{R_1} = I_{R_3}$ and $I_{R_2} = I_{R_4}$

Substituting eqn 1 into eqn 2 gives:

$$\left(I_{R_2} \frac{R_2}{R_1}\right) R_3 = I_{R_2} R_4$$

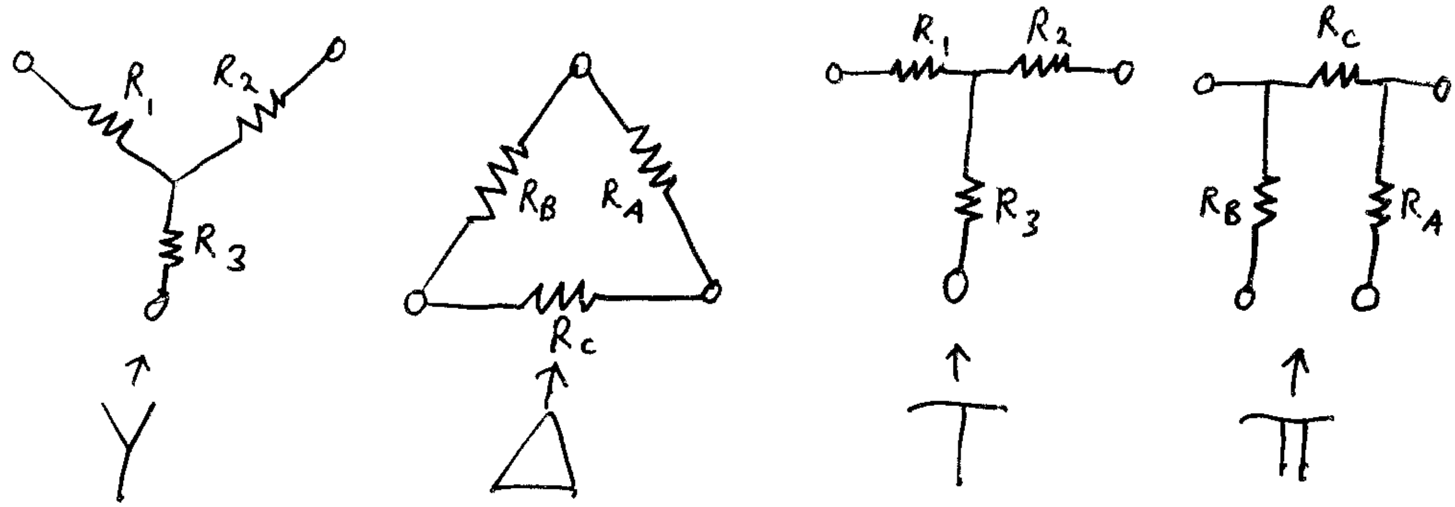
or re-arranging gives:

The condition for the bridge circuit to be balanced is: $\boxed{\dfrac{R_1}{R_3} = \dfrac{R_2}{R_4}}$

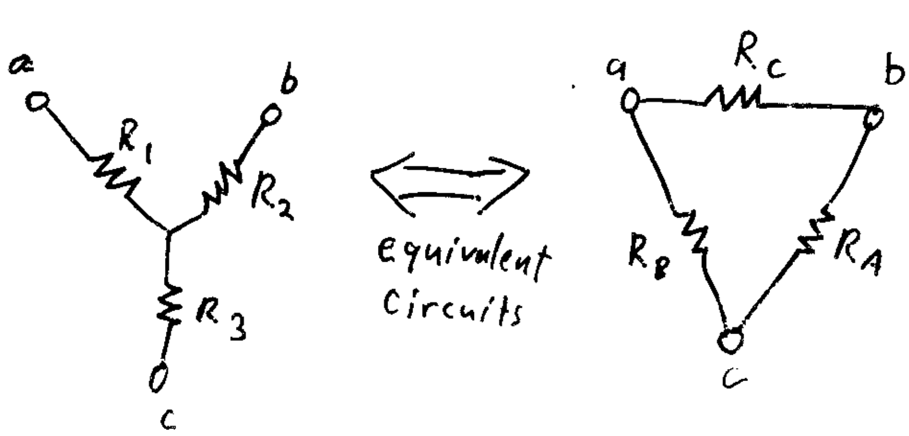
This concept is used in measurement equipment. i.e. If the bridge is adjusted to a balanced condition, an unknown bridge arm R_X can be computed from the 3 known bridge arms.

