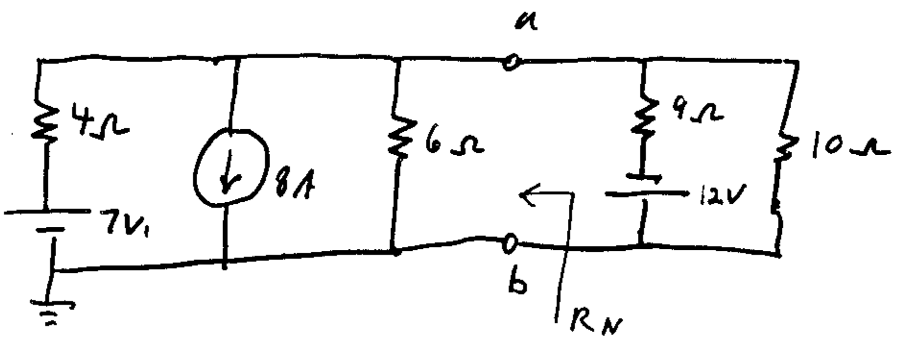
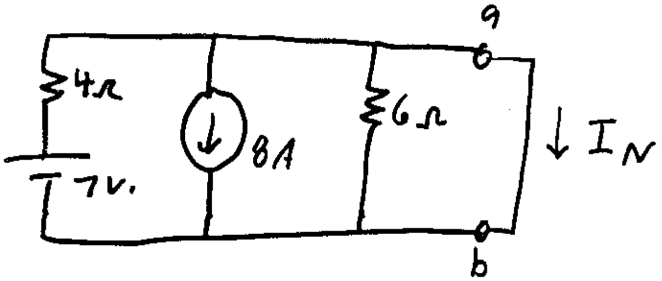
Part 2 ex/ Find the Norton equiv. ckt. to the left of terminals a and b.



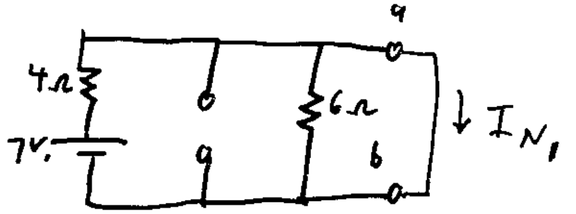
Solution:

1.) Remove the external circuitry and apply a short ckt.



2.) Solve for I_N which is the current through the short ckt since 2 sources are present use superposition theorem:

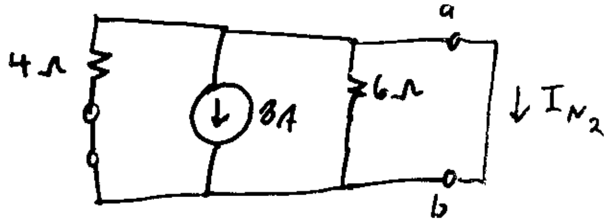
1. kill Current source (replace with open ckt.)



Note: 6Ω resistor is "shorted out"

So, $I_{N_1} = \frac{7V.}{4\Omega} = 1.75A.$

2. Kill voltage source (replace with short ckt.)



By CDR:

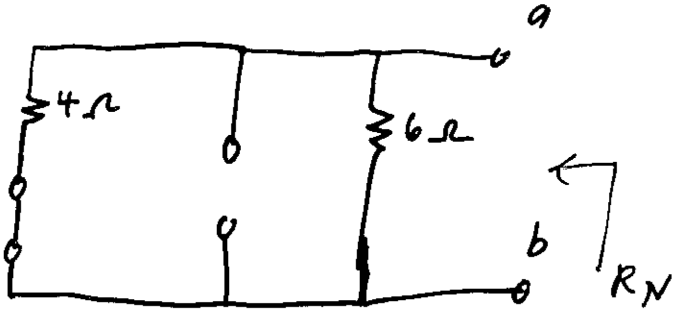
$$I_{N_2} = -(8A)\frac{4}{4+0} = -8A.$$

3. Combine to get $I_N = I_{N_1} + I_{N_2} = 1.75 + (-8) = \boxed{-6.25A}$

continued....

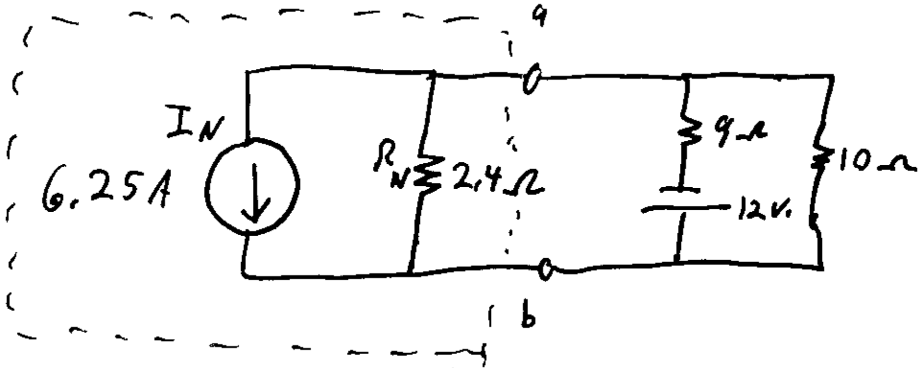
ex/ continued...

3.) kill sources and solve for R_N
(replace current source with open ckt.)
(replace voltage source with short ckt.)



$R_N = 4\,\Omega \parallel 6\,\Omega = \frac{(4)(6)}{4+6} = \frac{24}{10} = \boxed{2.4\,\Omega}$

4.) Redraw original ckt. with Norton Equiv. ckt. in place



Norton Equiv. ckt.

Note: In schematic I_N was shown as being forced out of terminal "a".
Since the computed I_N value was negative, the direction of the current source is drawn to force current out of terminal "b".

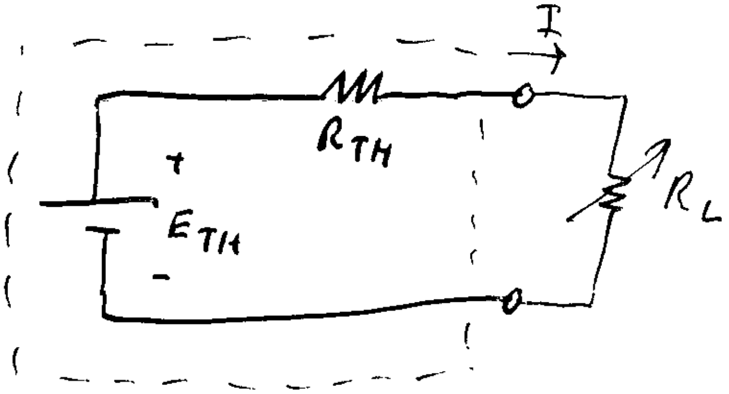
Maximum Power Transfer Theorem

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as "seen" by the load.

For maximum power transfer ;

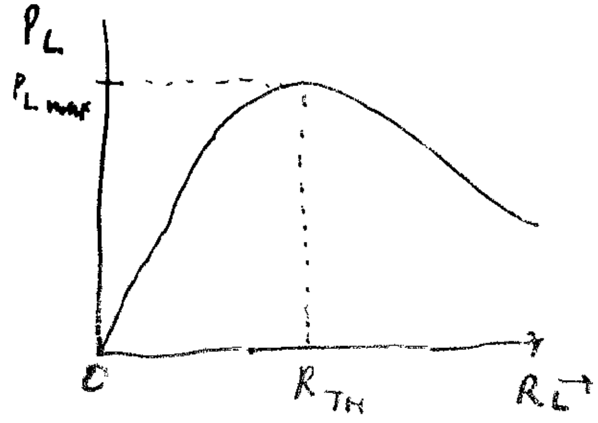
$$R_L = R_{TH}$$

ex/



$$I = \frac{E_{TH}}{R_{TH}+R_L} \quad , \quad P_L = I^2 R_L = \left(\frac{E_{TH}}{R_{TH}+R_L}\right)^2 R_L$$

$$P_L = \frac{E_{TH}^2 \, R_L}{(R_{TH}+R_L)^2}$$



Operating Efficiency:

$$\eta = \frac{P_L}{P_s} \times 100\%$$

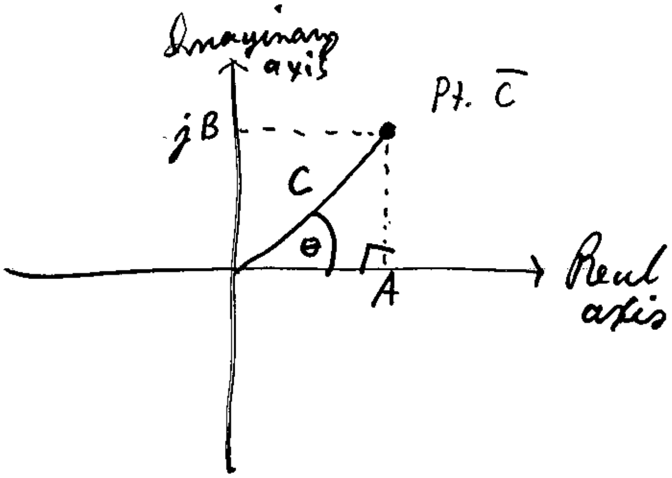
Phasor Review

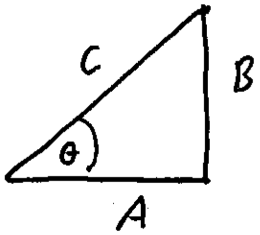
Complex Numbers :

$$\boxed{\begin{array}{ll} \bar{C} = A + jB & \leftarrow \text{rectangular form} \\ \text{OR} & \\ \bar{C} = C \angle \theta & \leftarrow \text{Polar form} \end{array}}$$

$$-\bar{C} = -C \angle \theta = C \angle \theta \pm \pi$$

Conversion Between Forms.





$$\boxed{\begin{array}{l} C = \sqrt{A^2 + B^2} \\ \theta = \tan^{-1} \dfrac{B}{A} \end{array}}$$

$$\boxed{\begin{array}{l} A = C \cos \theta \\ B = C \sin \theta \end{array}}$$

$$\boxed{j = \sqrt{-1}} \quad \boxed{j^2 = -1} \quad \boxed{\frac{1}{j} = -j}$$

Complex Conjugate:

$$\bar{C}^* = A - jB$$

OR

$$\bar{C}^* = C \angle -\theta$$

Math Operations with Complex Numbers

Given: $\bar{C}_1 = A_1 + jB_1$ and $\bar{C}_2 = A_2 + jB_2$

Addition: $\bar{C}_1 + \bar{C}_2 = (A_1 + A_2) + j(B_1 + B_2)$

Subtraction: $\bar{C}_1 - \bar{C}_2 = (A_1 - A_2) + j(B_1 - B_2)$

Addition and Subtraction are easiest in rectangular form.

