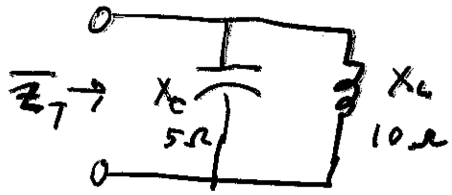
Equivalent Circuits

Equivalent circuits will have the same impedance and the same input current will result for the same applied voltage.

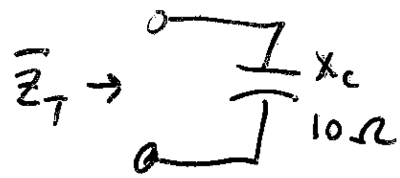
ex/



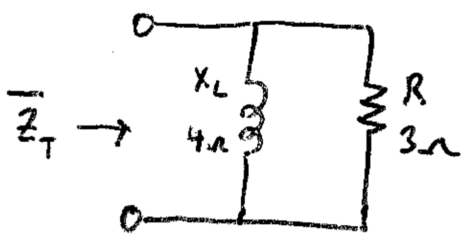
$$\overline{Z}_T = \frac{\overline{Z}_C \, \overline{Z}_L}{\overline{Z}_C + \overline{Z}_L}$$

$$\overline{Z}_T = \frac{(5\angle -90^\circ)(10\angle 90^\circ)}{5\angle -90^\circ + 10\angle 90^\circ} = \frac{50\angle 0^\circ}{5\angle 90^\circ} = 10\Omega\angle -90^\circ$$

The equivalent circuit is:



ex/

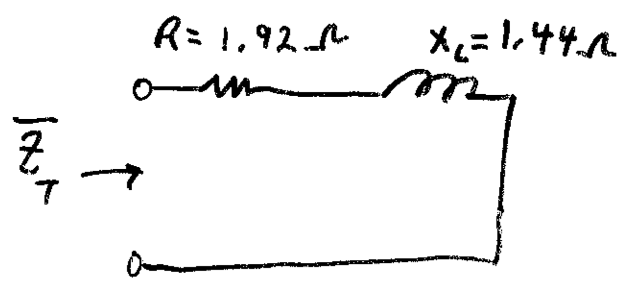


$$\overline{Z}_T = \frac{\overline{Z}_L \, \overline{Z}_R}{\overline{Z}_L + \overline{Z}_R} = \frac{(4\angle 90^\circ)(3\angle 0^\circ)}{4\angle 90^\circ + 3\angle 0^\circ}$$

$$\overline{Z}_T = \frac{12\angle 90^\circ}{5\angle 53.13^\circ} = 2.4\angle 36.87^\circ$$

$$\overline{Z}_T = 1.92 + j\,1.44\ \Omega$$

Equivalent ckt. is:



To find a parallel equivalent circuit it is easier to work with admittance instead of impedance.

1.) First total impedance: $\overline{Z}_T$

2.) Convert to total admittance: $\overline{Y}_T = \dfrac{1}{\overline{Z}_T}$

3.) Identify the Real part of $\overline{Y}_T$. G

Then $R = \dfrac{1}{G}$.

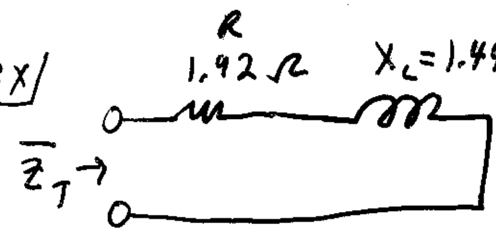
4.) Identify the Imaginary part of $\overline{Y}_T$. B

Then $X = \dfrac{1}{B}$

If B is negative then $X = X_L$ (inductive)

If B is positive then $X = X_C$ (Capacitive)

ex/



Find equivalent parallel ckt.

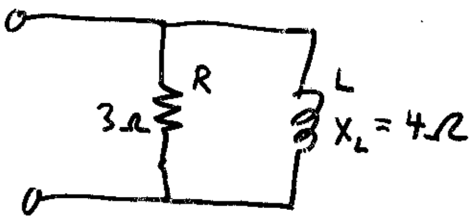
Sol: $\overline{Z}_T = 1.92 + j\,1.44 = 2.4\angle 36.87^\circ$

$\overline{Y}_T = \dfrac{1}{2.4\angle 36.87^\circ} = 0.4167\angle -36.87^\circ = 0.333 - j\,0.25$

since its negative its inductive

$R = \dfrac{1}{.333} = 3\,\Omega$, $X = \dfrac{1}{.25} = 4\,\Omega = X_L$

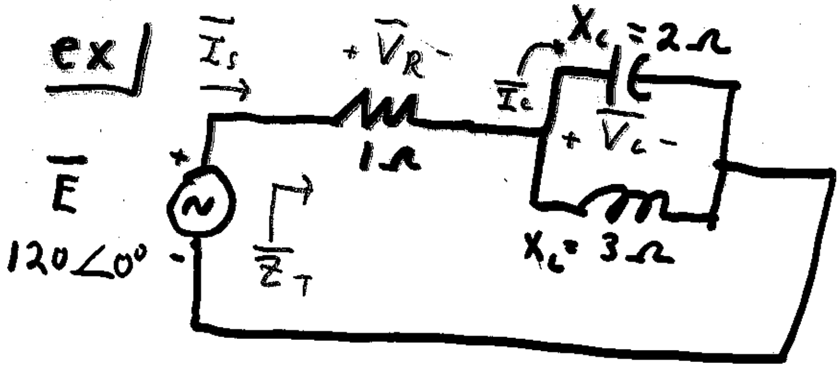
equivalent ckt. → $\overline{Z}_T$ →



Series - Parallel AC Networks

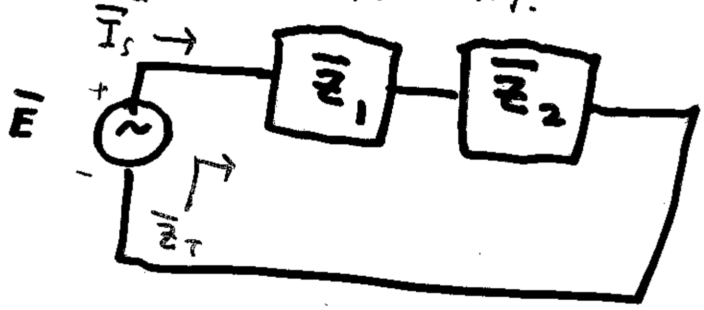
It is often useful to combine elements together into impedance blocks to simplify the circuit analysis.

ex



Find: $\overline{Z}_T$, $\overline{I}_s$, $\overline{V}_R$, $\overline{V}_c$, $\overline{I}_c$, P, F_p

Solution: Redraw Ckt.



$$\overline{Z}_T = \overline{Z}_1 + \overline{Z}_2$$

$$\overline{Z}_1 = 1\,\Omega\angle 0^\circ$$

$$\overline{Z}_2 = \overline{Z}_c \| \overline{Z}_L = \frac{(2\angle -90^\circ)(3\angle 90^\circ)}{-j2+j3} = \frac{6\angle 0^\circ}{j1} = \frac{6\angle 0^\circ}{1\angle 90^\circ} = 6\angle -90^\circ$$

$$\overline{Z}_T = 1\,\Omega - j6\,\Omega = \boxed{6.08\,\Omega\angle -80.54^\circ}$$

By Ohms Law:

$$\overline{I}_s = \frac{\overline{E}}{\overline{Z}_T} = \frac{120\angle 0^\circ}{6.08\angle -80.54^\circ} = \boxed{19.74\,A.\angle 80.54^\circ}$$

$$\overline{V}_R = \overline{I}_s\overline{Z}_1 = (19.74\angle 80.54)(1\,\Omega\angle 0^\circ) = \boxed{19.74\,V.\angle 80.54^\circ}$$

$$\overline{V}_c = \overline{I}_s\overline{Z}_2 = (19.74\angle 80.54^\circ)(6\,\Omega\angle -90^\circ) = \boxed{118.44\,V.\angle -9.46^\circ}$$

ex) .... continued

$$\overline{I}_c = \frac{\overline{V}_c}{\overline{Z}_c} = \frac{118.44\text{ V.} \angle -9.46^\circ}{2\Omega \angle -90^\circ} \doteq \boxed{59.22\text{ A.} \angle 80.54^\circ}$$