Y-Δ (T-π) AND Δ-Y (π-T) Conversions

Circuit analysis can sometimes be simplified by converting a circuit in one form to another equivalent form.



The Y and the T are identical circuits that are drawn differently.
The Δ and the π are identical circuits that are drawn differently.



<u>Δ → Y conversion</u>:
$$R_1 = \frac{R_B R_c}{R_A+R_B+R_c} \qquad R_2 = \frac{R_A R_c}{R_A+R_B+R_c} \qquad R_3 = \frac{R_A R_B}{R_A+R_B+R_c}$$

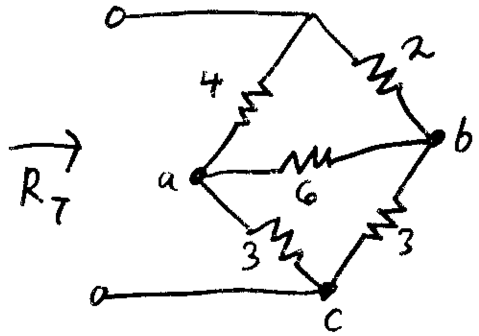
<u>Y → Δ Conversion</u>:
$$R_A = \frac{R_1R_2+R_1R_3+R_2R_3}{R_1} \qquad R_B = \frac{R_1R_2+R_1R_3+R_2R_3}{R_2}$$
$$R_c = \frac{R_1R_2+R_1R_3+R_2R_3}{R_3}$$

For a Y of three equal resistors, the value of each resistor of the equivalent Δ is equal to three times the value of any resistor of the Y.

$$\boxed{R_\Delta = 3 R_Y} \qquad \boxed{R_Y = \frac{R_\Delta}{3}}$$

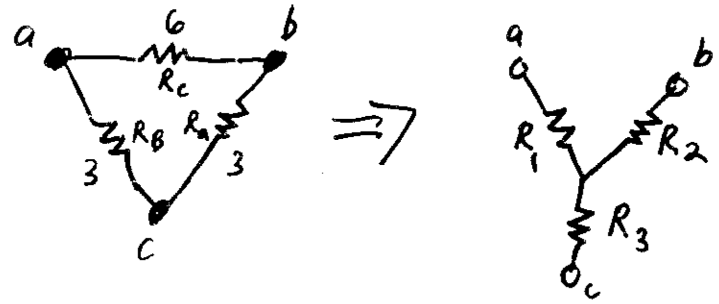
when $R_1 = R_2 = R_3 = R_Y$
$R_A = R_B = R_C = R_\Delta$

ex/ Find the total resistance R_T:



Note: None of the resistors are in parallel or series.

In order to simplify the network we can replace the Δ section with a Y Section



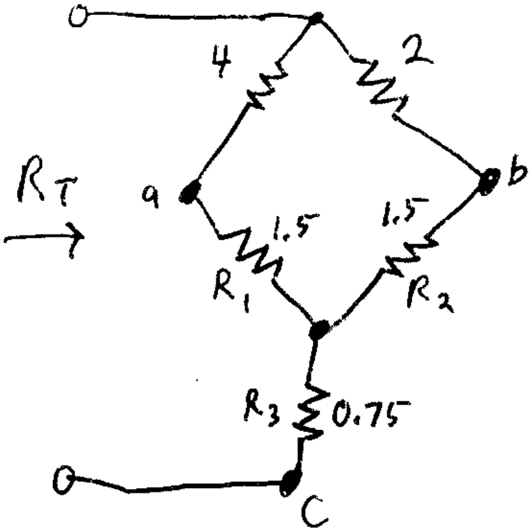
Using the conversion formulas to find R_1, R_2, R_3 gives:

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(3)(6)}{3+3+6} = \frac{18}{12} = 1.5\,\Omega$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(3)(6)}{12} = \frac{18}{12} = 1.5\,\Omega$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(3)(3)}{12} = \frac{9}{12} = 0.75\,\Omega$$

Now re-draw the original ckt. with the Y network inserted in place of the Δ network.