Multiplication: $\bar{C}_1 \cdot \bar{C}_2 = C_1 C_2 \angle(\theta_1 + \theta_2)$

Division: $\dfrac{\bar{C}_1}{\bar{C}_2} = \dfrac{C_1}{C_2} \angle(\theta_1 - \theta_2)$

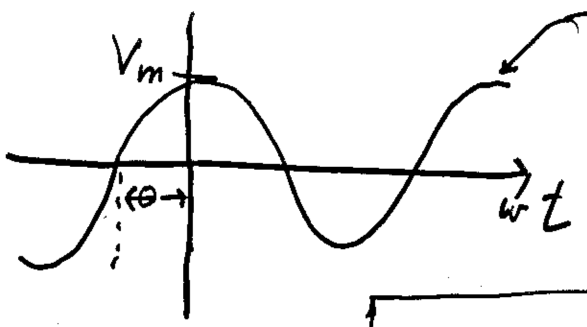
Multiplication and Division are easiest in Polar form.

$$\frac{1}{\bar{C}_1 \angle \theta_1} = \frac{1}{C} \angle -\theta_1$$

↖ note: 1 is really $1\angle 0^\circ$

Phasors

A Phasor is a complex number which can be used to represent a sinusoidal AC waveform.



$V_m \sin(\omega t+\theta)$

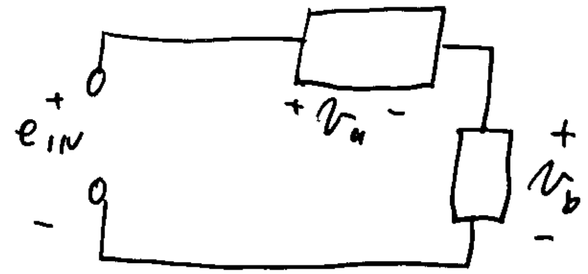
↑ peak value

↖ Time Domain Expression for Sinusoid

$$\boxed{V_m \sin(\omega t+\theta) \Longleftrightarrow V_m \angle \theta}$$

Time Domain — Phasor Domain

ex



Given: $f = 60Hz$

$v_a = 50 \sin(377t+30^\circ)$

$v_b = 30 \sin(377t+60^\circ)$

Find e_{IN}

Solution: By KVL $-e_{IN} + v_a + v_b = 0$

$e_{IN} = v_a + v_b$

Convert to Phasor Domain:

$v_a = 50\,(.707)\angle 30^\circ = 35.35\angle 30^\circ = 30.61 + j17.68 = \overline{V_a}$

$\uparrow$ converts peak to RMS

$v_b = 30\,(.707)\angle 60^\circ = 21.21\angle 60^\circ = 10.61 + j18.37 = \overline{V_b}$

$\overline{E}_{IN} = \overline{V_a} + \overline{V_b} = (30.61 + 10.61) + j(17.58 + 18.37)$

$\overline{E}_{IN} = 41.22 + j36.05 = 54.76\angle 41.17^\circ$ Now convert back to time doma

$e_{IN} = \sqrt{2}\,(54.76)\sin(377t + 41.17^\circ) = 77.43 \sin(377t + 41.17$

Impedance and the Phasor Diagram

Resistive Elements

For a purely resistive element the current and voltage are in phase.

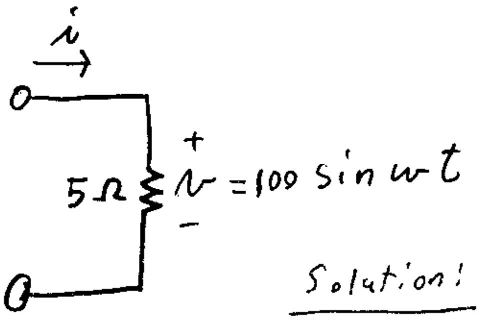
$\boxed{\overline{Z}_R = R\angle 0^\circ}$ where $\overline{Z}_R$ is the Impedance of a resistor.

$$\overline{I} = \frac{\overline{V}}{\overline{Z}_R} \quad \leftarrow \text{Ohm's Law for Phasors}$$

$$\overline{V} = \overline{I}\,\overline{Z}_R$$

A phasor is a is a rotating vector with a magnitude and phase angle associated with it.

ex/



Find the current i. Sketch i and v

Solution!

Solution: 1.) First convert v to Phasor form:

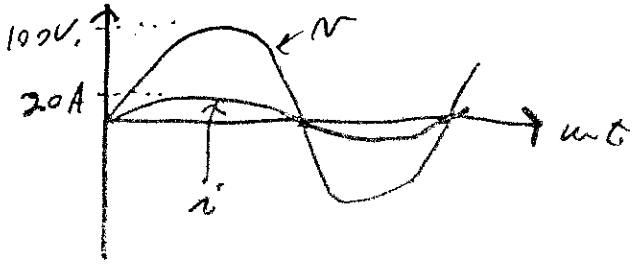
$$v = 100 \sin wt \Longrightarrow \overline{V} = 100\angle 0^\circ$$

2) Apply ohm's Law:

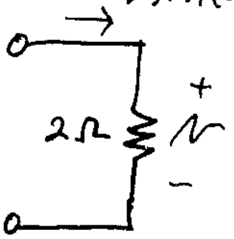
$$\overline{I} = \frac{\overline{V}}{\overline{Z}_R} = \frac{100\angle 0^\circ}{5\angle 0^\circ} = 20\angle 0^\circ$$

3.) Convert back to time domain:

$$i = 20 \sin wt \text{ Amps.}$$



ex/ $i = 4\sin(\omega t+30^\circ)$ Find v, sketch i and v



solution:

1.) Convert i to Phasor form:

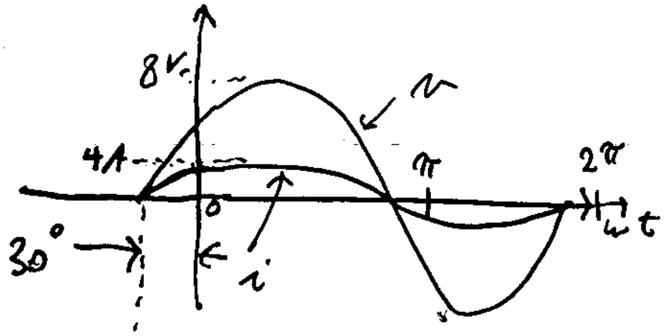
$$i = 4\sin(\omega t+30^\circ) \Longrightarrow \bar{I} = 4\angle 30^\circ \text{ Amps.}$$

2.) Apply Ohm's Law:

$$\bar{V} = \bar{I}\,\bar{Z}_R = (4\angle 30^\circ)(2\angle 0^\circ) = 8\angle 30^\circ \text{ volts.}$$

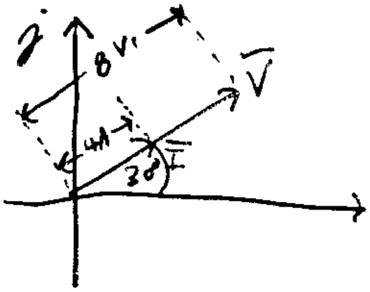
3.) Convert back to time domain:

$$v = 8\sin(\omega t+30^\circ) \text{ volts}$$



A Phasor Diagram is plot in the complex plane showing the magnitude and phase relationships of circuit quantities.

For the above example the Phasor Diagram would be:



Note: $\bar{I}$ and $\bar{V}$ are in phase since it's a resistive circuit

Inductive Reactance

For a purely inductive element the current lags the voltage by 90°.

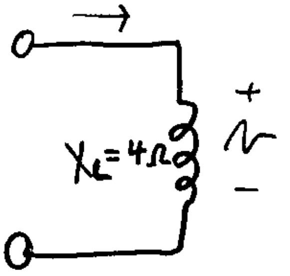
$$\boxed{\overline{Z}_L = X_L \angle 90^\circ}$$ where $\overline{Z}_L$ is the Impedance of an inductor

$$\boxed{X_L = 2\pi f L = \omega L}$$

In rectangular form; $\overline{Z}_L = j\omega L = j X_L$

Ohm's Law for Inductors; $\overline{I} = \dfrac{\overline{V}}{\overline{Z}_L}$, $\overline{V} = \overline{I}\,\overline{Z}_L$

ex $i = 5\sin(\omega t + 30^\circ)$



Find voltage v.
Sketch v and i.

Solution;

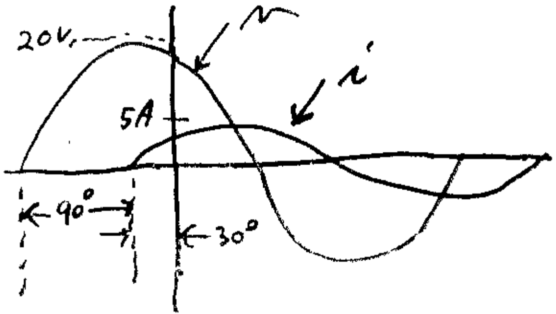
1.) Convert i and Z_L to phasor form:

$i = 5\sin(\omega t + 30^\circ) \Longrightarrow \overline{I} = 5\angle 30^\circ$

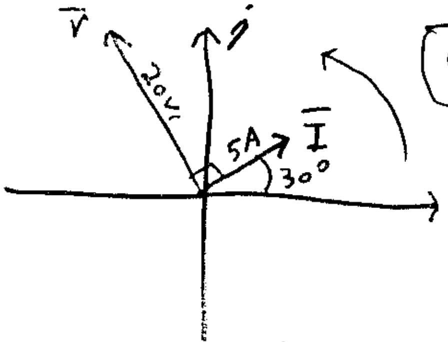
$\overline{Z}_L = X_L \angle 90^\circ = 4\angle 90^\circ$

2.) Apply Ohm's Law : $\overline{V} = \overline{I}\,\overline{Z}_L = (5\angle 30^\circ)(4\angle 90^\circ) = 20\angle 120^\circ$