The total power delivered to the ckt. from the source is dissapated by the resistor.

$$P = I_s^2 R = (19.74\text{A.})^2 (1\Omega) = \boxed{389.67\text{ W.}}$$

Power could also have been found from:

$$P = E\, I_s \cos\theta$$

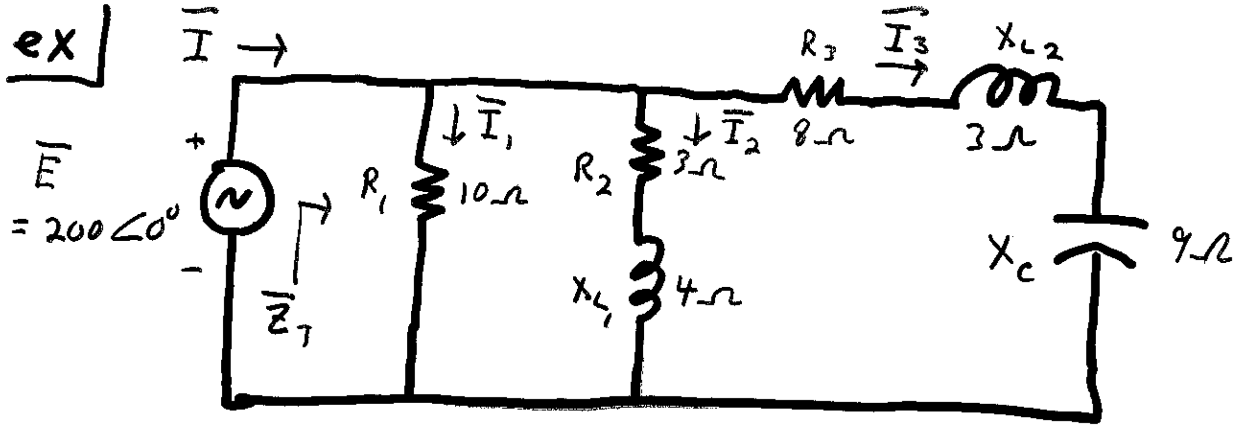
θ is the angle of $\overline{Z}_T$

The power factor F_p is:

$$F_p = \cos\theta = \cos(-80.54^\circ) = \boxed{0.164 \text{ leading}}$$

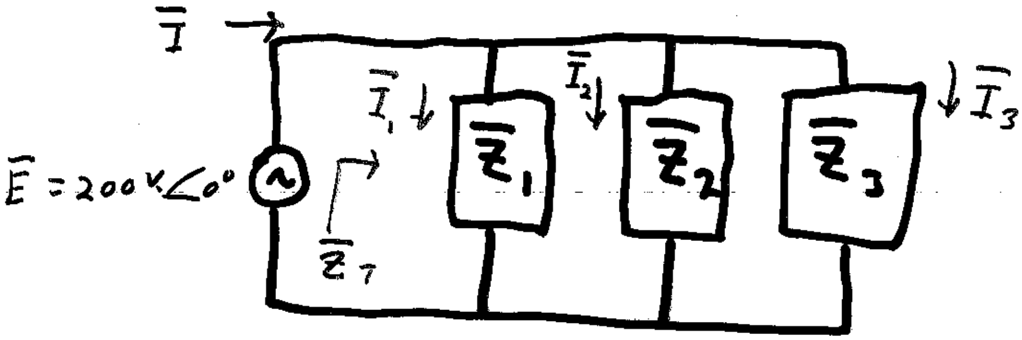
It's leading since the angle of $\overline{Z}_T$ is negative which means the net impedance is capacitive.

ex



Find: $\overline{I}$, $\overline{I}_1$, $\overline{I}_2$, $\overline{I}_3$, $\overline{Z}_T$

Solution: Redraw ckt.



$$\overline{Z}_1 = R_1 = 10\angle 0^\circ\ \Omega$$

$$\overline{Z}_2 = R_2 + jX_{L_1} = 3\Omega + j4\Omega$$

$$\overline{Z}_3 = R_3 + jX_{L_2} - jX_C = 8 + j3 - j9 = 8 - j6\ \Omega$$

For a parallel ckt. it's easier to work with admittance.

$$\overline{Y}_T = \overline{Y}_1 + \overline{Y}_2 + \overline{Y}_3 = \frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3} = \frac{1}{10} + \frac{1}{3+j4} + \frac{1}{8-j6}$$

$$\overline{Y}_T = 0.1 + \frac{1}{5\angle 53.13^\circ} + \frac{1}{10\angle -36.87^\circ} = 0.1 + 0.2\angle -53.13^\circ + 0.1\angle 36.87^\circ$$

$$\overline{Y}_T = 0.1 + 0.12 - j0.16 + 0.08 + j0.06 = 0.3 - j0.1 = 0.316\angle -18.435^\circ$$

By Ohm's Law:

$$\overline{I} = \overline{E}\,\overline{Y}_T = (200V\angle 0^\circ)(0.316\angle -18.435^\circ) =$$

$$\overline{I} = \boxed{63.2\,A\ \angle -18.435^\circ}$$

ex/ .... continued

$$\overline{I}_1 = \frac{\overline{E}}{\overline{Z}_1} = \frac{200\text{ V.}\angle 0^\circ}{10\,\Omega\angle 0^\circ} = \boxed{20\text{ A.}\angle 0^\circ}$$

$$\overline{I}_2 = \frac{\overline{E}}{\overline{Z}_2} = \frac{200\text{ V.}\angle 0^\circ}{5\,\Omega\angle 53.13^\circ} = \boxed{40\text{ A.}\angle -53.13^\circ}$$

$$\overline{I}_3 = \frac{\overline{E}}{\overline{Z}_3} = \frac{200\text{ V.}\angle 0^\circ}{10\,\Omega\angle -36.87^\circ} = \boxed{20\text{ A.}\angle 36.87^\circ}$$

We can check results using KCL:

$$\overline{I} = \overline{I}_1 + \overline{I}_2 + \overline{I}_3$$

$$63.2\text{A}\angle -18.435^\circ \stackrel{?}{=} 20\angle 0^\circ + 40\angle -53.13^\circ + 20\angle 36.87^\circ$$

$$60 - j20 = (20+j0) + (24-j32) + (16+j12)$$

$$60 - j20 = 60 - j20$$

checks ✓

Also,

$$\overline{Z}_T = \frac{1}{\overline{Y}_T} = \frac{1}{0.316\text{ S}\angle -18.435^\circ}$$

$$\overline{Z}_T = \boxed{3.165\,\Omega\angle 18.435^\circ}$$

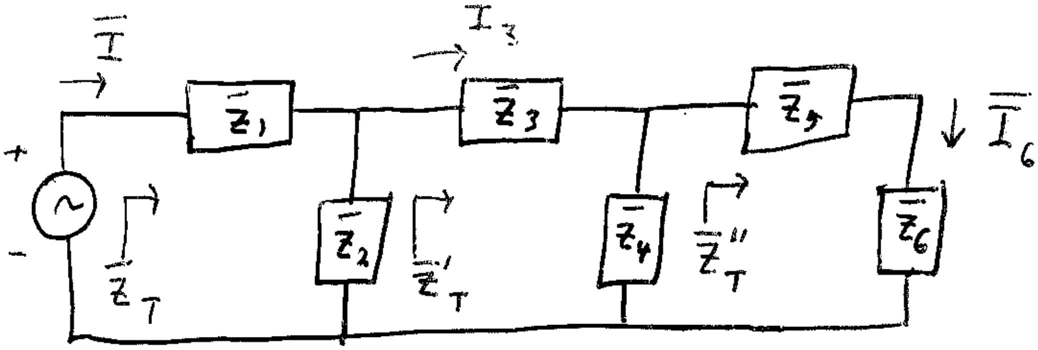
$$P = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

$$F_p = \cos\theta = \cos 18.435^\circ$$

Ladder Networks

ex/

$\bar{E} = 120\text{V} \angle 0^\circ$



Find $\bar{I}_6$

Solution Method:

Begin at far end and find $\bar{Z}_T$

$$\bar{Z}''_T = \bar{Z}_5 + \bar{Z}_6$$

$$\bar{Z}'_T = \bar{Z}_3 + \bar{Z}_4 \,||\, \bar{Z}''_T$$

$$\bar{Z}_T = \bar{Z}_1 + \bar{Z}_2 \,||\, \bar{Z}'_T$$

Then find total current $\bar{I}$: $\quad \bar{I} = \dfrac{\bar{E}}{\bar{Z}_T}$

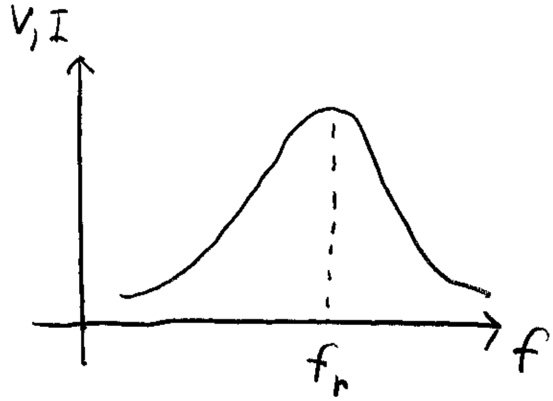
Use repeated application of CDR to find other currents:

$$\bar{I}_3 = \frac{\bar{Z}_2 \bar{I}}{\bar{Z}_2 + \bar{Z}'_T}$$

$$\bar{I}_6 = \frac{\bar{Z}_4 \bar{I}_3}{\bar{Z}_4 + \bar{Z}''_T}$$

Resonance

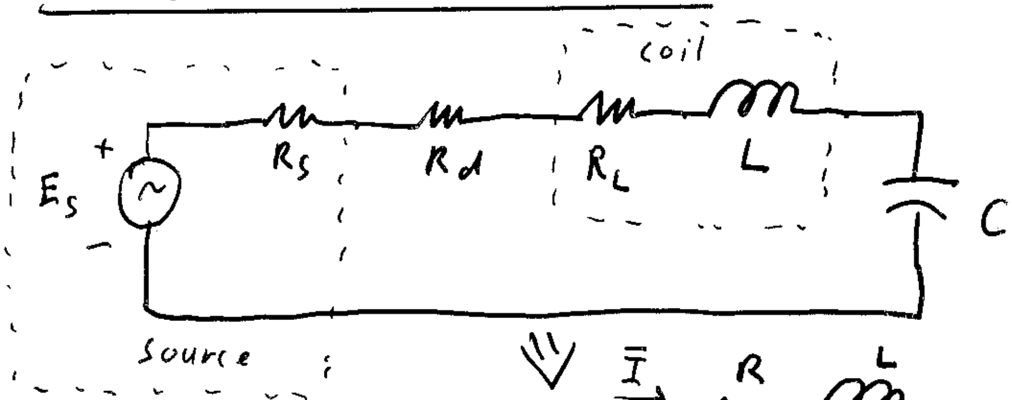
At a particular frequency f_r, an L-C circuit will exhibit resonance.

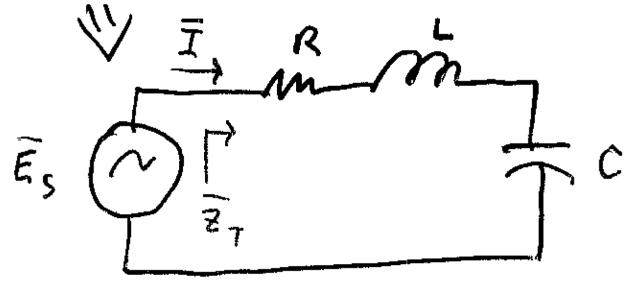


At the resonant frequency: $\boxed{X_L = X_C}$

The LC circuit is also called a <u>tuned circuit</u>.

Series Resonant Circuit





Equiv. ckt:

$R = R_S + R_d + R_L$

$$\bar{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

At the <u>resonant frequency</u> the inductive reactance and the capacitive reactance cancel each other out.

At Resonance: $\boxed{\begin{array}{l} X_L = X_C \\ \bar{Z}_{T_S} = R \end{array}}$

When $X_L = X_C$,

$$\omega L = \frac{1}{\omega C}$$

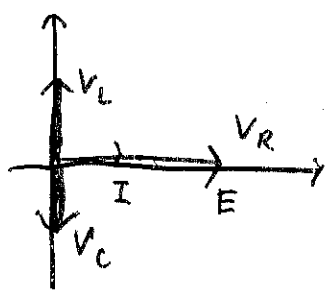
$$\omega^2 = \frac{1}{LC}$$

$f = $ Hz.
$L = $ Henries
$C = $ Farads

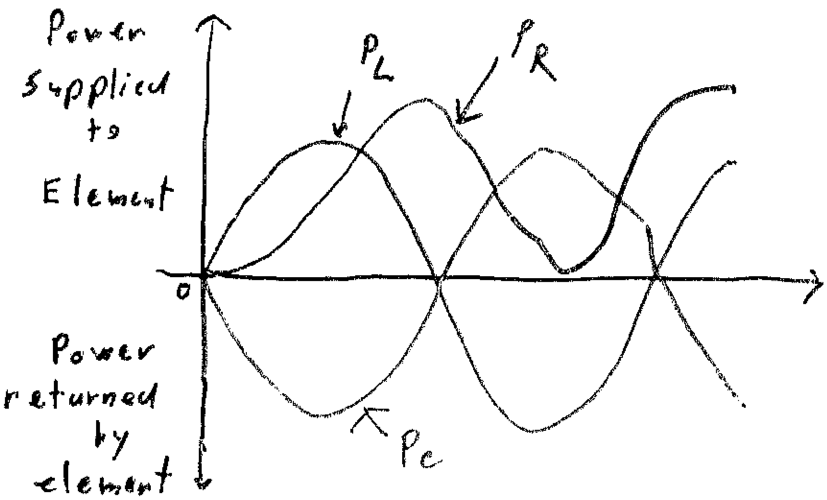
$$\boxed{\omega_s = \frac{1}{\sqrt{LC}} \qquad f_s = \frac{1}{2\pi\sqrt{LC}} \qquad \text{Resonant Frequency}}$$

$$\left.\begin{aligned} \bar{V}_L &= (I\angle 0^\circ)(X_L\angle 90^\circ) = IX_L\angle 90^\circ \\ \bar{V}_C &= (I\angle 0^\circ)(X_C\angle -90^\circ) = IX_C\angle -90^\circ \end{aligned}\right\} \text{ } 180^\circ \text{ out of phase}$$

$$V_{L_s} = V_{C_s}$$



$\boxed{\text{At resonance } \bar{Z}_T \text{ is Purely Real (the imaginary parts cancel out). So } F_{p_s} = \cos\theta = 1}$



Energy is exchanged between L and C during each half Cycle.