In this new equivalent ckt. it is possible to use series and parallel resistor combining rules to simplify the network.

$$R_T = 0.75 + (R_1 + 4) \,||\, (R_2 + 2)$$

$$R_T = 0.75 + \frac{(1.5+4)(1.5+2)}{(1.5+4)+(1.5+2)} = 0.75 + \frac{(5.5)(3.5)}{5.5 + 3.5}$$

$$R_T = 0.75 + 2.139$$

$$R_T = \boxed{2.889\ \Omega}$$

Superposition Theorem

The Superposition Theorem states that the current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the currents or voltages produce independently by each source.

Superposition Method:

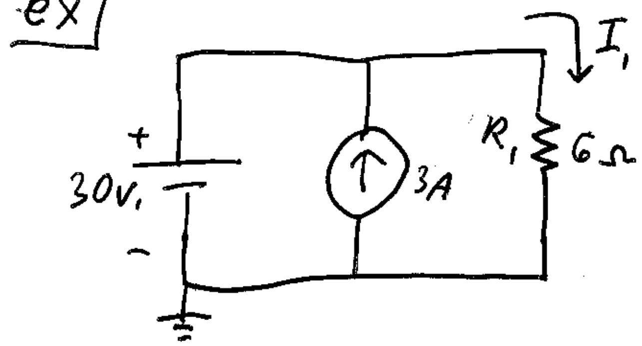
To find a particular I or V in a multisource network (n sources): $S_1, S_2 \dots S_n$

1. Kill all of the sources except S_1
   (to kill a voltage source, replace it with a short ckt.)
   (to kill a current source, replace it with an open ckt.)
2. Solve the resulting network for V and I, let these values equal: $V = V_1$, $I = I_1$
3. Kill all of the sources except S_2
4. Solve the resulting network for V and I, let these values equal: $V = V_2$, $I = I_2$
5. Repeat steps 3 and 4 for sources $S_3, S_4 \dots S_n$
6. The total values are the found:

$$V = V_1 + V_2 + V_3 + \dots V_n$$

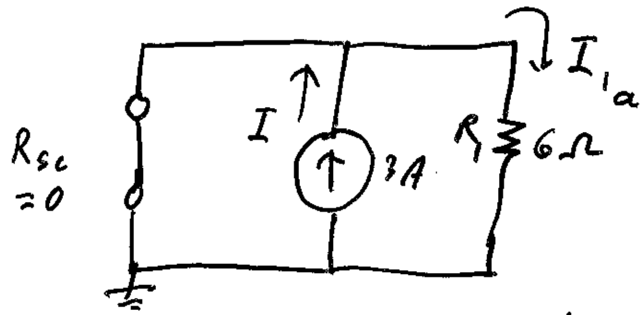
$$I = I_1 + I_2 + I_3 + \dots I_n$$

ex/



Find I_1 using Superposition Theorem:

1. kill 30V. source (replace it with a short ckt. v=0)

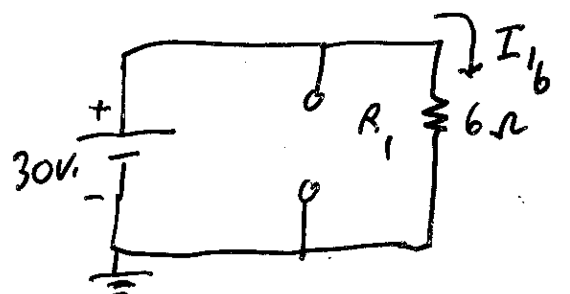


(Note: I_{1a} is the part of I_1 due to the 3A. Source)

2. Solve resulting network:

By Current Divider Rule: $I_{1_a} = I \dfrac{R_{sc}}{R_1 + R_{sc}} = 3 \dfrac{0\Omega}{6+0} = 0$ A.