3.) Convert back to time domain: $v = 20\sin(\omega t + 120^\circ)$



Time Domain graph



phasor Diagram

Can think of Phasors as rotating ccw

I lags V by 90°

Capacitive Reactance

$$\boxed{\text{For a purely capacitive element the current leads the voltage by } 90^\circ.}$$

$$\boxed{\overline{Z}_c = X_c \angle -90^\circ}$$

where $\overline{Z}_c$ is the impedance of a capacitor

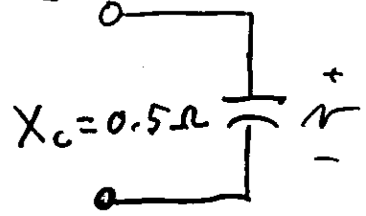
$$\boxed{X_c = \frac{1}{2\pi f C} = \frac{1}{\omega C}}$$

In rectangular form: $\overline{Z}_c = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C}$

Ohm's Law for Capacitors: $\overline{I} = \dfrac{\overline{V}}{\overline{Z}_c}$, $\overline{V} = \overline{I}\,\overline{Z}_c$

ex/ $i = 6 \sin(\omega t - 60^\circ)$

Find voltage v
Sketch v and i



solution:

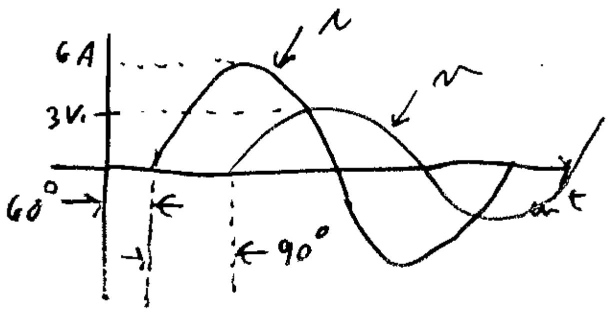
1.) Convert i and Z_c to phasor form

$\overline{I} = 6\angle -60^\circ$, $\overline{Z}_c = X_c \angle -90^\circ = 0.5\angle -9$

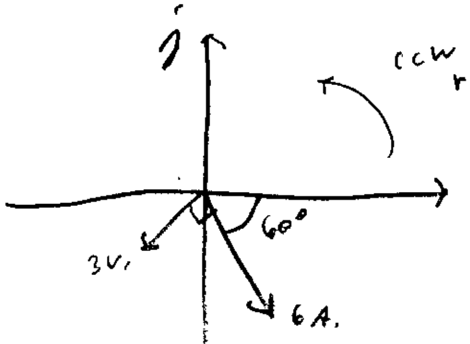
2.) Apply Ohm's Law : $\overline{V} = \overline{I}\,\overline{Z}_c = (6\angle -60^\circ)(0.5\angle -90^\circ)$

$\overline{V} = 3 \angle(-60 + (-90)) = 3\angle -150^\circ$

3.) Convert back to time domain: $v = 3 \sin(\omega t - 150^\circ)$



Time Domain Graph

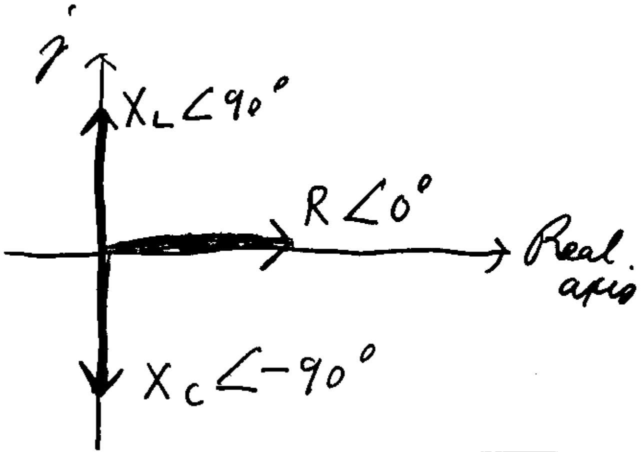


ccw rotation so I leads V by 90°

Phasor Diagram

Impedance Diagram

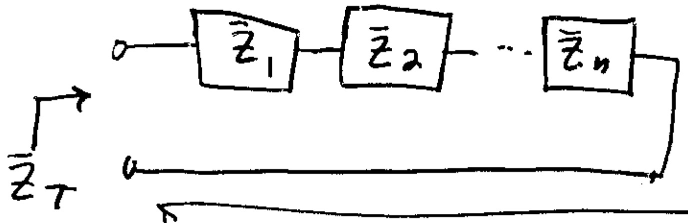
The complex quantities corresponding to the impedance of R, L, and C elements can be graphed in the complex plane.



For any configuration (series, parallel, series-parallel, etc.) the angle associated with the total impedance is the angle by which the source current lags the applied voltage.

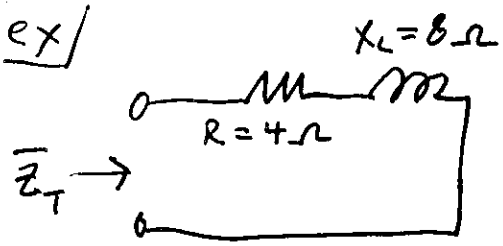
For inductive networks θ_T will be positive.
For capacitive networks θ_T will be negative.

Series Configuration



For Impedances in series the total impedance is: $\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 + \dots + \bar{Z}_n$

ex/



Draw the Impedance diagram. Find total impedance Z_T.

Solution: 1.) Convert impedances to phasor form:

$$\overline{Z}_R = 4\angle 0^\circ \quad , \quad \overline{Z}_L = 8\angle 90^\circ$$

2.) Apply series Z rule:

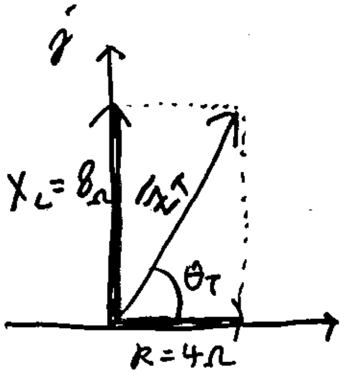
$$\overline{Z}_T = \overline{Z}_R + \overline{Z}_L = 4\angle 0^\circ + 8\angle 90^\circ$$

convert to rectangular form to add:

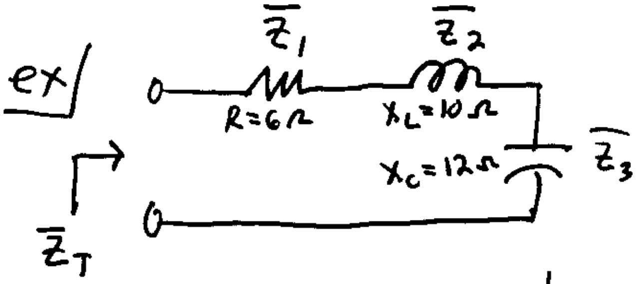
$$\overline{Z}_T = 4 + 8j$$

convert back to polar form:

$$\overline{Z}_T = 8.944\,\Omega \angle 63.43^\circ$$



ex/



Determine total $\overline{Z}$. Draw $\overline{Z}$ diagram.

Solution: 1.) Convert to phasor form:

$$\overline{Z}_1 = 6\angle 0^\circ \quad , \quad \overline{Z}_2 = 10\angle 90^\circ \quad , \quad \overline{Z}_3 = 12\angle -90^\circ$$

2.) Apply series Z rule:

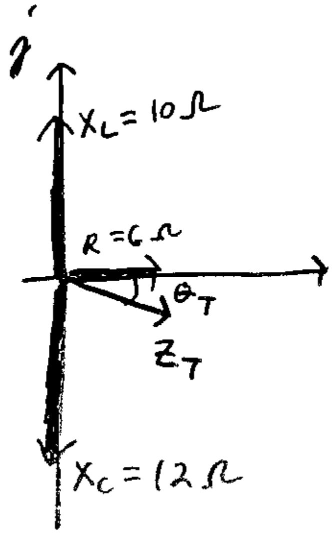
$$\overline{Z}_T = 6\angle 0 + 10\angle 90^\circ + 12\angle -90$$

convert to rect. form to add:

$$\overline{Z}_T = 6 + 10j - 12j = 6 - 2j$$

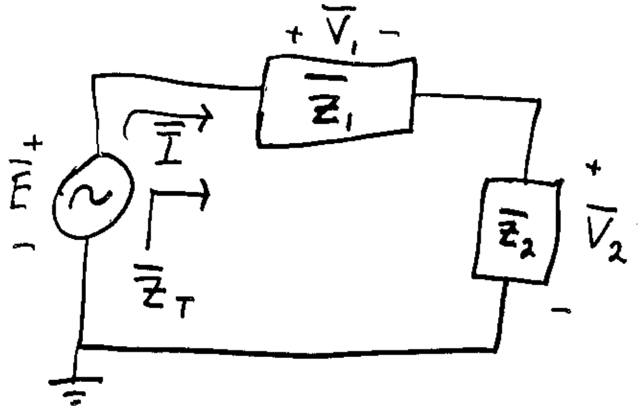
Convert back to polar form:

$$\overline{Z}_T = 6.325\,\Omega \angle -18.43^\circ$$



θ_T neg. → Z_T is capacitive

For the Series AC circuit the current is the same through each element.



Phasor Equations:

$$\overline{I} = \frac{\overline{E}}{\overline{Z}_T}$$

$$\overline{Z}_T = \overline{Z}_1 + \overline{Z}_2$$

$$\overline{V}_1 = \overline{I}\,\overline{Z}_1 \quad , \quad \overline{V}_2 = \overline{I}\,\overline{Z}_2$$

By KVL: $-\overline{E} + \overline{V}_1 + \overline{V}_2 = 0$

$$\overline{E} = \overline{V}_1 + \overline{V}_2$$

By Voltage Divider Rule: $\overline{V}_2 = \overline{E}\left(\frac{\overline{Z}_2}{\overline{Z}_1+\overline{Z}_2}\right)$

The Power supplied to the circuit is:

$$P = EI \cos\theta_T$$

Where: E is voltage in <u>RMS</u>

I is current in <u>RMS</u>

θ_T is the phase angle between $\overline{E}$ and $\overline{I}$

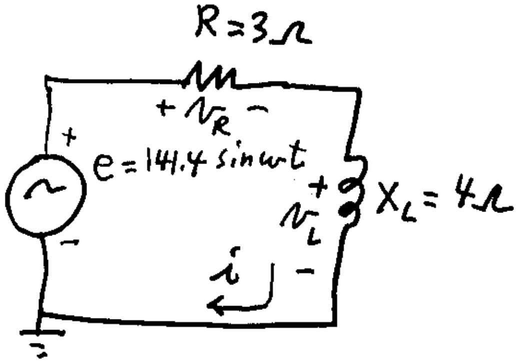
θ_T is also the angle of $\overline{Z}_T$.