The Quality Factor "Q"

The Q of a resonant circuit is measure of how much the reactive elements dominate over the resitive elements. i.e. how close it resembles a "loss less" tuned circuit.

$$\boxed{Q_s = \frac{\text{reactive Power} \leftarrow \text{VARS}}{\text{average Power} \leftarrow \text{watts}}}$$

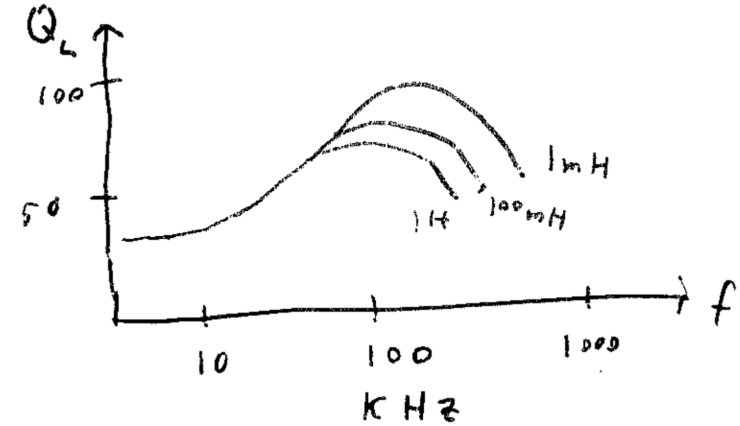
For the series RL circuit:

$$Q_s = \frac{I^2 X_L}{I^2 R} \qquad \boxed{Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R}}$$

For a coil where R_L is the wire's resistance:

$$Q_{coil} = \frac{X_L}{R_L}$$

ex/



For an RLC series circuit:

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R}$$

Also $f_s = \dfrac{1}{2\pi\sqrt{LC}}$

Substituting gives:

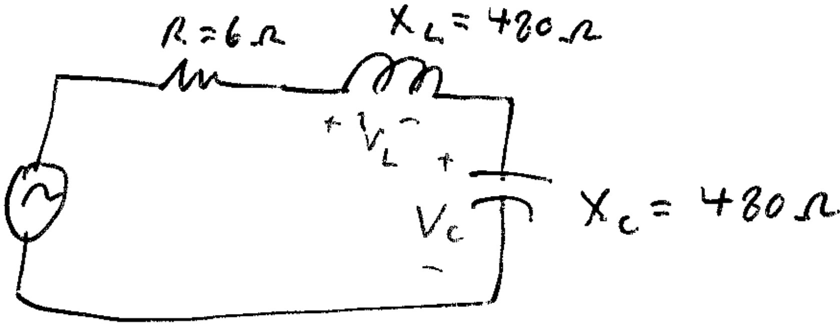
$$Q_s = \frac{2\pi\left(\frac{1}{2\pi\sqrt{LC}}\right)L}{R}$$

$$\boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$ ← for series RLC

At resonance $V_L = \dfrac{X_L E}{Z_T} = \dfrac{X_L}{R} E$

$\boxed{V_{L_s} = Q_s E}$ similarly $\boxed{V_{C_s} = Q_s E}$

ex/



$\bar{E} = 10\,V \angle 0°$

$$Q_s = \frac{X_L}{R} = \frac{480\,\Omega}{6\,\Omega} = 80$$

$V_L = V_C = Q_s E = (80)(10) = 800\,V.$ ←

Note: V_L and V_C much larger than applied volta

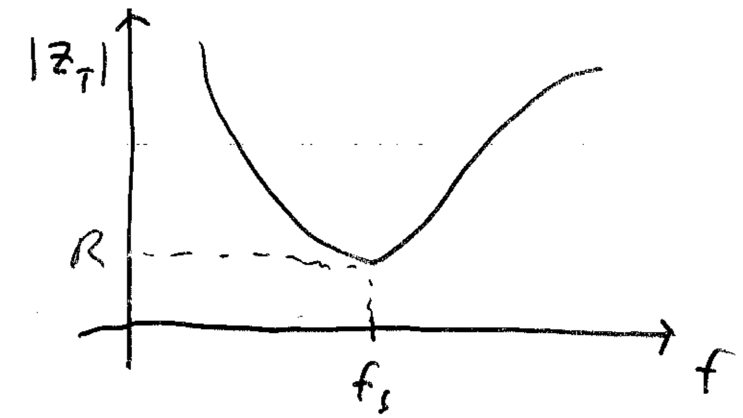
Z_T Versus Frequency

For the series RLC circuit

$$\bar{Z}_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

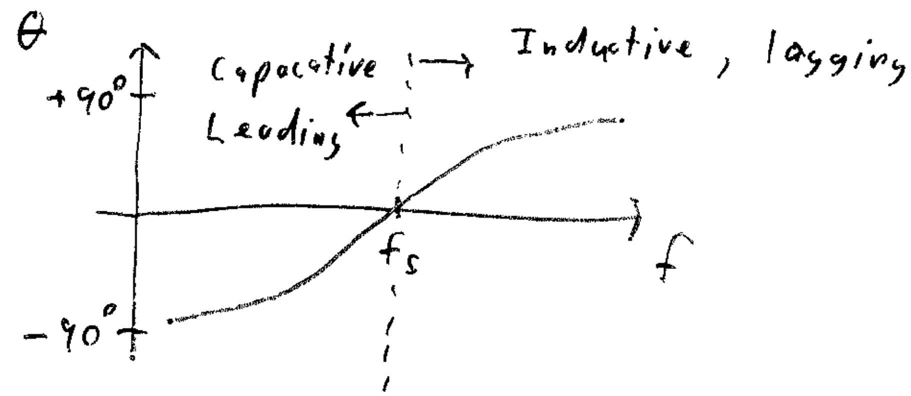
Magnitude of $\bar{Z}_T$:

$$|Z_T| = \sqrt{R^2 + (X_L - X_C)^2}$$



$|Z_T|$ is a minimum at resonance for the series RLC circuit

$$\theta = \tan^{-1} \frac{(X_L - X_C)}{R}$$



Selectivity

For the series RLC circuit:

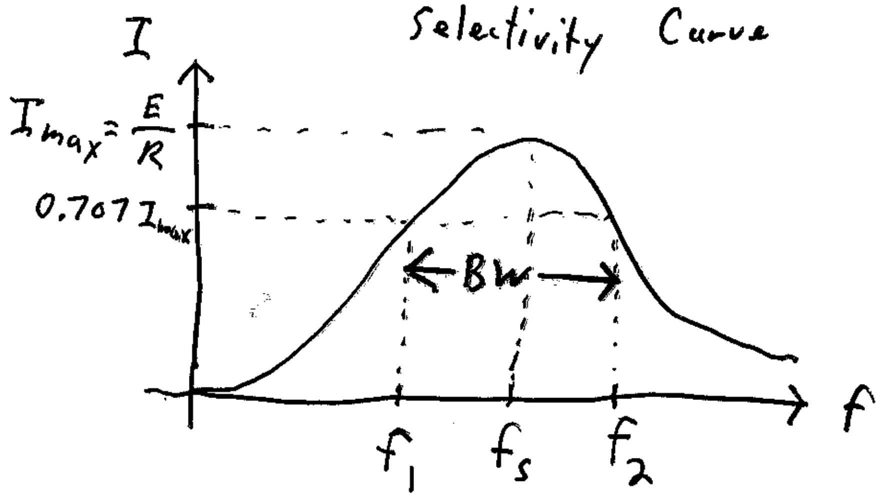
$P = I^2 R$

$\frac{1}{2} P = \left(\frac{I}{\sqrt{2}}\right)^2 R$

Power:

$$\boxed{P_{HPF} = \frac{1}{2} P_{max}}$$

$$\boxed{BW = f_2 - f_1}$$



f_s is the resonant or center frequency.

f_1 is the lower cutoff or half-power frequency.

f_2 is the upper cutoff or half-power frequency.

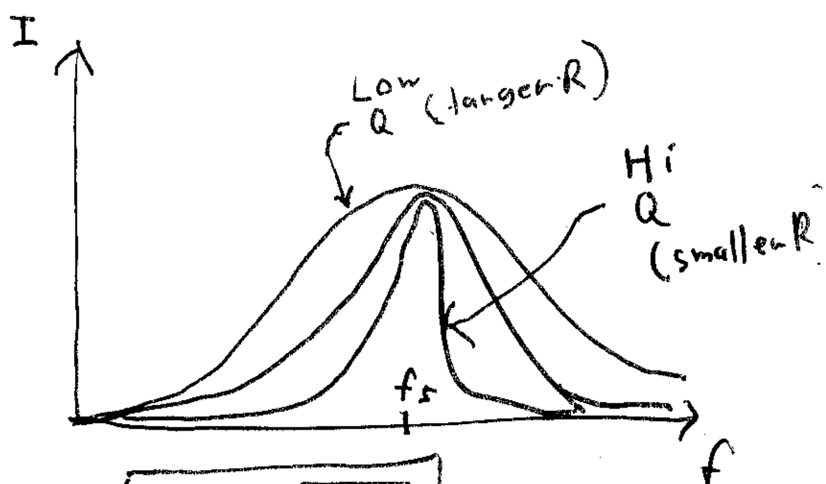
The range of frequencies between f_1 and f_2 is called the Bandwidth (BW).

