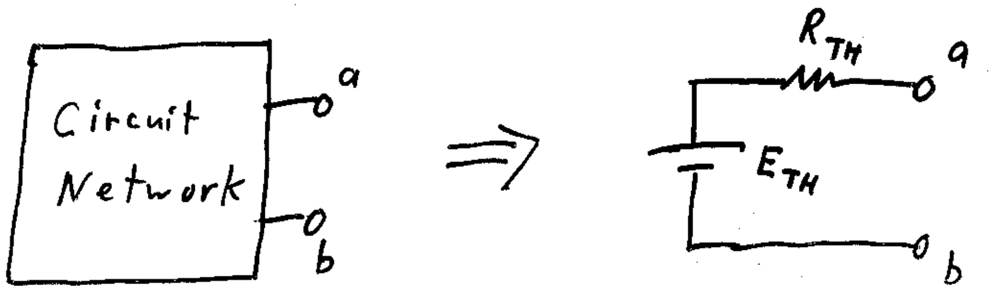
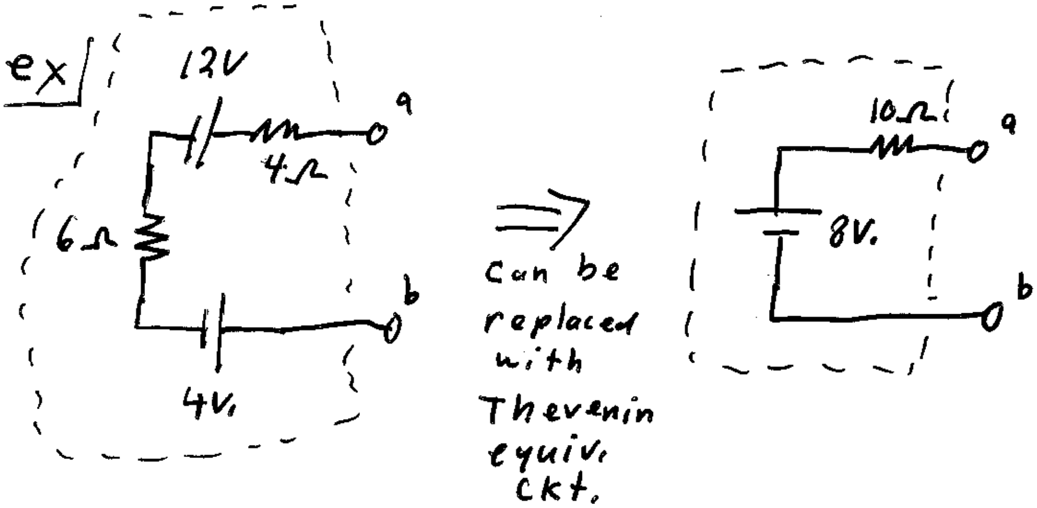
Thevenin's Theorem

> Thevenin's theorem states that any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.

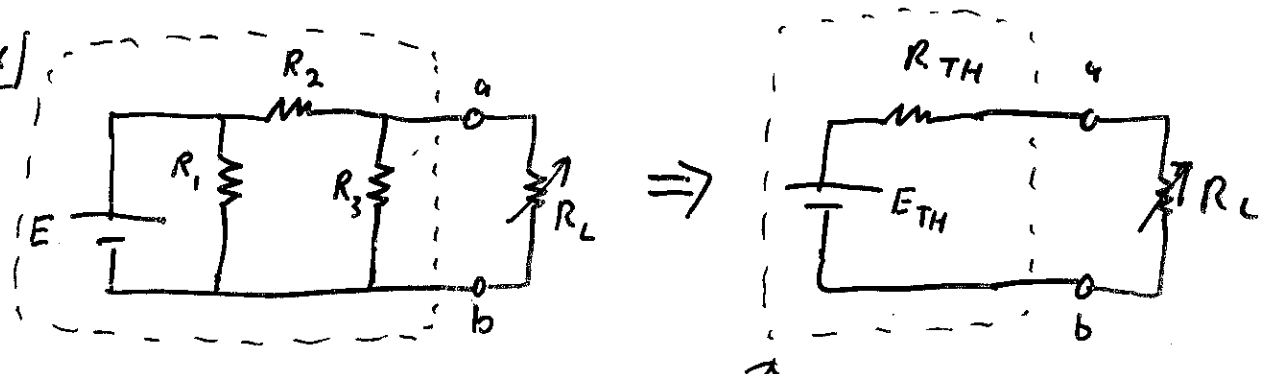


Thevenin Equivalent Circuit

ex/



ex/



Easier Ckt. to analyze

Proceedure to Find E_{TH} & R_{TH}

1.) Remove that portion of the network across which the Thevenin Equiv. ckt is to be found.