The <u>Power Factor</u> F_p is:

$$\boxed{F_p = \cos\theta_T} \qquad \text{so} \quad P = EI\,F_p$$

R-L Circuit

ex/



Convert to Phasors:

$e = 141.4 \sin \omega t \longrightarrow \overline{E} = 141.4\angle 0^\circ$

Convert to RMS

$\overline{E} = 141.4(.707)\angle 0^\circ = 100\angle 0^\circ$ volts RMS

Find total impedance:

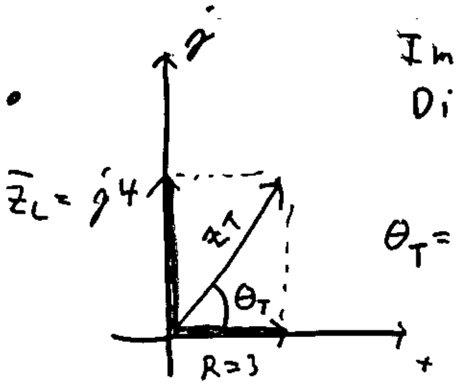
$$\overline{Z}_T = \overline{Z}_R + \overline{Z}_L = 3\angle 0^\circ + 4\angle 90^\circ$$

converting to rect. form: $\overline{Z}_T = 3 + j4\ \Omega$

converting to polar form: $\overline{Z}_T = 5\ \Omega\angle 53.13^\circ$

By Ohm's Law:

$$\overline{I} = \frac{\overline{E}}{\overline{Z}_T} = \frac{100\angle 0^\circ}{5\angle 53.13^\circ} = 20\text{ A.}\angle -53.13^\circ \text{ RMS}$$



Impedance Diagram

$\theta_T = 53.13^\circ$

$$\overline{V}_R = \overline{I}\,\overline{Z}_R = (20\angle -53.13)(3\angle 0^\circ)$$

$$\overline{V}_R = 60\text{ V.}\angle -53.13^\circ$$

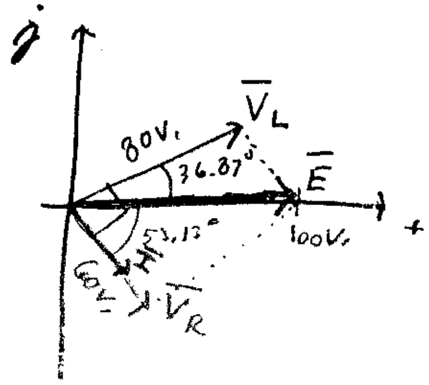
$$\overline{V}_L = \overline{I}\,\overline{Z}_L = (20\angle -53.13)(4\angle 90^\circ) = 80\angle 36.87^\circ \text{ V. RMS}$$

Check by KVL:

$$-\overline{E} + \overline{V}_R + \overline{V}_L = 0 \rightarrow \overline{E} = \overline{V}_R + \overline{V}_L = 60\angle -53.13^\circ + 80\angle 36.87^\circ$$

Converting to rect. form: $\overline{E} = 36 - j48 + 64 + j48$

$\overline{E} = 100 + j0 = 100\angle 0^\circ$ V. ✓ Checks



← Phasor Diagram

ex/ ... continued

Total Power delivered to R-L ckt:

$$P_T = E\,I\cos\theta_T$$ ← E & I are effective (RMS) values

$$P_T = (100\,V.)(20\,A.)\cos 53.13^\circ = 2000\,W.\,(.6) = 1200\,W.$$

Power dissipated by Resistor:

$$P_R = I^2 R = (20\,A.)^2(3\Omega) = 400(3) = 1200\,w.$$

Total Power dissipated by R-L ckt:

$$P_T = P_R + P_L = V_R I\cos\theta_R + V_L I\cos\theta_L$$

$$= (60)(20)\cos 0^\circ + (80)(20)\cos 90^\circ$$

$$= 1200\,w. + 0 = 1200\,w.$$

Note: Inductor dissipates zero power

Power Factor:

$$F_p = \cos\theta_T = \cos 53.13^\circ = 0.6 \text{ Lagging}$$

F_p is Lagging because I lags E in an inductive ckt.

Also: since $p = EI\,F_p$

$$F_p = \frac{P}{EI}$$ ← P called "Real Power" in watts

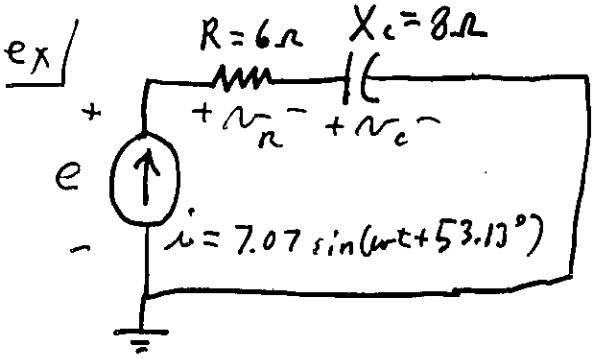
EI called "VA" or Apparent Power

Also $F_p = \frac{R}{Z_T}$

$$\cos\theta = \frac{P}{EI} = \frac{I^2R}{EI} = \frac{IR}{E} = \frac{R}{E/I} = \frac{R}{Z_T}$$

R - C Circuit

ex/



Convert to Phasors:

$i = 7.07\sin(\omega t + 53.13°) \longrightarrow \bar{I} = \underset{\text{Peak}}{7.07\angle 53.1}$

Convert to RMS:

$\bar{I} = 7.07(.707)\angle 53.13° = 5A.\underset{\text{RMS}}{\angle 53.13°}$

Find total impedance

$\bar{Z}_T = \bar{Z}_R + \bar{Z}_C = 6\angle 0° + 8\angle -90°$

converting to rect. form: $\bar{Z}_T = 6 - j8\ \Omega$

converting to polar form: $\bar{Z}_T = 10\Omega\angle -53.13°$

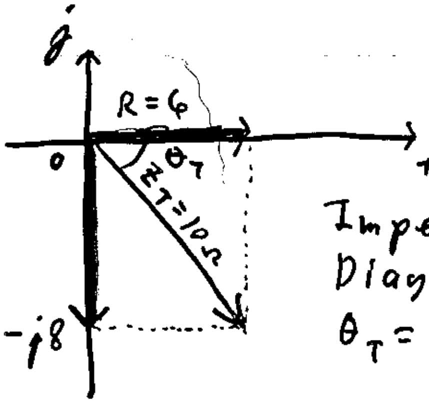
By Ohm's Law:

$\bar{E} = \bar{I}\,\bar{Z}_T = (5A\angle 53.13°)(10\Omega\angle -53.13°)$

$\bar{E} = 50V.\angle 0°$

$\bar{V}_R = \bar{I}\,\bar{Z}_R = (5A\angle 53.13°)(6\Omega\angle 0°)$

$\bar{V}_R = 30V.\angle 53.13°$



Impedance Diagram

$\theta_T = -53.13°$

$\bar{V}_c = \bar{I}\,\bar{Z}_c = (5A\angle 53.13°)(8\Omega\angle -90°) = 40V.\angle -36.87°$

Check by KVL:

$-\bar{E} + \bar{V}_R + \bar{V}_c = 0 \longrightarrow \bar{E} = \bar{V}_R + \bar{V}_c = 30\angle 53.13° + 40\angle -36.87°$

Convert to rect. form: $\bar{E} = 18 + j24 + 32 - j24 = 50 + j0$

$\bar{E} = 50\angle 0°$ ✓ checks

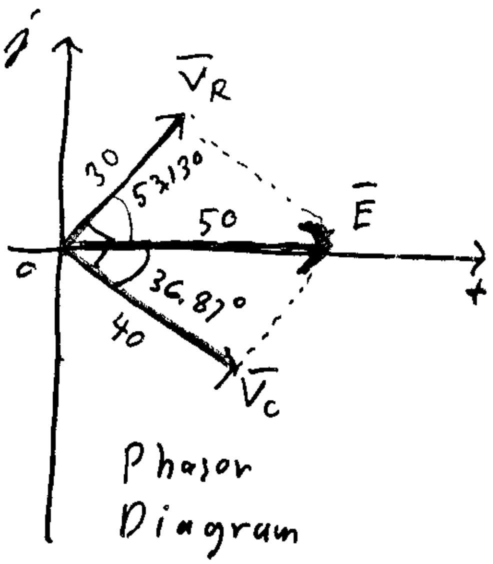
Convert to time domain:

$\underset{\text{RMS}}{\bar{E} = 50\angle 0°} \longrightarrow e = \sqrt{2}\,50\sin\omega t = 70.7\sin\omega t$

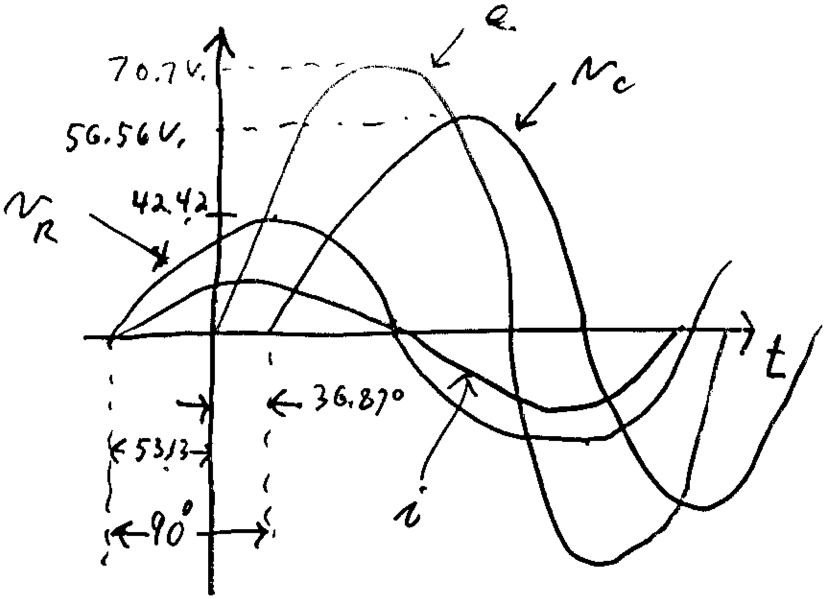
$\bar{V}_R = 30\angle 53.13° \longrightarrow v_R = \sqrt{2}\,30\sin(\omega t + 53.13) = 42.42\sin(\omega t + 53.13°)$