The BW depends upon the Q of the ckt.

$$\boxed{BW = \frac{f_s}{Q_s}}$$

$$\boxed{\frac{f_2 - f_1}{f_s} = \frac{1}{Q_s}}$$

fractional BW



$$\boxed{f_s = \sqrt{f_1 f_2}}$$

For the series RLC circuit:

$$\boxed{f_s = \frac{1}{2\pi\sqrt{LC}}} \qquad \boxed{Q_s = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

$$f_1 = \frac{1}{2\pi}\left[-\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

$$f_2 = \frac{1}{2\pi}\left[\frac{R}{2L} + \frac{1}{2}\sqrt{\left(\frac{R}{L}\right)^2 + \frac{4}{LC}}\right]$$

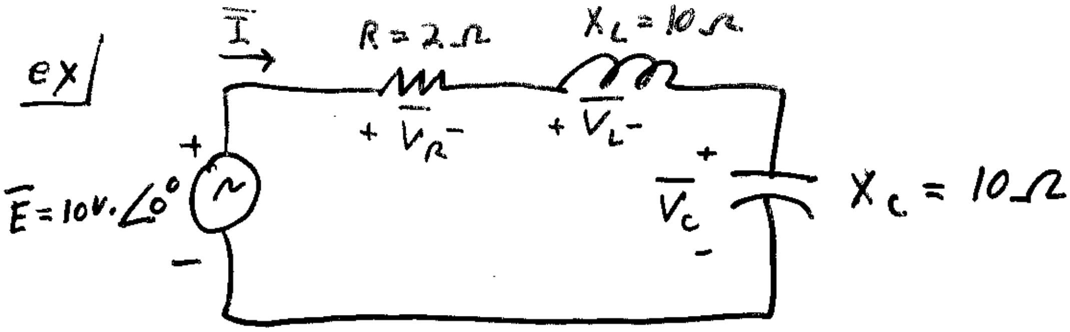
$$\boxed{BW = f_2 - f_1 = \frac{R}{2\pi L}}$$

For $Q \geq 10$ it can be assumed that f_s is in the center of the BW. This simplifies finding f_1 and f_2 to:

$$f_1 = f_s + \frac{BW}{2}$$

$$f_2 = f_s - \frac{BW}{2}$$

ex/



a) Find $\overline{I}$, $\overline{V_R}$, $\overline{V_L}$ and $\overline{V_C}$ for series resonant ckt.

Solution: $\overline{Z}_{T_s} = R = 2\,\Omega$

$$\overline{I} = \frac{\overline{E}}{\overline{Z}_{T_s}} = \frac{10\,V.\angle 0^\circ}{2\,\Omega\,\angle 0^\circ} = \boxed{5A\angle 0^\circ}$$

$$\overline{V_R} = \overline{E} = \boxed{10V.\ \angle 0^\circ}$$

$$\overline{V_L} = \overline{I}\,\overline{Z}_L = (5\angle 0^\circ)(10\angle 90^\circ) = \boxed{50V.\angle 90^\circ}$$

$$\overline{V_C} = \overline{I}\,\overline{Z}_C = (5\angle 0^\circ)(10\angle -90^\circ) = \boxed{50V.\ \angle -90^\circ}$$

b.) Find Q_s

solution:
$$Q_s = \frac{X_L}{R} = \frac{10\,\Omega}{2\,\Omega} = \boxed{5}$$

c.) If the resonant frequency is 5 kHz, find BW.

Solution:
$$BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000\text{ Hz}}{5} = \boxed{1\text{ kHz}}$$

d.) what is the power dissipated in the circuit at the half-power frequencies?

solution:
$$P_{HPF} = \frac{1}{2}P_{max} = \frac{1}{2}I_{max}^2 R = \left(\frac{1}{2}\right)(5A.)^2(2\,\Omega) = \boxed{25W.}$$

ex/ The bandwidth of a series resonant circuit is 400 Hz.

a.) If the resonant frequency is 4000 Hz, what is the Q?

solution:

$$BW = \frac{f_s}{Q_s} \quad \text{so} \quad Q = \frac{f_s}{BW} = \frac{4000\,Hz}{400\,Hz} = \boxed{10}$$

b.) If $R = 10\,\Omega$, what is the value of X_L at resonance?

Solution:

$$Q_s = \frac{X_L}{R} \quad \text{so} \quad X_L = Q_s R = (10)(10\,\Omega) = \boxed{100\,\Omega}$$

c.) Find the inductance L and Capacitance C of the circuit.

solution:

$$X_L = 2\pi f_s L \quad \text{so} \quad L = \frac{X_L}{2\pi f_s} = \frac{100\,\Omega}{2\pi(4000\,Hz)} = \boxed{3.98\,mH}$$

$$X_C = \frac{1}{2\pi f_s C} \quad \text{so} \quad C = \frac{1}{2\pi f_s X_C} = \frac{1}{2\pi(4000)(100)} = \boxed{0.398\,\mu F}$$

ex/ A series RLC circuit has a resonant frequency of 12,000 Hz.

a.) If $R = 5\,\Omega$ and X_L at resonance is $300\,\Omega$, find BW.

solution: $Q_s = \frac{X_L}{R} = \frac{300\,\Omega}{5\,\Omega} = 60$ Then $BW = \frac{f_s}{Q_s} = \frac{12{,}000\,Hz}{60} = \boxed{200\,Hz}$

b.) Find the cutoff frequencies.

Solution: Since $Q_s \geq 10$:

$$f_1 = f_s + \frac{BW}{2} = 12{,}000\,Hz + 100\,Hz = \boxed{12{,}100\,Hz}$$

$$f_2 = f_s - \frac{BW}{2} = 12{,}000 - 100 = \boxed{11{,}900\,Hz}$$

ex/ A series RLC circuit is designed to resonate at $\omega_s = 10^5$ rad/s, have a BW = $0.15\,\omega_s$ and draw 16 W. from a 120 V. source at resonance.

a.) Determine the value of R

solution: $P = \frac{E^2}{R}$ so, $R = \frac{E^2}{P} = \frac{(120\,V.)^2}{16\,W} = \boxed{900\,\Omega}$

b.) Find BW in Hertz.

solution: $f_s = \frac{\omega_s}{2\pi} = \frac{10^5\,rad/s}{2\pi} = 15{,}915\ Hz$

$BW = 0.15 f_s = 0.15(15{,}915) = \boxed{2387\ Hz.}$

c.) Find the values of L and C.

solution: $BW = \frac{R}{2\pi L}$ so, $L = \frac{R}{2\pi BW} = \frac{900\,\Omega}{2\pi(2387\,Hz.)} = \boxed{60\ mH.}$

Then $f_s = \frac{1}{2\pi\sqrt{LC}}$ so, $C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2(15{,}915)^2(60\times10^{-3})} = \boxed{1.67\,nF}$

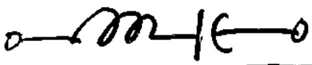
d.) What is the Q_s of the circuit?

solution: $Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15{,}915)(60\,mH)}{900\,\Omega} = \boxed{6.67}$

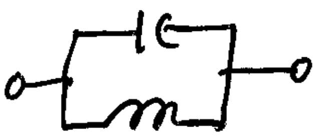
e.) Determine the fractional BW.

solution: $\frac{f_2 - f_1}{f_s} = \frac{BW}{f_s} = \frac{1}{Q_s} = \frac{1}{6.67} = \boxed{0.15}$

L-C Filters



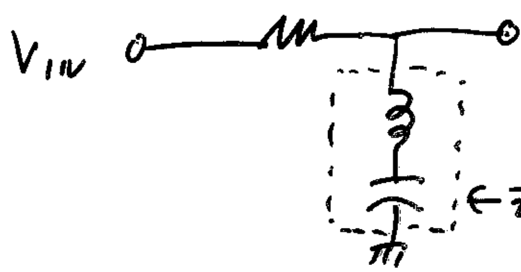
The series LC ckt. looks like a short circuit (Minimum Z) at resonance.