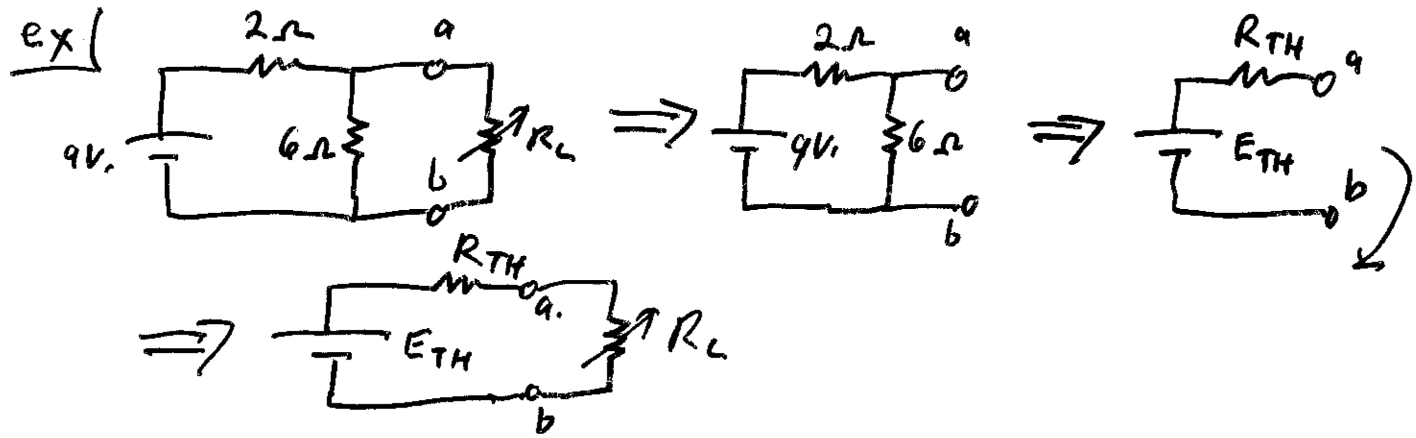
2.) Label terminals "a" and "b" which correspond to the location where the Thevenin Equiv. ckt will be connected to the network.

3.) Calculate E_{TH} as the voltage between terminals "a" and "b" in the removed portion of the network which is to be "Thevenized". E_{TH} is also called the "open ckt." voltage found when any load has been removed from across terminals "a" and "b".

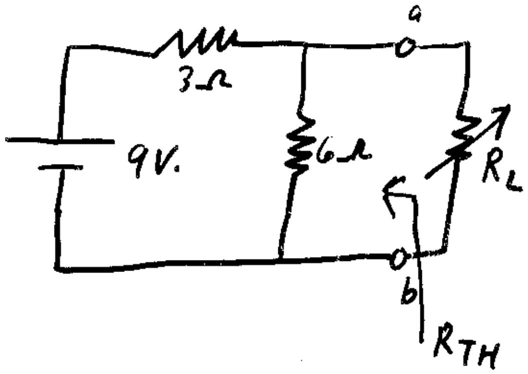
4.) Calculate R_{TH} by killing all sources (replace voltage sources with short ckts. and current sources with open ckts.) and then combining the resistors to a single total resistance.

5.) Draw the Thevenin Equiv. ckt. in place of the original ckt. in the network.

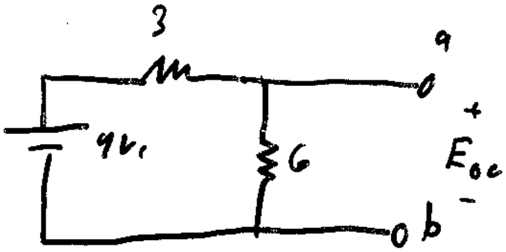
ex)



ex)



Find Thevenin Equiv. Ckt. "looking" to the left of Terminals a & b.
Then find I_L for $R_L = 2\Omega$, 10Ω, 100Ω



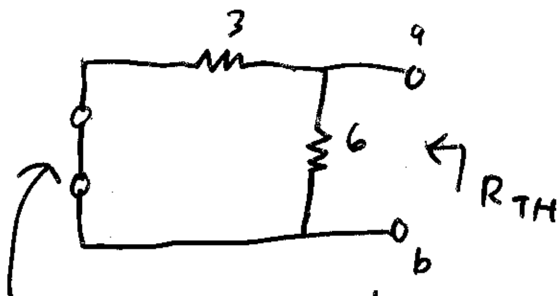
1.) Remove ckt. from rest of network

2.) Find E_{oc} (open ckt.)

$E_{TH} = E_{oc}$

$$E_{TH} = E_{oc} = \frac{9\,(6)}{3+6} = \boxed{6V.}$$

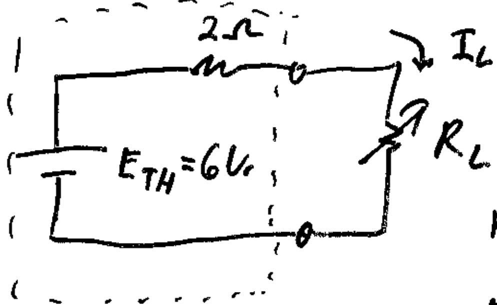
3.) Kill sources to find R_{TH}



$$R_{TH} = 3\Omega \parallel 6\Omega = \frac{(3)(6)}{3+6} = \boxed{2\Omega}$$

Replace voltage source with a short ckt.

4.) Put Thevenin Equiv. Ckt. in place of original Ckt.



Thevenin Equiv. Ckt.