3. Start with original ckt. and now kill 3A. Source; (replace 3A. source with open ckt. I=0)



(Note: I_{1_b} is the part of I_1 due to 30V. source)

4. Solve resulting network:

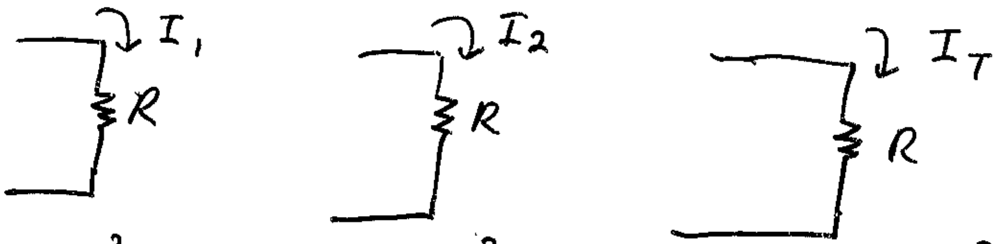
$$I_{1b} = \frac{30V.}{6\Omega} = 5A.$$

Now combine results:

$$I = I_{1a} + I_{1_b} = 0 + 5 = \boxed{5A}$$

Note: Superposition principle does not work for Power computations.

ex)



$P_1 = I_1^2 R$ $\quad$ $P_2 = I_2^2 R$ $\quad$ $P_T = I_T^2 R$

If superposition applied: $\quad P_T = P_1 + P_2$

Since $I_T = I_1 + I_2$

then $P_T = (I_1 + I_2)^2 R = (I_1^2 + I_2^2 + 2 I_1 I_2) R$

$$P_T = I_1^2 R + I_2^2 R + 2 I_1 I_2 R$$

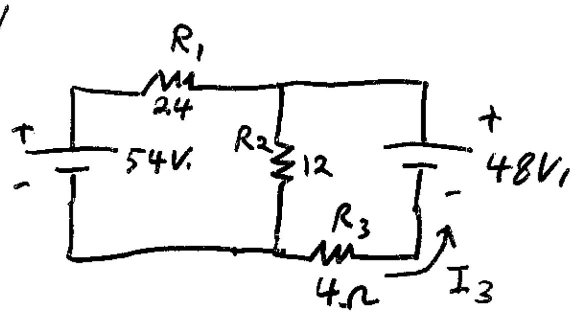
$$P_T = P_1 + P_2 + 2 I_1 I_2 R \neq P_1 + P_2$$

Superposition only applies for <u>linear</u> functions. Power is a <u>square law</u> and therefore not linear.

So:

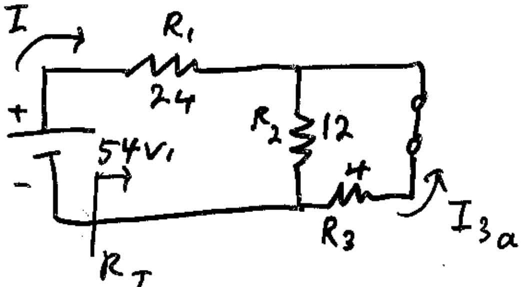
$\boxed{\text{The total power delivered to a resistive element must be determined using the total current through or voltage across the element and cannot be determined by a simple sum of the power levels established by each source.}}$

ex/



Find I_3 using superposition.