← Also called a "Tank Circuit"

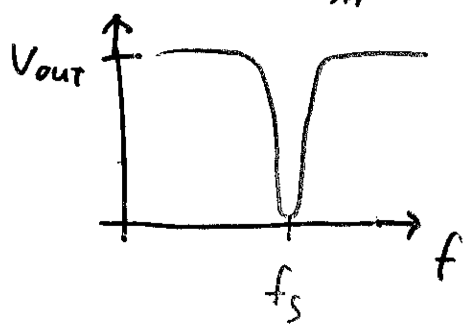
The parallel LC ckt. looks like an open circuit (Maximum Z) at resonance.

ex/



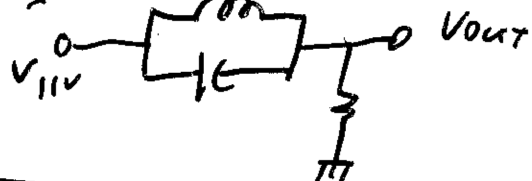
$$V_{OUT} = V_{IN}\left(\frac{\bar{Z}_T}{R+\bar{Z}_T}\right)$$

when $\bar{Z}_T = 0$, $V_{OUT} = 0$

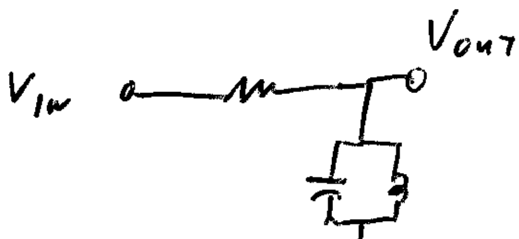


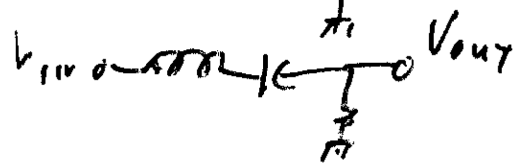
called a "Band Stop" or Notch Filter

OR

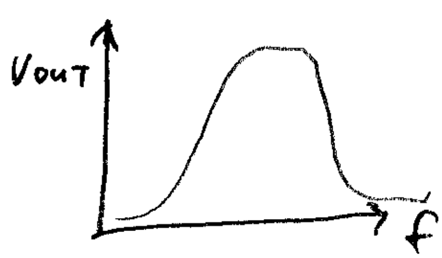


ex/

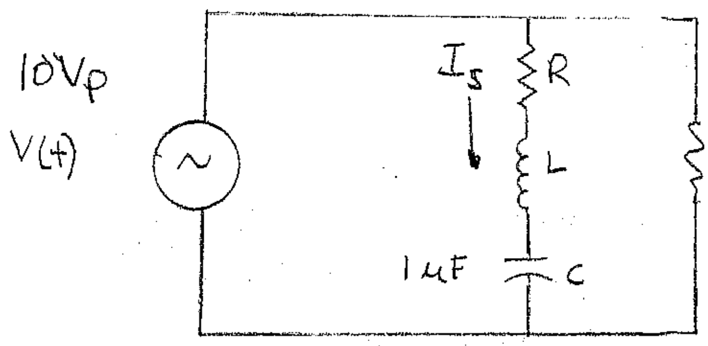




"Band Pass" Filters



Quality Factor
Series Resonant Circuits



$V(t) = 10\sin(2\pi f t)$

$R_L = 300\,\Omega$

$Q_s = 5 \quad f_s = \frac{1}{2\pi\sqrt{LC}}$

Set $f_s = 100$ Hz find L and R

$\frac{1}{2\pi f_s} = \sqrt{LC} \quad \left(\frac{1}{2\pi f_s}\right)^2 = LC \quad \left(\frac{1}{2\pi f_s}\right)^2 \frac{1}{C} = L \quad$ L (Henrey) C (Farards)

$\left(\frac{1}{2\pi(100\,Hz)}\right)^2 \frac{1}{1\times10^{-6}} = L \Rightarrow L = 2.54\,H$

For $Q_s = 5 \quad Q_s = \frac{1}{R}\sqrt{\frac{L}{C}} \Rightarrow R = \frac{1}{Q_s}\sqrt{\frac{L}{C}} \quad C = 1\times10^{-6}F \quad L = 2.54\,H$

$R = \frac{1}{5}\sqrt{\frac{2.54}{1\times10^{-6}}} = 318.5\,\Omega$ INCREASE Q_s to 10, 20, 40, 80

Keep f_s at 100 Hz Recompute R for new values of Q.

$R_{10} = \frac{1}{10}\sqrt{\frac{2.54}{1\times10^{-6}}} = 159.4\,\Omega \quad R_{20} = \frac{1}{20}\sqrt{\frac{2.54\,H}{1\times10^{-6}F}} = 79.7\,\Omega$

$R_{40} = \frac{1}{40}\sqrt{\frac{2.54\,H}{1\times10^{-6}F}} = 39.9\,\Omega \quad R_{80} = \frac{1}{80}\sqrt{\frac{2.54\,H}{1\times10^{-6}F}} = 20\,\Omega$

Compute I_s when $V(t) = 10\sin(2\pi 60 t)$ for each Series Resonant CIRCUIT

$Q = 5 \quad \frac{1}{R_L} = \frac{1}{300} = 0.0033\,℧$

$Z_s = 318.5 + j\,2\pi 60(2.54\,H) + \frac{1}{2\pi 60\, 1\times10^{-6}\, j}$

$Z_s = 318.5 + j\,958\,\Omega - j\,2653\,\Omega$

Ex

$Z_5 = 318.5 - 1695j = 1725\angle -79.1^\circ$

$Y_5 = 5.797 \times 10^{-4} \angle 79.1^\circ$

Z_{10} $159.4 - 1695j$ REACTIVE components do not CHANGE

$Z_{10} = 1702.5\angle -84.6^\circ$

$Y_{10} = 5.5 \times 10^{-4} \angle 84.6^\circ$

$Z_{20} = 79.7 - 1695j = 1697\angle -87.3^\circ$

$Y_{20} = 5.893 \times 10^{-4} \angle 87.3^\circ$

$Z_{40} = 39.9 - 1695j$

$Z_{40} = 1696\angle -88.6^\circ$ $\quad Y_{40} = 5.9 \times 10^{-4} \angle 88.6^\circ$

$Z_{80} = 20 - 1695j = 1695.1\angle -89.3^\circ$ $\quad Y_{80} = 5.99 \times 10^{-4} \angle 89.3^\circ$

Find Total Source Current

$I_s = V_s Y_T$ $\quad Y_{T5} = 0.0033\angle 0^\circ + 5.797 \times 10^{-4} \angle 79.1^\circ$

$V_{s\,Rms} = 7.07\angle 0^\circ$ $\quad Y_{T5} = 0.00346\angle 9.5^\circ$

$I_T = 7.07\angle 0^\circ (0.00346\angle 9.5^\circ) = 0.0244\ A\angle 9.5^\circ$

$Q_s = 5$ $\quad \bar{I}_s = \dfrac{\bar{Y}_5}{\bar{Y}_{T5}} \bar{I}_T$ $\quad I_s = \dfrac{5.797 \times 10^{-4} \angle 79.1^\circ}{0.00346\angle 9.5^\circ} 0.0244\angle 9.5^\circ$

$\underline{I_s = 0.0041\ A\angle 79.1^\circ}$

$Q_s = 10$ $\quad Y_{T10} = 0.0033\angle 0^\circ + 5.5 \times 10^{-4} \angle -84.6^\circ$