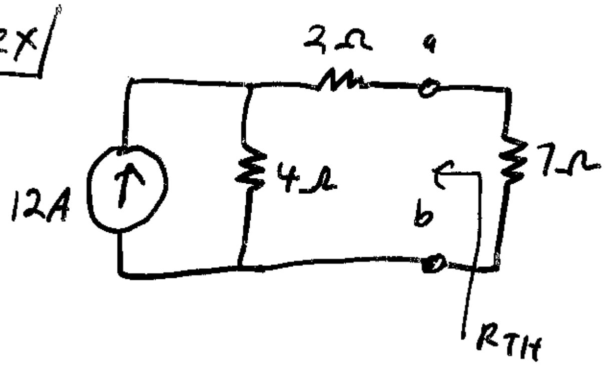
Then $I_L = \dfrac{6V.}{2 + R_L}$

For $R_L = 2\Omega$: $I_L = \dfrac{6}{2+2} = \boxed{1.5A.}$

For $R_L = 10\Omega$: $I_L = \dfrac{6}{2+10} = \boxed{0.5A}$

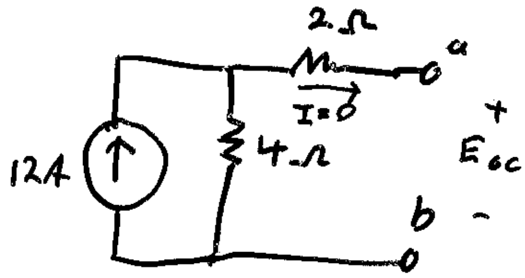
For $R_L = 100\Omega$: $I_L = \dfrac{6}{2+100} = \boxed{0.059A}$

ex/



Find Thevenin Equiv. Ckt.
"look" to the left of terminals a & b.

1.) Remove ckt. from network

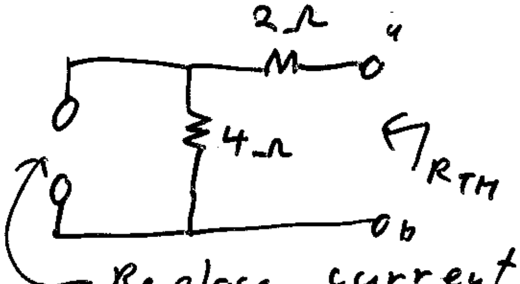


2.) Find $E_{oc} = E_{TH}$

$V_{2\Omega} = (2)(0) = 0$

$E_{oc} = E_{4\Omega} = (12A)(4\Omega) = \boxed{48V.}$

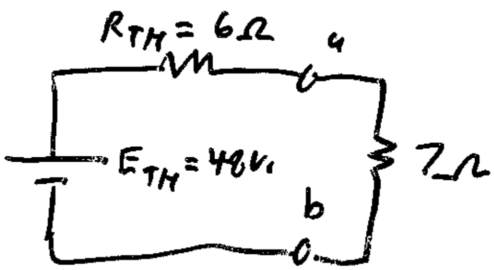
3.) kill sources to find R_{TH}:



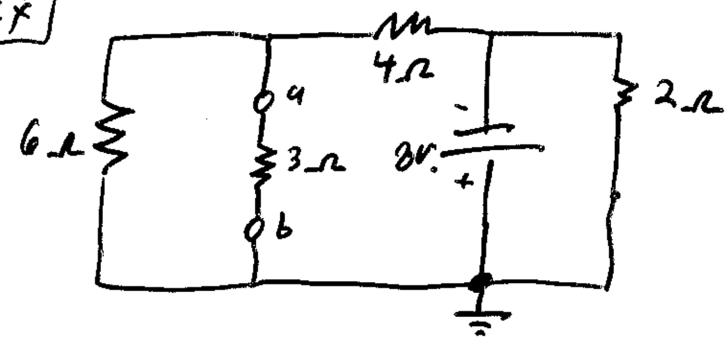
$R_{TH} = 2\Omega + 4\Omega = \boxed{6\Omega}$

Replace current source with open ckt.

4.) Put Thevenin Equiv. Ckt. in place of original ckt.

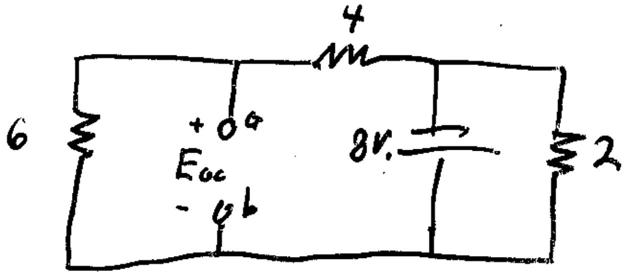


ex



Find Thevenin Equiv. ckt. "seen" by the 3Ω resistor.

1.) Remove ckt. from network

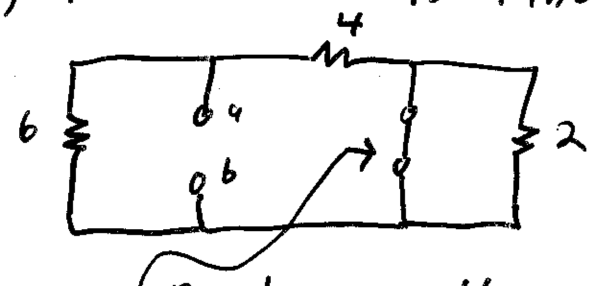


2.) Find $E_{oc} = E_{TH}$

By Voltage Divider Rule:

$$E_{TH} = E_{oc} = (-8V.)\frac{6}{4+6} = \boxed{-4.8V.}$$

3.) Kill sources to find R_{TH}:

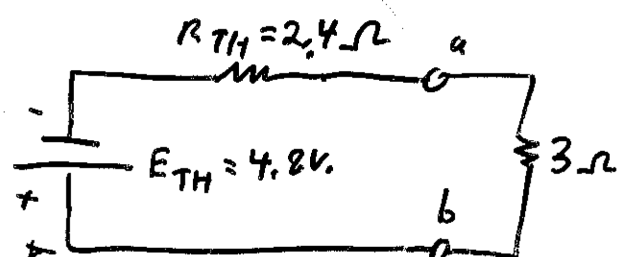


$R_{TH} = 6Ω \parallel 4Ω$

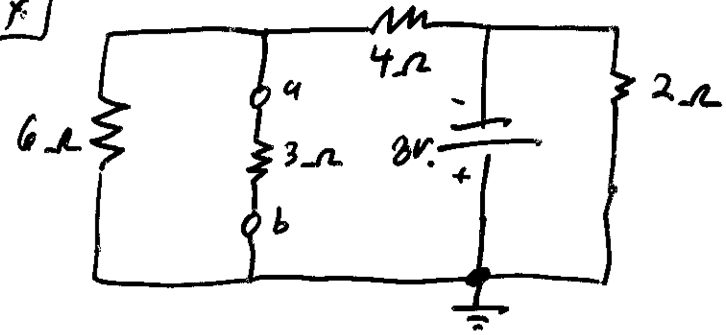
$$R_{TH} = \frac{(6)(4)}{6+4} = \boxed{2.4Ω}$$

Replace voltage source with short ckt.

4.) Put Thevenin Equiv. ckt. in place of original ckt.

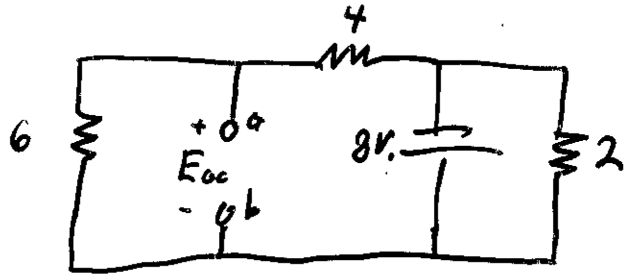


note polarity from original ckt., the positive source terminal is connected to terminal "b" ex.



Find Thevenin Equiv. ckt. "seen" by the 3Ω resistor.

1.) Remove ckt. from network

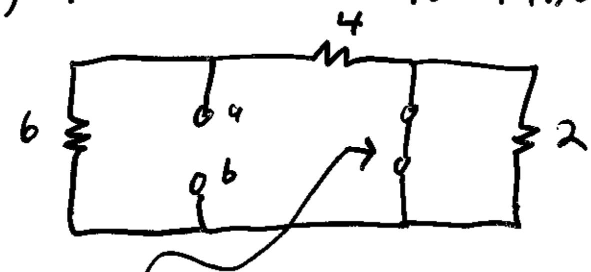


2.) Find $E_{oc} = E_{TH}$

By Voltage Divider Rule:

$$E_{TH} = E_{oc} = (-8V.)\frac{6}{4+6} = \boxed{-4.8V.}$$

3.) Kill sources to find R_{TH}:

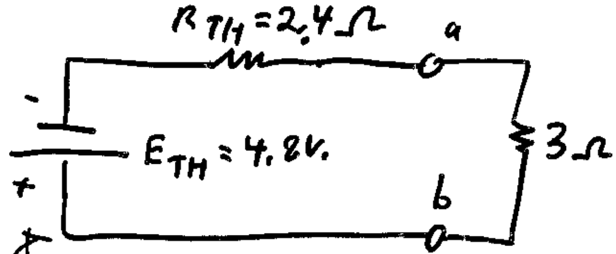


Replace voltage source with short ckt.

$R_{TH} = 6\Omega \parallel 4\Omega$

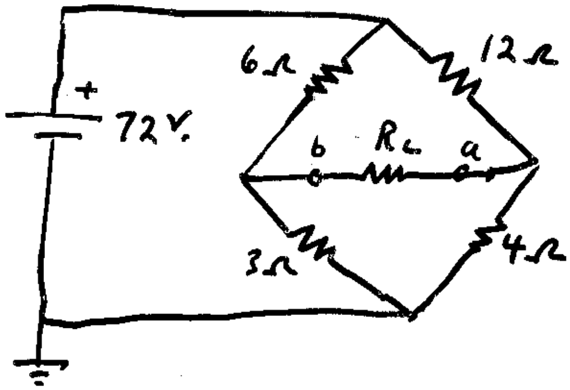
$$R_{TH} = \frac{(6)(4)}{6+4} = \boxed{2.4\Omega}$$

4.) Put Thevenin Equiv. ckt. in place of original ckt.



note polarity from original ckt., the positive source terminal is connected to terminal "b"

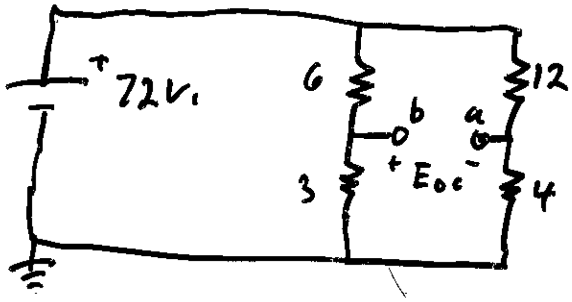
ex



Find the Thevenin Equiv. ckt. "seen" by the R_L resistor.