1.) kill 48V. source:



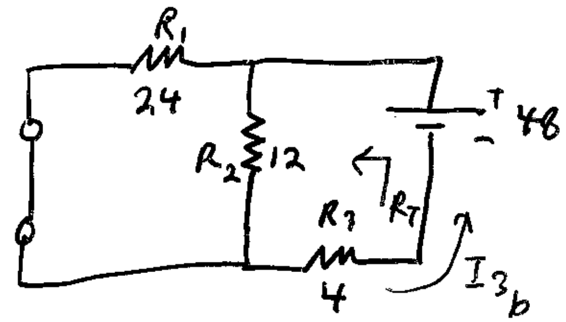
$$R_T = R_1 + R_2 \| R_3$$

$$R_T = 24 + \frac{(4)(12)}{4+12} = 24+3 = 27\Omega$$

$$I = \frac{54}{R_T} = \frac{54}{27} = 2A.$$

By CDR: $I_{3a} = -I\frac{R_2}{R_2+R_3} = -2\frac{(12)}{12+4} = \frac{-24}{16} = -1.5A$

2.) kill 54V. source:



$$R_T = R_3 + R_1 \| R_2$$

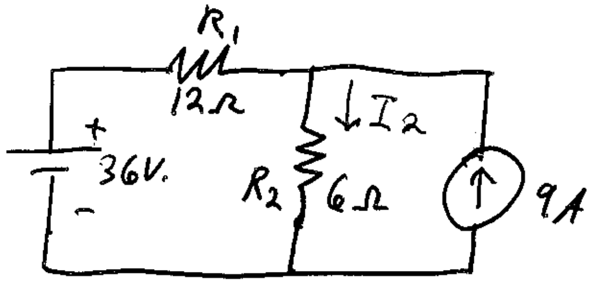
$$R_T = 4 + \frac{(24)(12)}{24+12} = 4+8 = 12\Omega$$

$$I_{3_b} = \frac{48}{R_T} = \frac{48}{12} = 4A.$$

4.) Combine results:

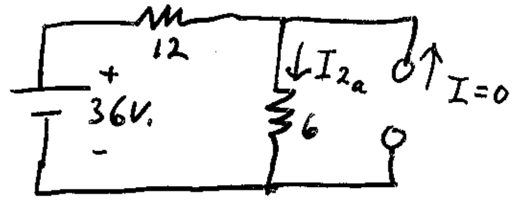
$$I_3 = I_{3a} + I_{3_b} = -1.5A + 4A = \boxed{2.5A}$$

ex



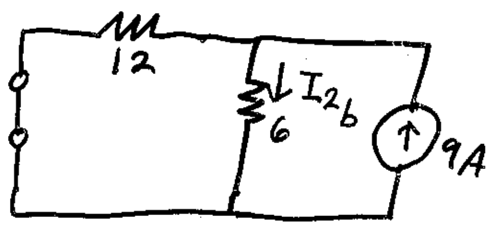
Find I_2 using Superposition.

1.) Kill 9A. source:



$$I_{2a} = \frac{36V.}{12\Omega + 6\Omega} = 2A.$$

2.) kill 36V. source;



By CDR;

$$I_{2b} = (9A.)\frac{12}{12+6} = 6A.$$

3.) combine results:

$$I_2 = I_{2a} + I_{2b} = 2A. + 6A. = \boxed{8A.}$$

$$P_{6\Omega} = I^2 R = (8)^2 6 = 384\,w.$$

If we <u>misuse</u> the superposition principle for combining power we get:

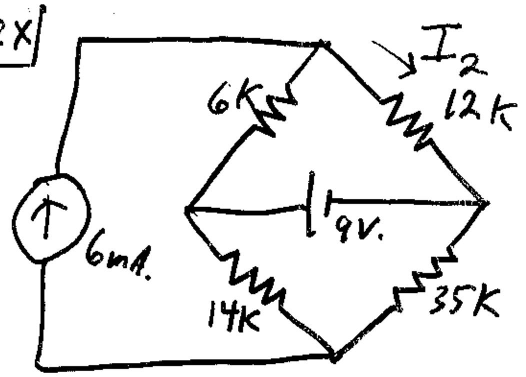
$$P_a = (I_{2a})^2 R = (2)^2 6 = 24\,w.$$