$\bar{V}_c = 40\angle -36.87° \longrightarrow v_c = \sqrt{2}\,40\sin(\omega t - 36.87) = 56.56\sin(\omega t - 36.87°)$

ex1 .... continued



Phasor Diagram



Time Domain wave forms

Note: v_R and i are in phase
i leads v_c by 90°

Total Power :

$$P_T = E I \cos\theta_T = (50V.)(5A)\cos 53.13° = 250(.6) = 150 \text{ watts}$$

RMS Values (E, I); angle of $\overline{Z}_T$ (θ_T)

Or alternately ; $P_T = I^2 R = (5A)^2 6\Omega = 25(6) = 150$ watts

Or alternately ;

$$P_T = P_R + P_C = V_R I \cos\theta_R + V_c I \cos\theta_c$$
$$= (30)(5)\cos 0° + (40)(5)\cos 90° = 150 + 0 = 150 \text{ watts}$$

Power Factor : $F_p = \cos\theta_T = \cos 53.13° = 0.6$ leading

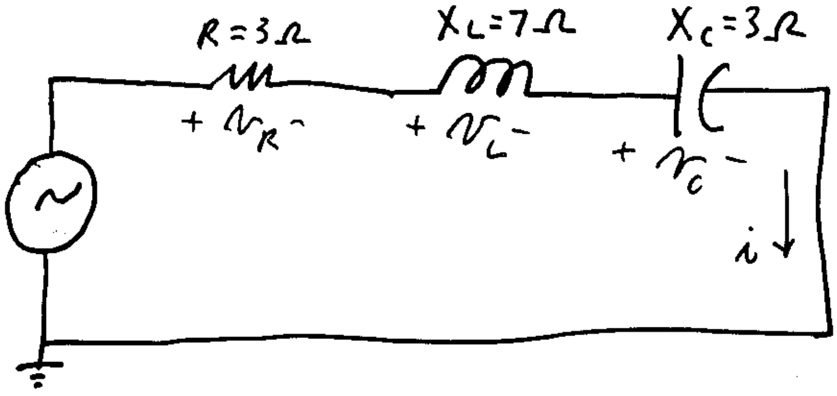
since i leads v

Or alternately: $F_p = \cos\theta = \dfrac{R}{Z_T} = \dfrac{6}{10} = 0.6$ leading

R-L-C Circuit

ex/

$e = 70.7 \sin \omega t$



Convert to Phasors:

$e = 70.7 \sin \omega t \rightarrow \overline{E} = 70.7 \angle 0^\circ$

Convert to RMS:

$\overline{E} = 70.7(.707) \angle 0^\circ = 50 \angle 0^\circ$

Find total impedance:

$\overline{Z}_T = \overline{Z}_R + \overline{Z}_L + \overline{Z}_C = 3 \angle 0^\circ + 7 \angle 90^\circ + 3 \angle -90^\circ$

Convert to rect. form to add:

$\overline{Z}_T = 3 + j0 + j7 + (-j3) = 3 + j4$

Convert to polar form to multiply or divide:

$\overline{Z}_T = 5 \angle 53.13^\circ$

By Ohm's Law:

$$\overline{I} = \frac{\overline{E}}{\overline{Z}_T} = \frac{50 \angle 0^\circ}{5 \angle 53.13^\circ} = 10\,A. \angle -53.13^\circ$$

$$\overline{V}_R = \overline{I}\,\overline{Z}_R = (10 \angle -53.13)(3 \angle 0^\circ) = 30\,V. \angle -53.13^\circ$$

$$\overline{V}_L = \overline{I}\,\overline{Z}_L = (10 \angle -53.13)(7 \angle 90^\circ) = 70\,V. \angle 36.87^\circ$$

$$\overline{V}_C = \overline{I}\,\overline{Z}_C = (10 \angle -53.13)(3 \angle -90^\circ) = 30\,V. \angle -143.13^\circ$$

(Note that V_L is larger than the source voltage.)

By KVL: $\overline{E} = \overline{V}_R + \overline{V}_L + \overline{V}_C$

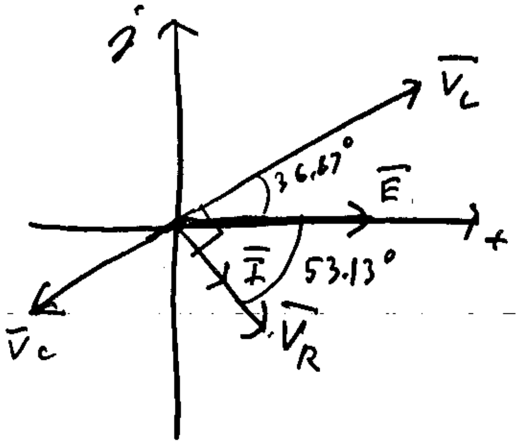
In time domain:

$$i = \sqrt{2}(10)\sin(\omega t - 53.13^\circ) = 14.14 \sin(\omega t - 53.13^\circ)$$

$$v_R = \sqrt{2}(30)\sin(\omega t - 53.13^\circ) = 42.42 \sin(\omega t - 53.13^\circ)$$

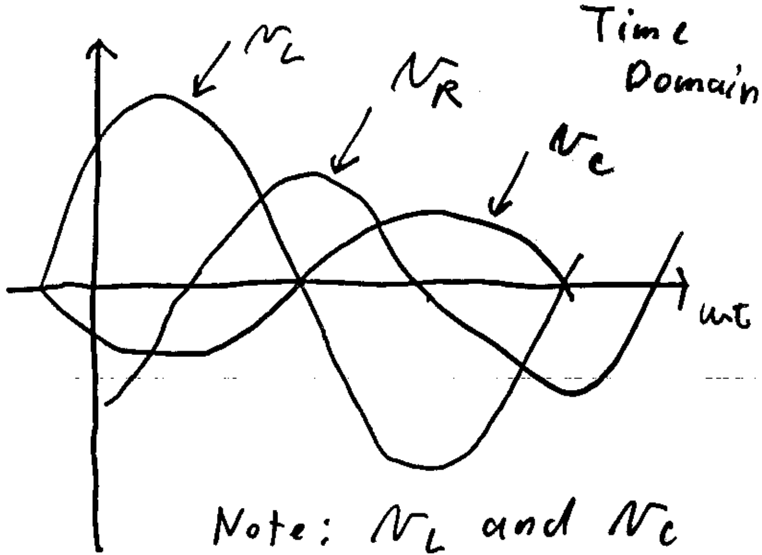
$$v_L = \sqrt{2}(70)\sin(\omega t + 36.87^\circ) = 98.98 \sin(\omega t + 36.87^\circ)$$

$$v_c = \sqrt{2}(30)\sin(\omega t - 143.13^\circ) = 42.42 \sin(\omega t - 143.13^\circ)$$



Phasor Diagram

Note: $\overline{I}$ lags $\overline{E}$ by 53.13°



Note: v_L and v_c are 180° out of phase

Total Power: $P_T = EI\cos\theta_T = (50)(10)\cos 53.13^\circ = 300$ watts

Also: $P_T = I^2 R = (10)^2(3) = 300$ watts

All of the power is dissipated by the resistor; Capacitors and Inductors do not dissipate any power

Power Factor:

$$F_p = \cos\theta_T = \cos 53.13^\circ = 0.6 \text{ lagging}$$

$$\text{Also: } F_p = \frac{R}{Z_T} = \frac{3}{5} = 0.6 \text{ lagging}$$

Note: In this example the inductive reactance dominates over the capacitive reactance making the total impedance "inductive" with a lagging power factor.

Voltage Divider Rule

This rule applies for AC ckts. as it did for D.C. ckts

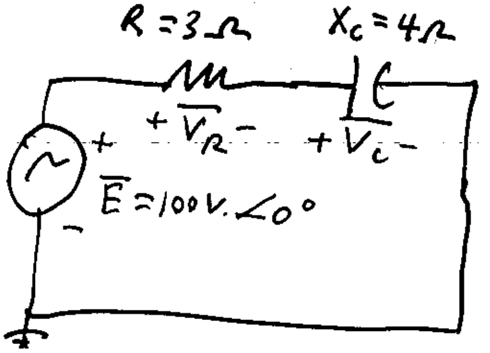
$$\boxed{\overline{V}_x = \frac{\overline{E}\ \overline{Z}_x}{\overline{Z}_T}}$$

where: $\overline{V}_x$ is the voltage across one or more series elements. that have impedance $\overline{Z}_x$

$\overline{E}$ is the total voltage across the series ckt.

$\overline{Z}_T$ is the total impedance of the series ckt.

ex/



Find $\overline{V}_c$ and $\overline{V}_R$

solution:

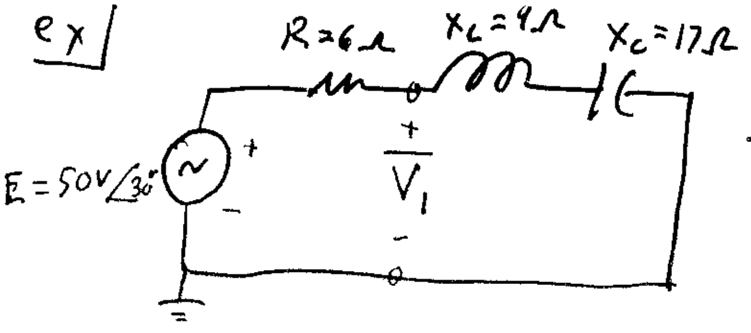
By Voltage Divider Rule:

$$\overline{V}_R = \frac{\overline{Z}_R\overline{E}}{\overline{Z}_c + \overline{Z}_R} = \frac{(3\angle 0^\circ)(100\angle 0^\circ)}{3\angle 0^\circ + 4\angle -90^\circ}$$

$$\overline{V}_R = \frac{300\angle 0^\circ}{3 - j4} = \frac{300\angle 0^\circ}{5\angle -53.13^\circ} = \boxed{60\text{V.}\angle +53.13^\circ}$$

$$\overline{V}_c = \frac{\overline{Z}_c\ \overline{E}}{\overline{Z}_c + \overline{Z}_R} = \frac{(4\angle -90^\circ)(100\angle 0^\circ)}{5\angle -53.13^\circ} = \boxed{80\text{V.}\angle -36.87^\circ}$$

ex/



Find $\overline{V}_1$

solution:

BY VDR

$$\overline{V}_1 = \frac{(\overline{Z}_L + \overline{Z}_c)\overline{E}}{\overline{Z}_T}$$