1.) Remove Ckt. from network

2.) Find $E_{oc} = E_{TH}$



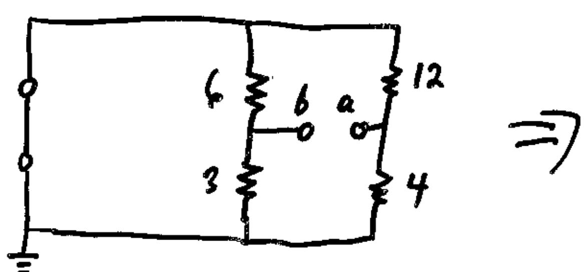
By Voltage Divider Rule:

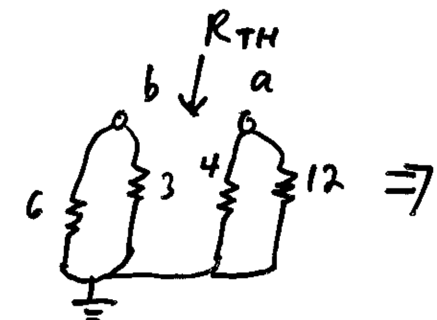
$$V_b = 72\,\frac{3}{6+3} = \frac{72}{3} = 24V.$$

$$V_a = 72\,\frac{4}{12+4} = \frac{72}{4} = 18\,V.$$

$$E_{oc} = V_b - V_a = 24 - 18 = \boxed{6V. = E_{TH}}$$

3.) kill sources to find R_{TH}:

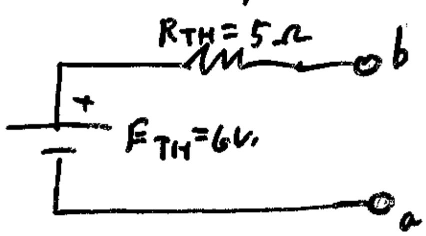




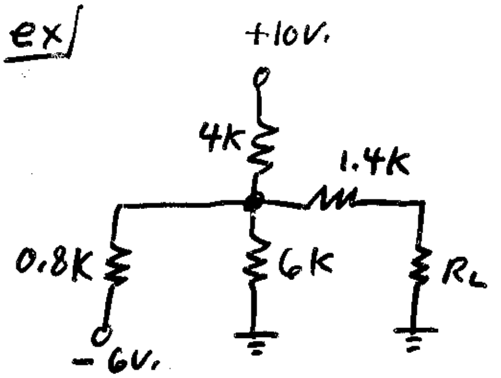
$$R_{TH} = 6 \| 3 + 4 \| 12 = \frac{6 \cdot 3}{6+3} + \frac{4 \cdot 12}{4+12}$$

$$R_{TH} = 2 + 3 = \boxed{5\,\Omega}$$

4.) Draw Thevenin Equiv. Ckt.



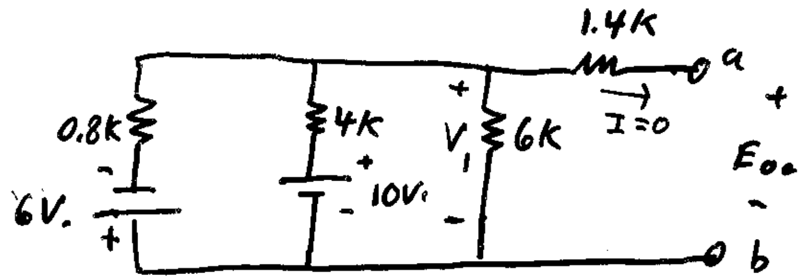
← note battery polarity for "b" more positive than terminal "a" ex/



Find the Thevenin Equiv. ckt. "seen" by the load.

1.) Remove the ckt. from the network

2.) Find $E_{oc} = E_{TH}$

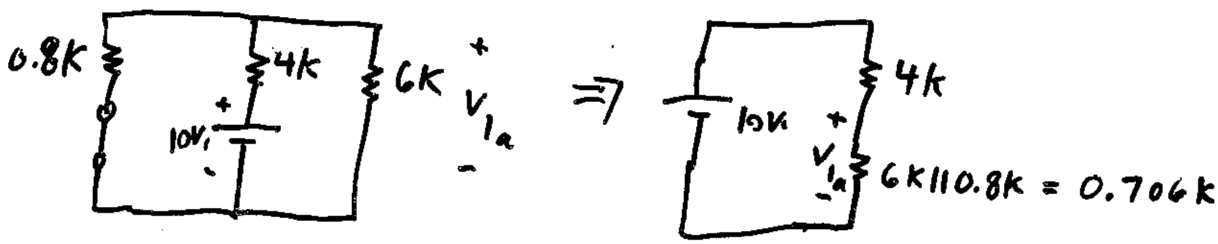


Since $I = 0$

$E_{oc} = V_1$

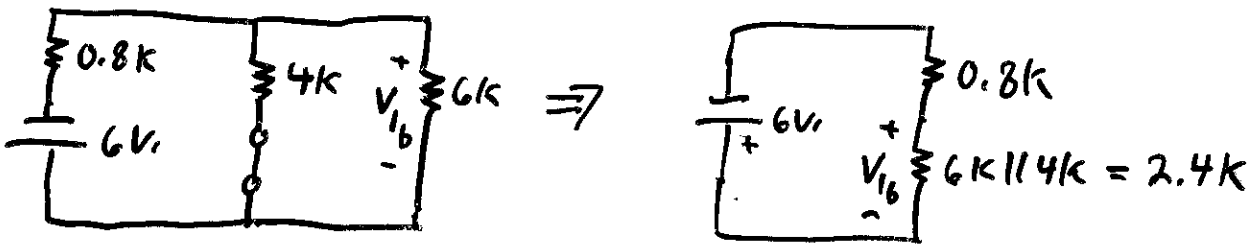
We can find V_1 using Superposition Theorem:

kill 6V. source:



By VDR: $V_{1a} = 10V. \left(\frac{0.706k}{4k + 0.706k} \right) = 1.5V.$

Kill 10V. source:



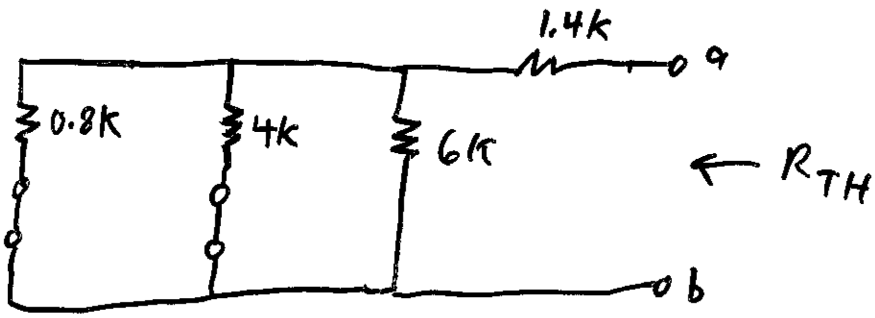
By VDR: $V_{1b} = (-6V.) \frac{2.4k}{2.4K + 0.8K} \approx -4.5V.$

Combine V_{1a} and V_{1b}:

$V_1 = V_{1a} + V_{1b} = 1.5 - 4.5 \approx -3V.$

$\boxed{E_{oc} = E_{TH} = -3V.}$

3.) Kill sources to find R_{TH}



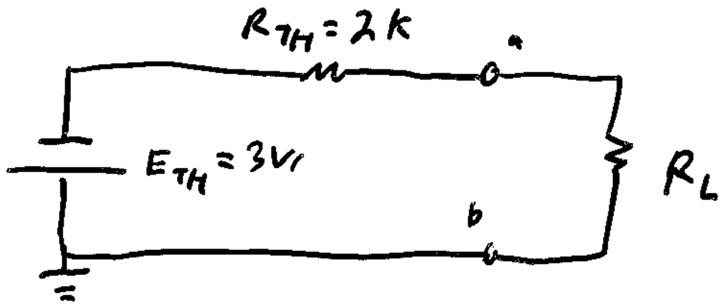
$$R_{TH} = 1.4K + 0.8K \parallel 4K \parallel 6K$$

$$R_{TH} = 1.4K + 0.8K \parallel 2.4K$$

$$R_{TH} = 1.4K + 0.6K$$

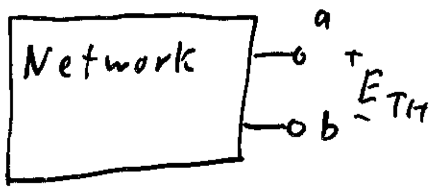
$$R_{TH} = \boxed{2K}$$

4.) Draw Thevenin Equiv. Ckt.



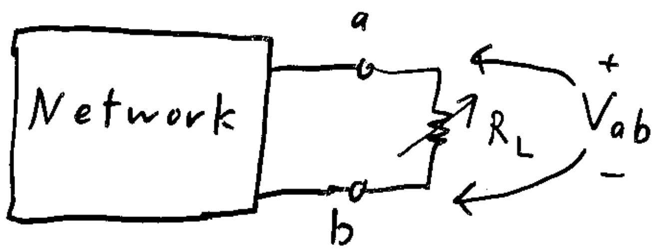
Experimental Proceedures

Direct Measurement of E_{TH} and R_{TH}



1.) Measure E_{TH} as $E_{open\ ckt.}$ with no R_L connected.

2.) Connect a variable resistance (Decade Box) whose value is larger than the suspected R_{TH} value.

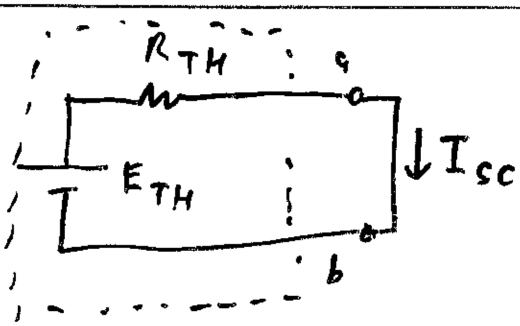


3.) While measuring V_{ab} with a voltmeter or oscilloscope, decrease R_L until $V_{ab} = \frac{E_{TH}}{2}$