$$P_b = (I_{2b})^2 R = (6)^2 6 = 216\,w.$$

$$P_a + P_b = 240\,w. \neq 384\,w.$$

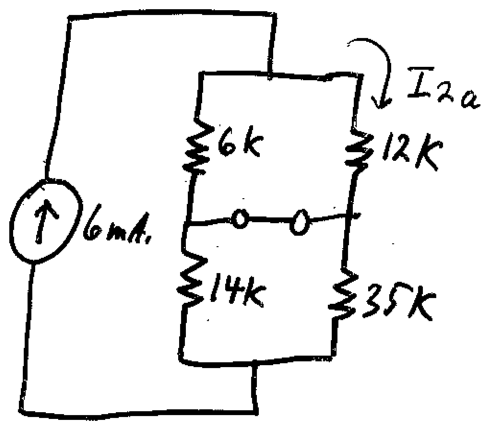
erroneous value

ex



Find I_2 using Superposition:

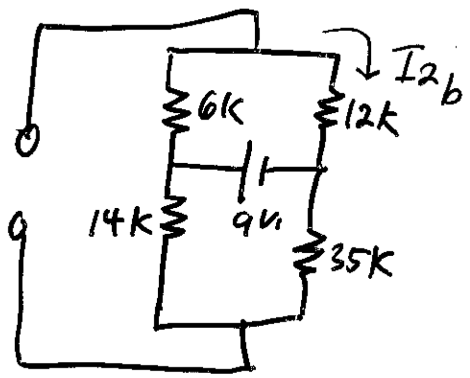
1.) Kill 9V. source:



By CDR:

$$I_{2a} = (6\,mA)\frac{6k}{6k+12k} = 2\,mA$$

2.) Kill 6mA. source:

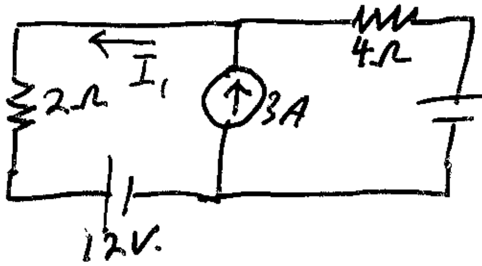


$$I_{2b} = \frac{9V.}{6k+12k} = 0.5\,mA$$

3.) Combine Results:

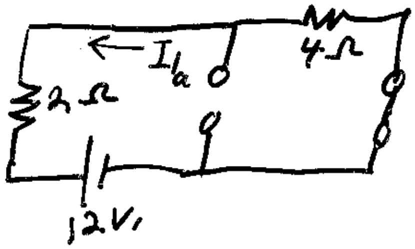
$$I_2 = I_{2a} + I_{2b} = 2mA. + 0.5mA = \boxed{2.5\ mA.}$$

ex/



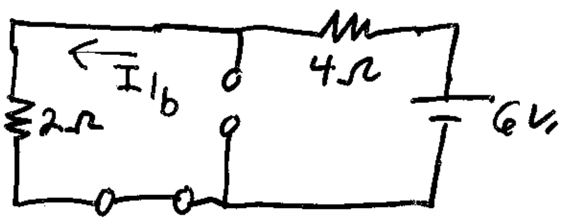
Find I_1 using Superposition:

1.) kill 3A and 6V. sources: (leave only 12V. source active)



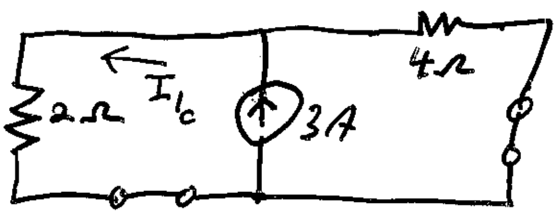
$$I_{1a} = -\frac{12V.}{2\Omega + 4\Omega} = -2A.$$

2) kill 3A. and 12V. sources:



$$I_{1b} = \frac{6V.}{2+4} = 1A.$$

3.) kill 12V. and 6V. sources:



By CDR:

$$I_{1c} = (3A)\frac{4}{2+4} = 2A.$$

4.) Combine Results:

$$I_1 = I_{1a} + I_{1b} + I_{1c} = -2 + 1 + 2 = \boxed{1A.}$$