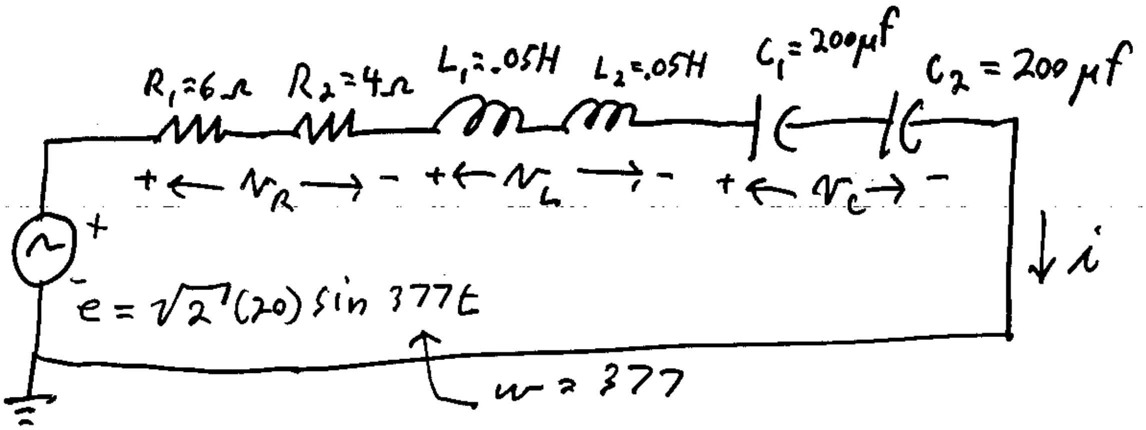
ex/ .... continued

$$\overline{V}_1 = \frac{((9\Omega\angle 90^\circ)+(17\Omega\angle -90^\circ))(50\angle 30^\circ)}{6\Omega\angle 0^\circ + 9\Omega\angle 90^\circ + 17\Omega\angle -90^\circ}$$

$$\overline{V}_1 = \frac{(j9 - j17)\ (50\angle 30^\circ)}{6 + j9 - j17}$$

$$\overline{V}_1 = \frac{(-j8)(50\angle 30^\circ)}{6 - j8} = \frac{(8\angle -90)(50\angle 30^\circ)}{(10\angle -53.13^\circ)} = \boxed{40V.\angle -6.87^\circ}$$

ex/



a.) Calculate $\overline{I}$, $\overline{V}_R$, $\overline{V}_L$, $\overline{V}_c$ in phasor form

Combining series elements: $R_T = R_1 + R_2 = 6 + 4 = 10\Omega$

$L_T = .05H + .05H = 0.1\ H$ $\qquad C_T = \frac{200\mu f}{2} = 100\mu f \leftarrow$

Note:
Capacitors
in series
combine
like Resista
in parall

Find X_L and X_c using w:

$$X_L = wL = (377\ \text{rad/sec})(0.1H) = 37.7\Omega$$

$$X_c = \frac{1}{wC} = \frac{1}{(377\ \text{rad/sec})(100\times 10^{-6} f.)} = 26.53\Omega$$

Find Total series impedance:

$$\overline{Z}_T = R\angle 0^\circ + X_L\angle 90^\circ + X_c\angle -90^\circ = 10\Omega + j37.7 - j26.53$$

$$\overline{Z}_T = 10 + j11.17 = 15\Omega\angle 48.16^\circ$$

EX ... continued

$$\overline{I} = \frac{E}{\overline{Z}_T} = \frac{20\angle 0}{15\angle 48.16^\circ} = \boxed{1.33A\angle -48.16}$$

Than: $\overline{V}_R = \overline{I}\,\overline{Z}_R = (1.33\angle -48.16^\circ)(10\angle 0^\circ) = \boxed{13.3V.\angle -48.16^\circ}$

$$\vec{V}_L = \overline{I}\,\overline{Z}_L = (1.33\angle -48.16^\circ)(37.7\angle 90^\circ) = \boxed{50.14V.\angle 41.84^\circ}$$

$$\overline{V}_C = \overline{I}\,\vec{Z}_c = (1.33\angle -48.16^\circ)(26.53\angle -90^\circ) = \boxed{35.28\angle -138.16^\circ}$$

b.) Calculate total power factor

The angle between $\overline{E}$ and $\overline{I}$ is also the angle of $\overline{Z}_T$

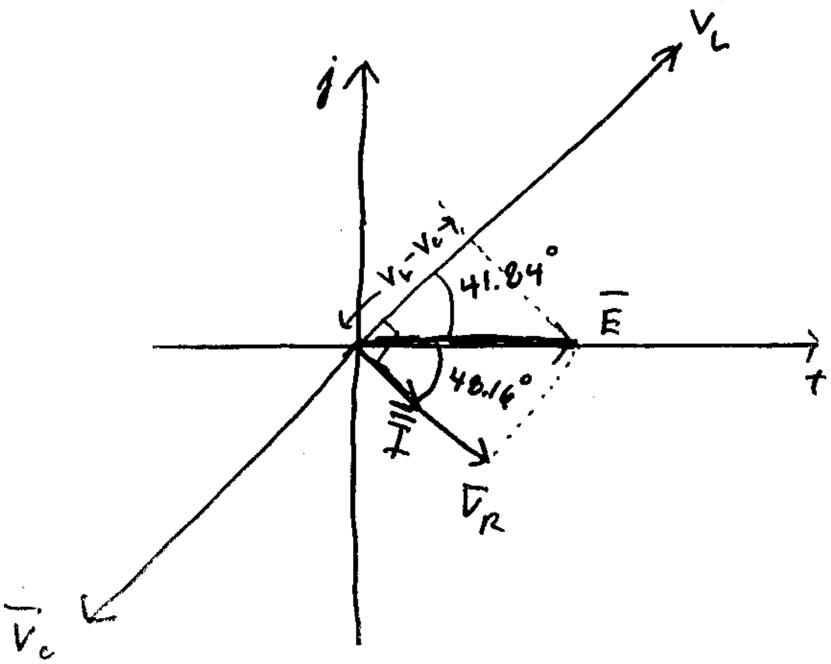
$$F_p = \cos\theta_{Z_T} = \cos 48.16^\circ = \boxed{0.667 \text{ lagging}}$$

Note: "lagging" because Z_T is inductive with a positive angle, i lags v

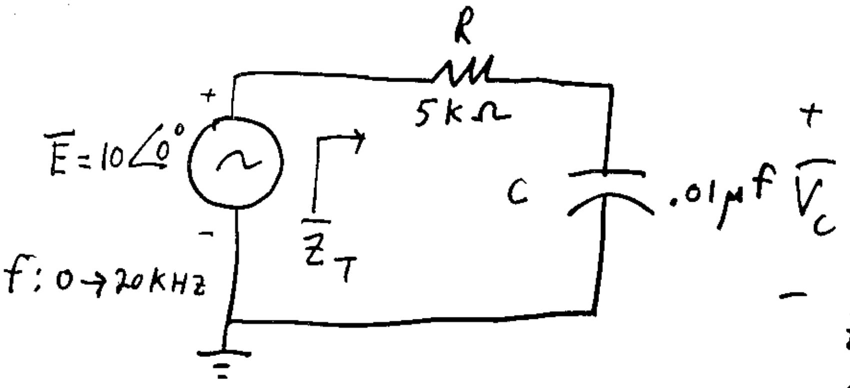
c.) Calculate the total power in watts delivered to the ckt.

$$P = EI\cos\theta = (20v.)(1.33A)(0.667) = \boxed{17.74 \text{ watts}}$$

d.) Draw Phasor Diagram

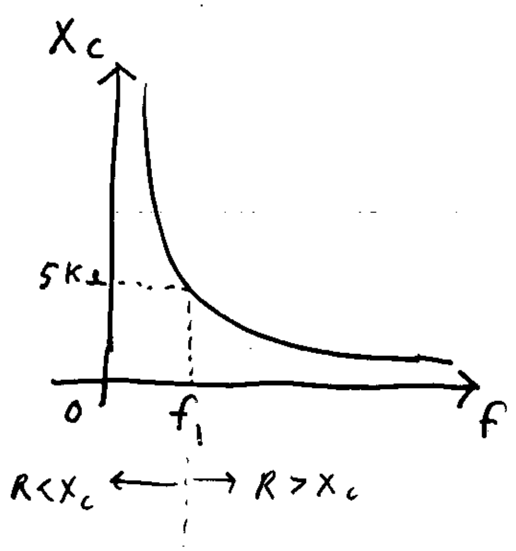


Frequency Response of the RC Circuit



The impedance $\overline{Z}_T$ will vary with frequency. This is due to X_C decreasing as frequency is increased.

$$X_C = \frac{1}{2\pi f C}$$



At some frequency f_1, X_C will equal R:

$$X_C = \frac{1}{2\pi f_1 C} = R$$

$$\boxed{f_1 = \frac{1}{2\pi R C}}$$

f_1 called <u>cut-off frequency</u> or 3-DB freq.

<u>ex/</u> $f_1 = \frac{1}{2\pi(5k)(.01\mu f)} = 3183.1 \text{ Hz}$

$$\overline{Z}_T = R - jX_C = Z_T\angle\theta_T = \sqrt{R^2+X^2}\angle -\tan^{-1}\frac{X_C}{R}$$

<u>For $f = 100\text{ Hz}$:</u> $X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(100\text{Hz})(.01\mu f)} = 159.16 \text{ kΩ}$

$$Z_T = \sqrt{R^2+X_C^2} = \sqrt{(5k)^2+(159.16k)^2} = 159.24\text{kΩ}$$

$$\theta_T = -\tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{159.16K}{5K} = -88.2°$$

$$\overline{Z}_T = 159.24k\angle -88.2°$$

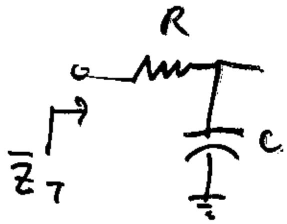
ex/ ..... continued

$\bar{Z}_T$ computed for other freq. values:

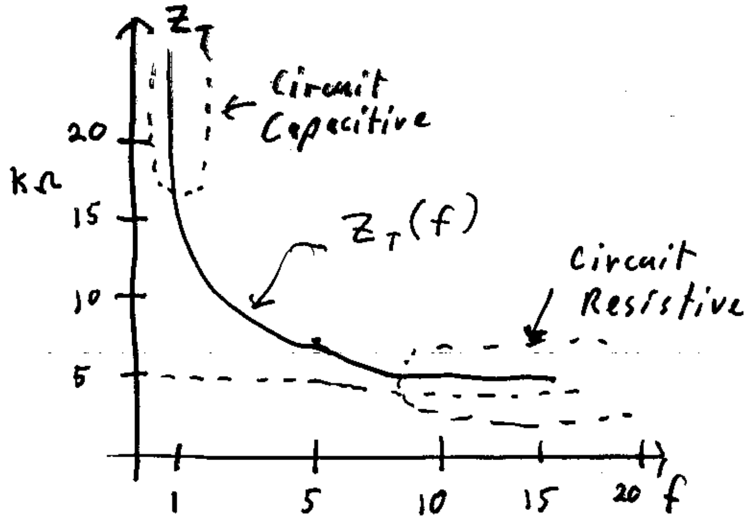
$f = 1\,KHz \rightarrow \bar{Z}_T = 5.93K\angle -32.48^\circ$

$f = 10\,KHz \rightarrow \bar{Z}_T = 5.25K\angle -17.66^\circ$

$f = 20\,KHz \rightarrow \bar{Z}_T = 5.06\,K\angle -9.04^\circ$

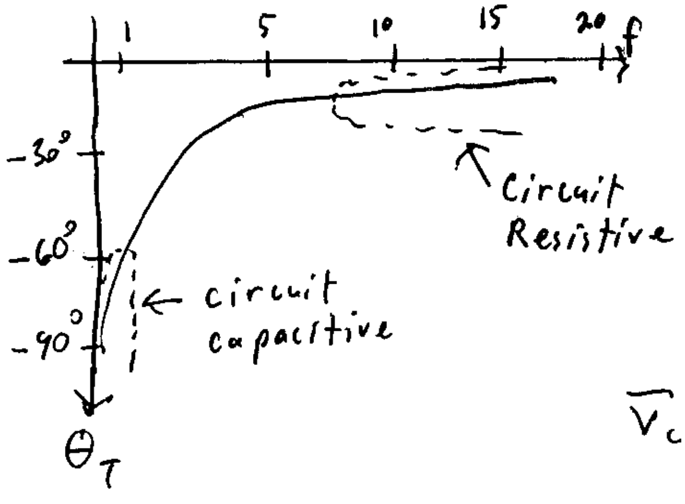


Plotting Gives:



Since $\bar{Z}_T = R - jX_c$, at low freq. X_c is large so the capacitance dominates

At high freq. X_c is small so the Resistance dominate



By the voltage divider rule:

$$\bar{V}_c = \frac{\bar{Z}_c \bar{E}}{\bar{Z}_c + \bar{Z}_R}$$

$$\bar{V}_c = \frac{(X_c\angle -90^\circ)(E\angle 0^\circ)}{R - jX_c}$$

$$\bar{V}_c = \frac{X_c E\angle -90^\circ}{\sqrt{R^2+X_c^2}\angle -\tan^{-1}\frac{X_c}{R}}$$

$$\bar{V}_c = \frac{X_c E}{\sqrt{R^2+X_c^2}}\angle -90^\circ + \tan^{-1}\frac{X_c}{R}$$

For $f = 0Hz$ (D.C.) $\rightarrow X_c = \frac{1}{2\pi 0 C} = \frac{1}{0} \leftarrow$ very large (acts like open ckt.)

so $\bar{V}_c = \bar{E} = 10V.\angle 0^\circ$

ex| .... continued

From the equation : $V_c = \dfrac{X_c E}{\sqrt{R^2+X_c^2}}$

For f=0 Hz. → $X_c^2 >> R^2$ so $\sqrt{R^2+X_c^2} \approx \sqrt{X_c^2} = X_c$

then $V_c = \dfrac{X_c E}{X_c} = E$

also $\theta_c = -\tan^{-1}\dfrac{R}{X_c} = -\tan^{-1} 0 = 0^\circ$

<u>for f = 1kHz</u> : $X_c = \dfrac{1}{2\pi f c} = \dfrac{1}{2\pi(10^3)(.01\times10^{-6})} = 15.92k\Omega$

$$\sqrt{R_2+X_c^2} = \sqrt{(5k)^2+(15.92k)^2} = 16.69k\Omega$$

$$V_c = \frac{X_c E}{\sqrt{R^2+X_c^2}} = \frac{15.92K}{16.69K}(10) = 9.54V.$$

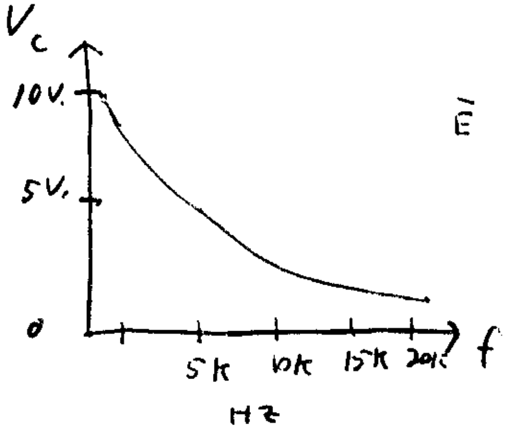
$$\theta_c = -\tan^{-1}\frac{R}{X_c} = -\tan^{-1}\frac{5k}{15.9k} = -17.46^\circ$$

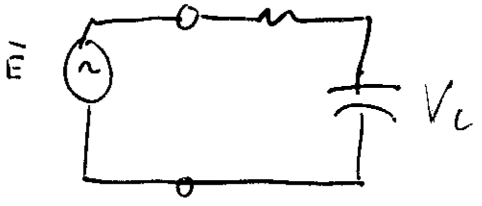
$$\overline{V_c} = 9.53V. \angle -17.46^\circ$$

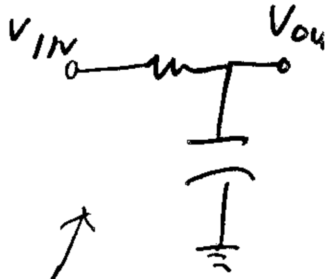
Similarly : for f = 5kHz → $\overline{V_c} = 5.37V. \angle -57.38^\circ$

f = 10KHz → $\overline{V_c} = 3.03V. \angle -72.34^\circ$

f = 20K Hz → $\overline{V_c} = 1.57V. \angle -80.79^\circ$







This RC circuit acts as a <u>LOW PASS filter</u>

It passes low freq. and attenuates high freq.

Parallel A.C. Circuits

Admittance and Susceptance

For D.C ckts we defined Conductance G as:

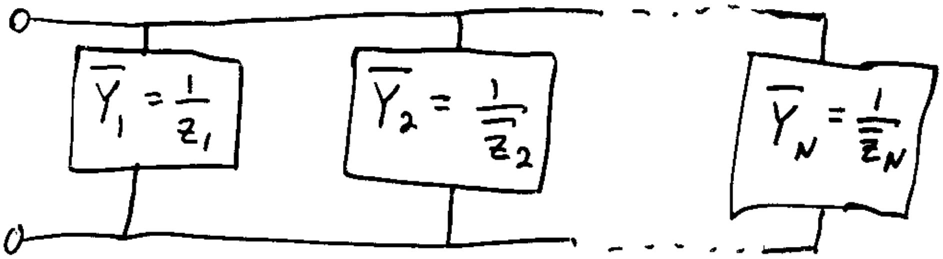
$$\boxed{G = \frac{1}{R}}$$

Similarly for A.C. ckts we define:

Admittance → $\boxed{\overline{Y} = \frac{1}{\overline{Z}}}$ ← z = impedance

Susceptance → $\boxed{B = \frac{1}{X}}$ ← x = reactance

The recipricol quantities simplify analysis of parallel ckts.



$\boxed{\overline{Y}_T = \overline{Y}_1 + \overline{Y}_2 + \dots + \overline{Y}_N}$ Admittances in parallel add together

Also:

$\boxed{\frac{1}{\overline{Z}_T} = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \dots + \frac{1}{\overline{Z}_N}}$ ← parallel impedance combining rule

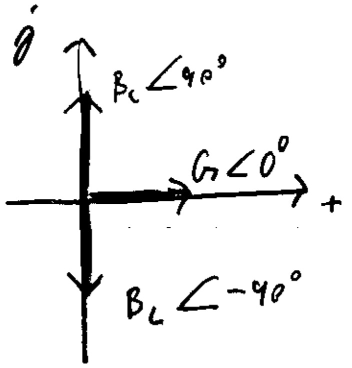
For 2 elements in parallel:

$$\boxed{\overline{Z}_T = \frac{\overline{Z}_1 \overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2}}$$

For a Resistor: $\boxed{\overline{Y}_R = \frac{1}{\overline{Z}_R} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ}$

For an Inductor: $\boxed{\overline{Y}_L = \frac{1}{\overline{Z}_L} = \frac{1}{X_L\angle 90^\circ} = \frac{1}{X_L}\angle -90^\circ = B_L\angle -90^\circ}$

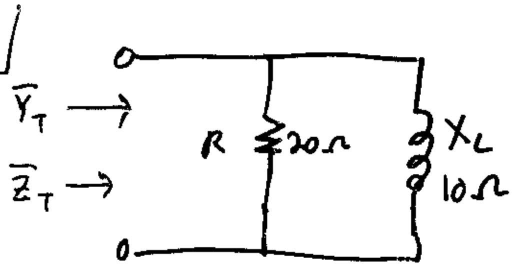
For a Capacitor: $\boxed{\overline{Y}_C = \frac{1}{\overline{Z}_C} = \frac{1}{X_C\angle -90^\circ} = \frac{1}{X_C}\angle 90^\circ = B_C\angle 90^\circ}$



← Admittance Diagram.

Note: L and C interchange roles between parallel and series ckts.

ex/



a.) Find $\overline{Y}$ of each branch.