$Y_{T10} = 0.00334\angle 9.3^\circ$

$I_T = 7.07\angle 0^\circ (0.00334\angle 9.3^\circ) = 0.024\angle 9.3^\circ\ A$

Ex

$$I_s = \frac{Y_{10}}{Y_{T10}} I_T \qquad I_s = \frac{5.5\times10^{-4}\angle 84.6^\circ}{0.00334\angle 9.3^\circ}\, 0.024\angle 9.3^\circ$$

$$I_s = 0.00395\angle 84.6^\circ$$

$Q_s = 20$ $\quad Y_{T20} = 0.0033\angle 0^\circ + 5.89\times10^{-4}\angle +87.3^\circ = 0.00338\angle 10.03^\circ$

$Q_s = 40$ $\quad Y_{T40} = 0.0033\angle 0^\circ + 5.9\times10^{-4}\angle 88.6^\circ = 0.00337\angle 10.1^\circ$

$Q_s = 80$ $\quad Y_{T80} = 0.0033\angle 0^\circ + 5.99\times10^{-4}\angle 89.3^\circ = 0.00336\angle 10.3^\circ$

$Q_s = 20$ $\quad I_{T20} = 7.07\angle 0^\circ (0.00338\angle 10.03^\circ) = 0.0238\angle 10.03^\circ\ A$

$Q_s = 40$ $\quad I_{T40} = 7.07\angle 0^\circ (0.00337\angle 10.1^\circ) = 0.0238\angle 10.1^\circ$

$Q_s = 80$ $\quad I_{T80} = 7.07\angle 0^\circ (0.00336\angle 10.3^\circ) = 0.02375\angle 10.3^\circ$

$Q_s = 20$ $\quad I_s = \dfrac{Y_{20}}{Y_{T20}} I_{T20} = \dfrac{5.893\times10^{-4}\angle 87.3}{0.00338\angle 10.03^\circ}\, 0.0238\angle 10.03^\circ$

$$I_s = 0.0041\angle 87.3^\circ\ A$$

$Q_s = 40$ $\quad I_s = \dfrac{Y_{40}}{Y_{T40}} I_{T40} \quad \dfrac{5.9\times10^{-4}\angle 88.6^\circ}{0.00337\angle 10.1^\circ}(0.0238\angle 10.1^\circ)$

$$I_s = 0.0042\angle 88.6^\circ\ A$$

$Q_s = 80$ $\quad I_s = 0.004\angle 89.3^\circ\ A$

RESONANT CIRCUIT APPLICATIONS

SHUNT FILTERS FOR HARMONIC CANCELLATION

HARMONIC - INTEGER MULTIPLE OF BASE Frequency

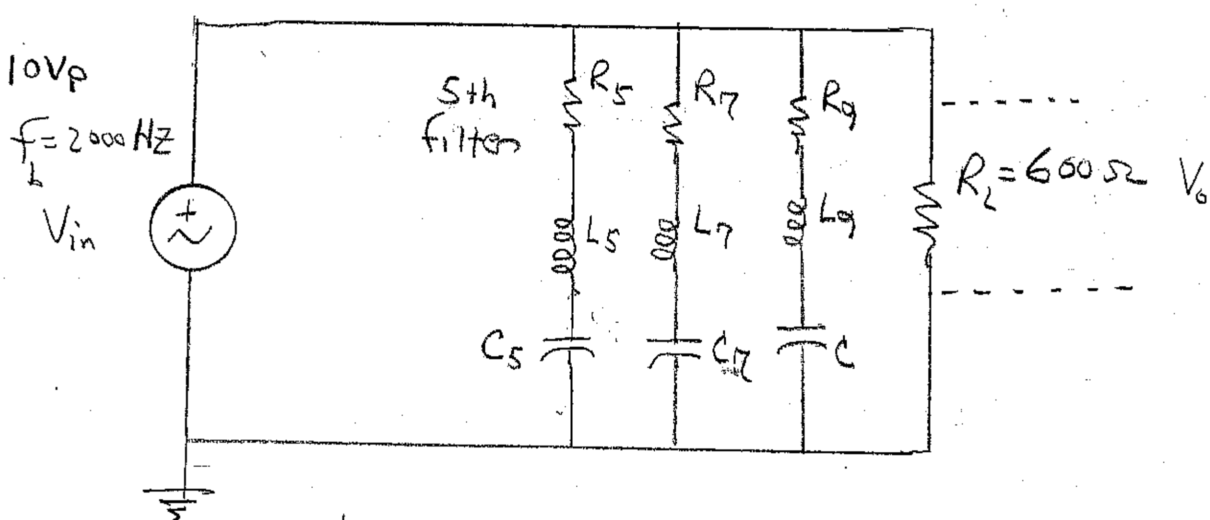
Ex.) base frequency $100\ Hz = f_b$

1st harmonic	100 Hz	$1 \times f_b$	odd harmonic
2nd harmonic	200 Hz	$2 \times f_b$	even harmonic
3rd harmonic	300 Hz	$3 \times f_b$	
⋮			
nth harmonic	100 n Hz	$n \times f_b$	

Harmonics occur in power systems & communications Circuits

Active Circuits - Transistors produce harmonics

Ex.) Design a set of series resonent circuits to cancel the 5th 7th and 9th harmonic of a sinusoidal tone of 2000 Hz for the circuit below.



Filter Q = 20

Ex cont. Inductors available have a value of 100 mH with a negligable winding Resistance

Size the other components and find V_o when V_{in} is $10V_p$ at 2000 Hz

Size Components for 5th harmonic

$f_5 = 5 \times f_1 = 5(2000\,Hz) = 10{,}000\,Hz$

At Resonance filter should act as a short to 10 kHz currents.

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad (1) \qquad Q_s = \frac{1}{R}\sqrt{\frac{L}{C}} \quad (2)$$

100 mH = L_5 only L available

Solve (1) for C

$$f_s 2\pi\sqrt{LC} = 1$$

$$\sqrt{L_5C_5} = \frac{1}{2\pi f_s}$$

$$L_5C_5 = \left(\frac{1}{2\pi f_s}\right)^2$$

$$C_5 = \left(\frac{1}{2\pi f_s}\right)^2 \frac{1}{L_5}$$

$$C_5 = \left[\frac{1}{2\pi(10000)}\right]^2 \frac{1}{100 \times 10^3\,H}$$

$$C_5 = 2.54 \times 10^{-9}\,F$$

Solve (2) for R

$$Q_s = \frac{1}{R_5}\sqrt{\frac{L_5}{C_5}}$$

$$R_5 Q_s = \sqrt{\frac{L_5}{C_5}}$$

$$R_5 = \frac{1}{Q_s}\sqrt{\frac{L_5}{C_5}} \qquad Q_s = 20 \text{ for All filters}$$

$$R_5 = \frac{1}{20}\sqrt{\frac{100 \times 10^{-3}}{2.54 \times 10^{-9}}}$$

$$R_5 = 313.7\,\Omega$$

for 5th harmonic

$L_5 = 100\,mH \quad C_5 = 0.0025\,uF \quad R_5 = 313.7\,\Omega$

Ex Cont.