4.) Read or measure this R_L value, then

$$R_{TH} = R_L$$

R_{TH} Determination from $I_{short\ ckt}$ & $V_{open\ ckt.}$



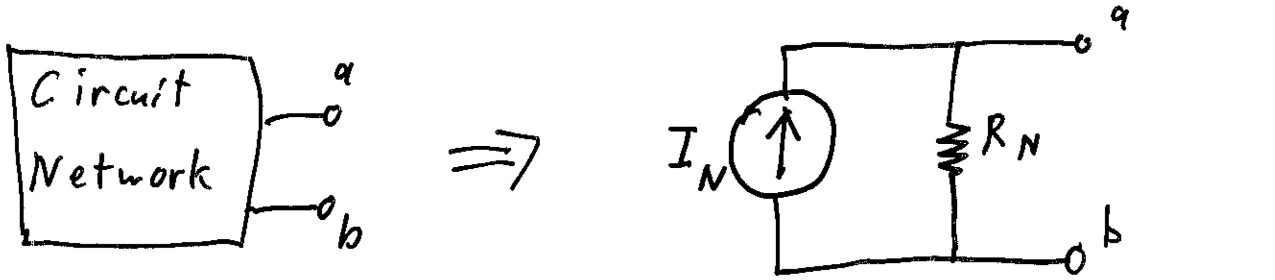
Apply a short ckt. between terminals a & b. Measure I_{sc}.

$I_{sc} = E_{TH}/R_{TH}$ so

$$R_{TH} = \frac{E_{oc}}{I_{sc}}$$

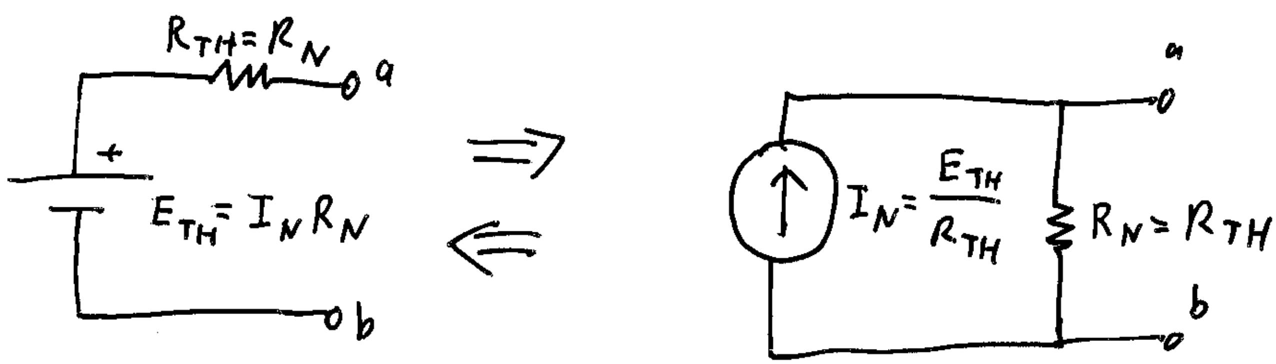
Norton's Theorem

Norton's theorem states that any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor.

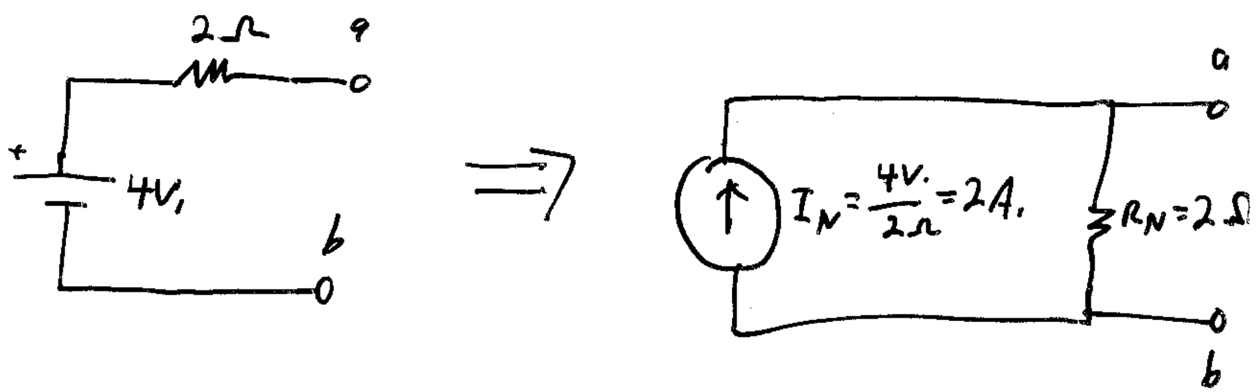


Norton Equivalent Circuit

ex/ Converting between Thevenin and Norton equivalent circuits:



ex/



Procedure to Find I_N and R_N

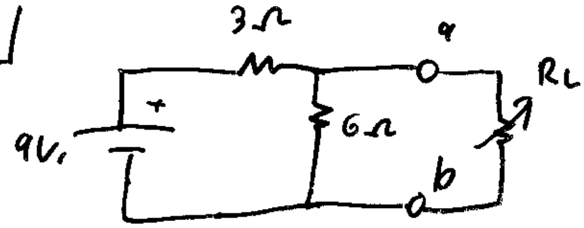
1.) Remove that portion of the network across which the Norton Equivalent circuit is to be found.

2.) Label terminals "a" and "b" which correspond to the location where the Norton Equiv. ckt. will be connected to the network.