Sol: $\overline{Y}_R = G\angle 0^\circ = \frac{1}{R}\angle 0^\circ = \frac{1}{20}\angle 0^\circ$

$\overline{Y}_R = .05\ S$ ← units of Seimen or $\frac{1}{\Omega}$

$\overline{Y}_L = B_L\angle -90^\circ = \frac{1}{X_L}\angle -90^\circ = \frac{1}{10}\angle -90^\circ = 0.1\angle -90^\circ = -j(.1)$

b.) Find $\overline{Y}_T$

Sol: $\overline{Y}_T = \overline{Y}_R + \overline{Y}_L = (.05) - j0.1\ S$

c.) Find $\overline{Z}_T$

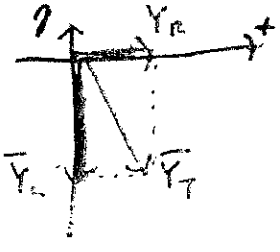
Sol: $\overline{Z}_T = \frac{1}{\overline{Y}_T} = \frac{1}{(.05)-j0.1} = \frac{1}{.112\angle -63.42^\circ}$

$\overline{Z}_T = 8.93\angle 63.43$

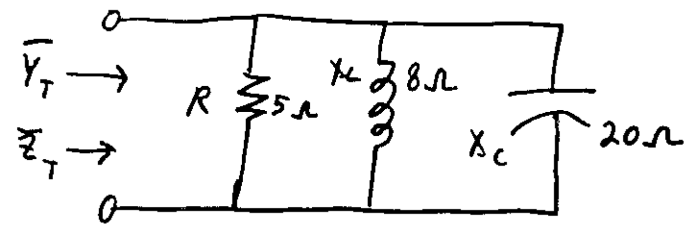
also $\overline{Z}_T = \frac{\overline{Z}_R\,\overline{Z}_L}{\overline{Z}_R + \overline{Z}_L}$

d.) Draw Admittance Diagram

Sol:



ex/



a.) Find $\overline{Y}$ of each branch

Sol: $\overline{Y}_R = \frac{1}{R}\angle 0° = \frac{1}{5\Omega}\angle 0°$

$\overline{Y}_R = 0,25\angle 0° = 0,2\,S + j0$

$\overline{Y}_L = \frac{1}{X_L}\angle -90° = \frac{1}{8\Omega}\angle -90° = 0.125\ S\angle -90° = 0 - j0.125\ S.$

$\overline{Y}_C = \frac{1}{X_C}\angle 90° = \frac{1}{20\Omega}\angle 90° = 0.05\ S\angle 90° = 0 + j.05\ S.$

b.) Find $\overline{Y}_T$

Sol: $\overline{Y}_T = \overline{Y}_R + \overline{Y}_L + \overline{Y}_C = 0.2 - j0.125 + j.05 = 0.2 - j.075$

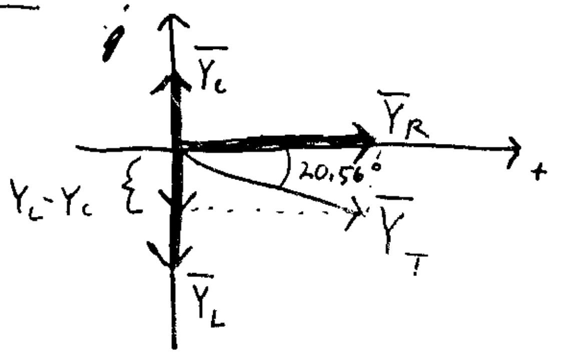
$= 0.2136\angle -20.56$

c.) Find $\overline{Z}_T$

Sol: $\overline{Z}_T = \frac{1}{\overline{Y}_T} = \frac{1}{0.2136}\angle -20.56 = 4.68\Omega\angle 20.56$

d.) Draw the admittance diagram

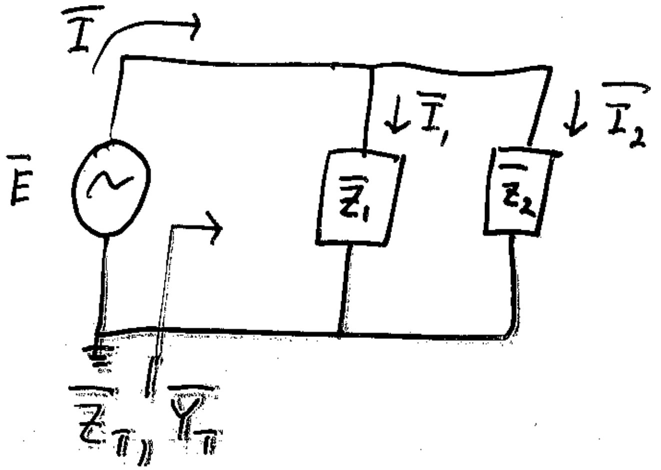
sol:



The total $\overline{Y}_T$ is <u>inductive</u> because the smaller X_L value will draw more of the current than the larger X_C value.

It is easier to use <u>Admittances</u> for <u>parallel</u> circuits instead of Impedances.

Parallel A.C. Networks



By Ohm's Law:

$$\overline{I} = \frac{\overline{E}}{\overline{Z}_T} = \overline{E}\,\overline{Y}_T$$

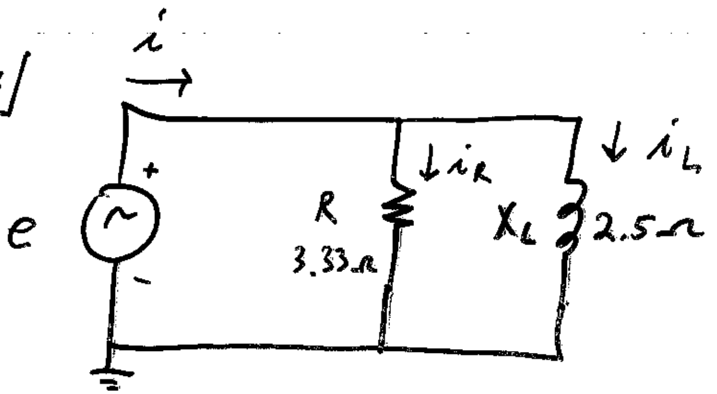
$$\overline{I}_1 = \frac{\overline{E}}{\overline{Z}_1} = \overline{E}\,\overline{Y}_1$$

$$\overline{I}_2 = \frac{\overline{E}}{\overline{Z}_2} = \overline{E}\,\overline{Y}_2$$

By KCL: $\overline{I} = \overline{I}_1 + \overline{I}_2$

Also $P = EI\cos\theta_T$

ex/



Find i, i_R, i_L

Given: $e = \sqrt{2}(20)\sin(\omega t + 53.13°)$

Solution:

$$\overline{Y}_T = \overline{Y}_R + \overline{Y}_L = \frac{1}{3.33}\angle 0° + \frac{1}{2.5\angle 90°} = 0.3\angle 0° + 0.4\angle -90°$$

$$\overline{Y}_T = 0.3 - j0.4 = 0.5\angle -53.13°$$

$$\overline{I} = \overline{E}\,\overline{Y}_T = (20\angle 53.13°)(0.5\angle -53.13°) = 10\text{ A}\angle 0°$$

$$i = (\sqrt{2})\,10\sin\omega t$$

$$\overline{I}_R = \overline{E}\,\overline{Y}_R = (20\angle 53.13°)(0.3\angle 0°) = 6\text{A}\angle 53.13°$$

$$i_R = (\sqrt{2})\,6\sin(\omega t + 53.13°)$$

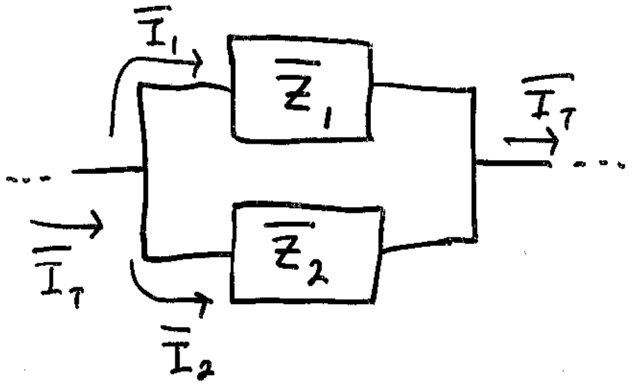
$$\overline{I}_L = \overline{E}\,\overline{Y}_L = (20\angle 53.13)(0.4\angle -90°) = 8\text{A.}\angle -36.87°$$

$$i_L = (\sqrt{2})\,8\sin(\omega t - 36.87°)$$

$P = (20)(10)\cos 53.13°$

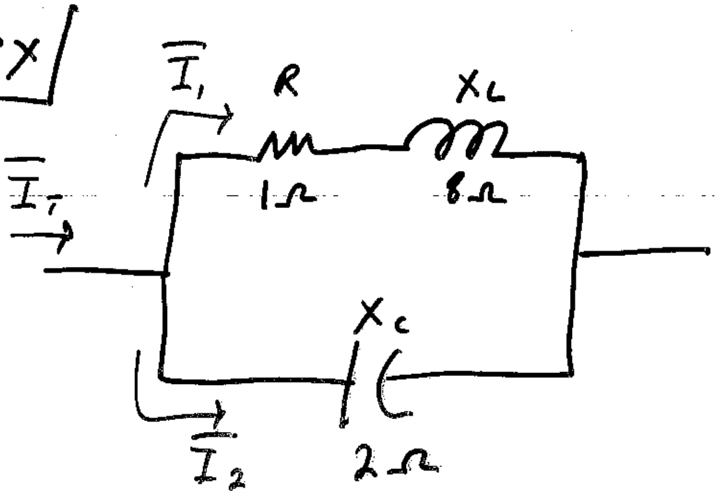
$P = 120$ W.

Current Divider Rule



$$\boxed{\begin{aligned} \overline{I}_1 &= \overline{I}_T \frac{\overline{Z}_2}{\overline{Z}_1 + \overline{Z}_2} \\ \overline{I}_2 &= \overline{I}_T \frac{\overline{Z}_1}{\overline{Z}_1 + \overline{Z}_2} \end{aligned}}$$

ex/



Find $\overline{I}_1$ and $\overline{I}_2$

Given $\overline{I}_T = 5A. \angle 30°$

<u>solution:</u>

By Current Divider Rule:

$$\overline{I}_1 = \overline{I}_T \frac{\overline{Z}_c}{\overline{Z}_c + \overline{Z}_{RL}} = \frac{(5\angle 30°)\quad(2\angle -90°)}{2\angle -90° + (1\angle 0° + 8\angle 90°)}$$

$$\overline{I}_1 = \frac{10\angle -60°}{-j2+1+j8} = \frac{10\angle -60°}{1+j6} = \frac{10\angle 60°}{6.08\angle 80.54°} = 1.64A.\angle -140.54°$$

$$\overline{I}_2 = \overline{I}_T \frac{\overline{Z}_{RL}}{\overline{Z}_c + \overline{Z}_{RL}} = \frac{(5\angle 30°)(1+j8)}{6.08\angle 80.54°} = \frac{(5\angle 30°)(8.06\angle 82.87°)}{6.08\angle 80.54°}$$

$$\overline{I}_2 = 6.625A.\angle 32.33°$$

Can check by KCL: $\overline{I}_T = \overline{I}_1 + \overline{I}_2$