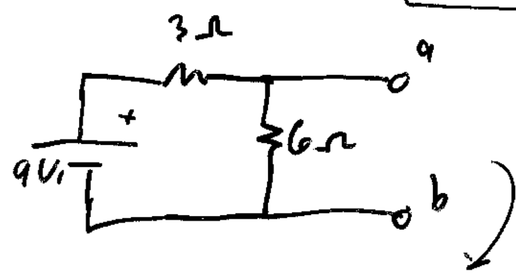
3.) Apply a short ckt. between terminals "a" and "b" in the removed portion of the network and calculate I_N as the current flowing through the short ckt. $\boxed{I_N = I_{sc}}$

4.) Calculate R_N by killing all sources (replace voltage sources with short ckts. and current sources with open ckts.). Remove the applied short ckt. across terminals "a" and "b" (from step 3. above) and calculate the total resistance between "a" and "b" by combining all the resistors to a single value equal to R_N. $\boxed{(R_N = R_{TH})}$

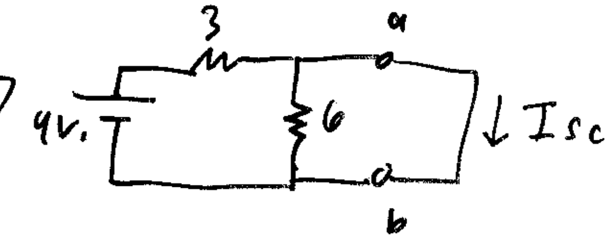
ex/



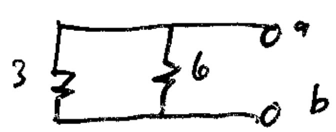
⇒



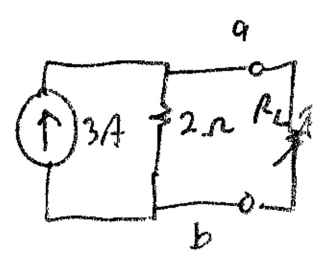
⇒



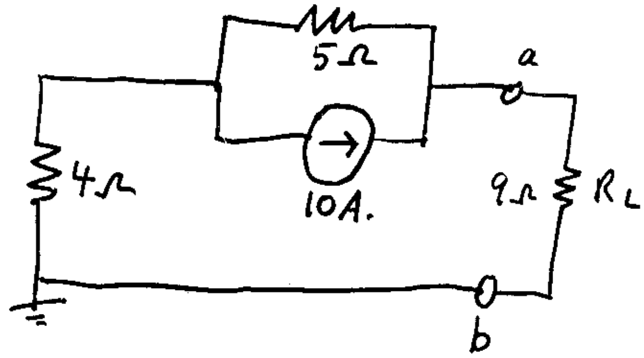
$$I_{sc} = \frac{9V.}{3\Omega} = 3A = I_N$$



$R_N = 3 \parallel 6 = 2\Omega$ ⇒

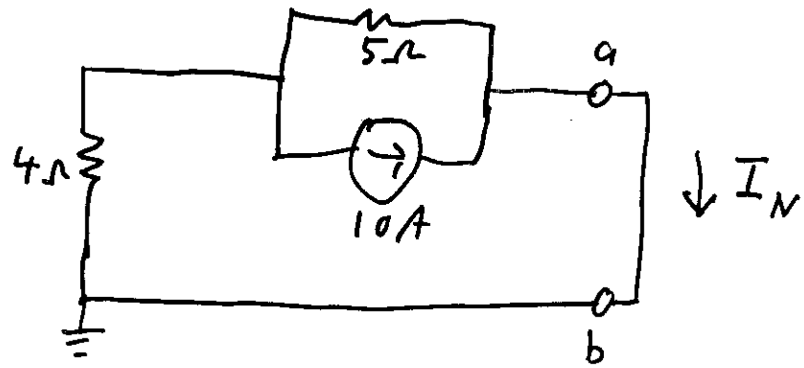


ex | Find the Norton equivalent ckt. "seen by" the 9 Ω resistor.



Solution:

1.) Remove R_L and apply a short ckt. between terminals a and b.

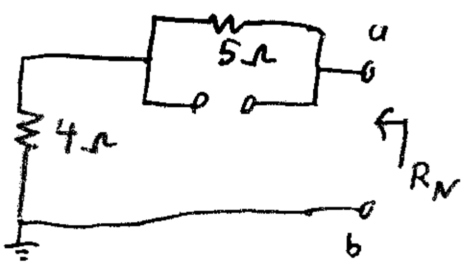


2.) Solve for I_N which is the current through the short ckt.

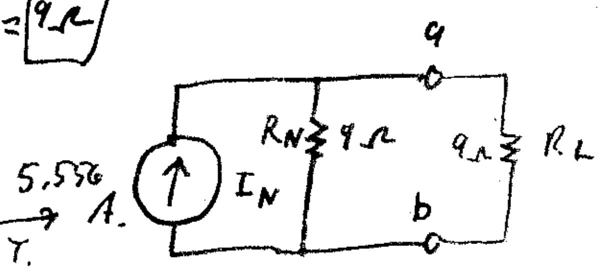
By Current Divider rule:

$$I_N = (10A)\frac{5}{5+4} = \frac{50}{9} = \boxed{5.556\ A.}$$

3.) Kill sources and solve for resistance between terminals a and b.
(replace current source with an open ckt.)



$R_N = 5+4 = \boxed{9\,\Omega}$



Norton Equiv. CKT.