Size Components For 7th Harmonic

$f_s = 7f_b = 7(2000\text{ Hz}) = 14{,}000\text{ Hz}$

At Resonance filter will be short to 14 KHz currents

$L_7 = 100\text{ mH}$ only L available

$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

Solve for C

$$C_7 = \left[\frac{1}{2\pi f_s}\right]^2 \frac{1}{L_7} = \left[\frac{1}{2\pi(14{,}000)}\right]^2 \frac{1}{100\times10^{-3}\text{H}}$$

$$C_7 = \frac{1.294\times10^{-10}}{100\times10^{-3}} = 1.294\times10^{-9}\text{F}$$

$$R_7 = \frac{1}{Q_s}\sqrt{\frac{L_7}{C_7}} = \frac{1}{20}\sqrt{\frac{100\times10^{-3}\text{H}}{1.294\times10^{-9}\text{F}}} = 440\ \Omega$$

for 7th Harmonic filter

$L_7 = 100\text{ mH}$ $C_7 = 0.0013\ \mu\text{F}$ $R_7 = 440\ \Omega$

Size Components for 9th Harmonic

$f_s = 9f_b = 9(2000\text{ Hz}) = 18{,}000\text{ Hz}$

At Resonance filter will short 18 kHz current

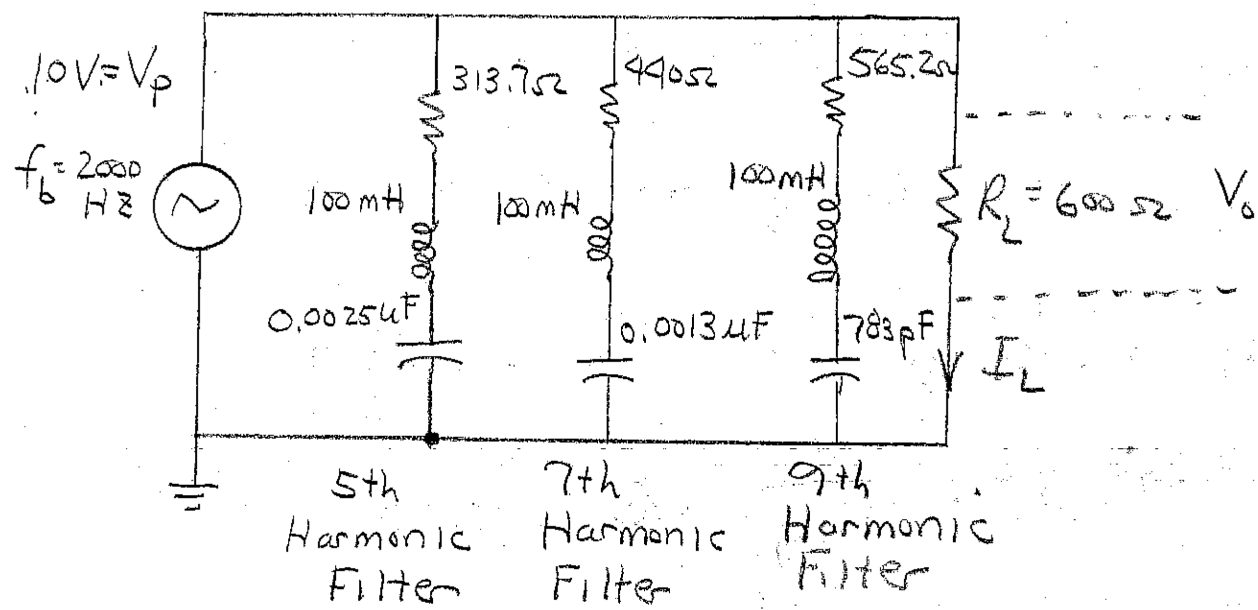
$$C_9 = \left[\frac{1}{2\pi f_s}\right]^2 \frac{1}{L_9} = \left[\frac{1}{2\pi(18\times10^3)}\right]^2 \frac{1}{100\times10^{-3}} = 7.83\times10^{-10}\text{ F}$$

$$R_9 = \frac{1}{20}\sqrt{\frac{100\times10^{-3}\text{H}}{7.83\times10^{-10}\text{F}}} = 565.2\ \Omega$$

Ex cont | Components for 9th Harmonic

$L_9 = 100\ mH \quad C_9 = 783\ pF \quad R = 565.2\Omega$

CIRCUIT WITH VALUES



Find CURRENT I_L (Rms) when $f = f_b$

$V(t) = 10\sin(2\pi\, 2000 t)$ to Phasor $\quad V_{Rms} = \frac{10}{\sqrt{2}} = 7.07$

$V_{in} = 7.07\angle 0^\circ \qquad f_b = 2000\ Hz$

$Z_5 = R_5 + 2\pi f L_5 j + \frac{1}{2\pi f C_5 j} \qquad R_5 = 313.7\Omega$

$X_{L5} = 2\pi(2000)(100\times 10^{-3})j = 1256j\ \Omega$

$X_{C5} = \frac{1}{2\pi\, 2000\,(0.0025\times 10^{-6})j} = -31847j\ \Omega$

$Z_5 = 313.7 + 1256j - 31847j = 313.7 - 30591j\ \Omega$

$Y_5 = \frac{1}{Z_5} = \frac{1}{313.7 - 30591j} = 3.351\times 10^{-7} + 3.269\times 10^{-5}j\ \mho$

Ex cont.

$$Z_7 = R_7 + 2\pi f_b L_7 j + \frac{1}{2\pi f_b C_7 j} \qquad R_7 = 440\,\Omega$$

$$X_{L7} = 2\pi(2000)\,100\times10^{-3} = 1256j\ \Omega$$

$$X_{C7} = \frac{1}{2\pi(2000)(0.0013\times10^{-6})j} = -61244j\ \Omega$$

$$Z_7 = 440\,\Omega + 1256j - 61244j = 440 - 59988j\ \Omega$$

$$Y_7 = \frac{1}{Z_7} = \frac{1}{440 - 59988j} = 1.223\times10^{-7} + 1.667\times10^{-5}j\ \mho$$

$$Z_9 = R_9 + 2\pi f_b L_9 j + \frac{1}{2\pi f_b C_9 j} \qquad R_9 = 565.2\,\Omega$$

$$X_{L9} = 2\pi(2000)\,100\times10^{-3} = 1256j$$

$$X_{C9} = \frac{1}{2\pi(2000)(783\times10^{-12})j} = -101683j$$

$$Z_9 = 565.2 + 1256j - 101683j = 565.2 - 100{,}427j$$

$$Y_9 = 5.604\times10^{-8} + j9.957\times10^{-6}\ \mho$$

$$Y_L = \frac{1}{R_L} = \frac{1}{600\,\Omega} = 0.001667\ \mho$$

$$I_s = \frac{V_s}{Z_T} = V_s\frac{1}{Z_T} = V_s Y_T$$

$$Y_T = Y_5 + Y_7 + Y_9 + Y_L$$

Ex Cont.

$$Y_T = (3.351\times10^{-7} + 3.269\times10^{-5}j) + (1.223\times10^{-7} + 1.667\times10^{-5}j) + \ldots$$

$$\ldots + (5.604\times10^{-8} + 9.957\times10^{-6}j) + (0.001667 + j0)$$

$$Y_T = 0.0016695 + 5.9317\times10^{-5}j = 0.001669\angle 2.04^\circ$$

$$I_S = V_S Y_T = (7.07\angle 0^\circ)(0.001669\angle 0.035^r) = 0.0118\,A\angle 2.04^\circ$$

Find I_L by current division using admittances

Y_f = sum of all filter branches

Y_L = admittance of load

$$I_L = \frac{Y_L}{Y_L + Y_f} I_S \Rightarrow I_L = \frac{0.001667\angle 0^\circ}{0.001669\angle 2.04^\circ}\, 0.0118A\angle 2.04^\circ$$

$$Y_L + Y_f = Y_T \qquad I_L = 0.9988\angle -2.04\,(0.0118A\angle 2.04^\circ)$$

$$I_L = 0.01178\,A\angle 0^\circ \text{ Through Load}$$

All Filter Branches Are High Impedance At Frequency of interest. Almost all current Flows through load R.

CREATE IMPEDANCE PLOT WITH MATHCAD

Plot the total impedance of the circuit as a function of frequency

$N := 5$ Plot the values to 100 kHz

$i := 60$ This is the number of points that are generated in each decade

$k := 140 .. i \cdot N$ This is the index for the frequency variable.

Generate the points

$f_k := 10^{k \cdot \frac{1}{i}}$ Define the filter values and compute the impedances as a function of frequency

$R_L := 600 \quad R_5 := 314 \quad R_7 := 440 \quad R_9 := 565$

$L_5 := 100 \cdot 10^{-3} \quad L_7 := L_5 \quad L_9 := L_5$

$C_5 := 0.0025 \cdot 10^{-6} \quad C_7 := 0.0013 \cdot 10^{-6} \quad C_9 := 783 \cdot 10^{-12}$

$$Z_5(f) := R_5 + 2 \cdot \pi \cdot f \cdot L_5 \cdot j + \frac{1}{2 \cdot \pi \cdot f \cdot C_5 \cdot j}$$ Fifth harmonic filter impedance

$$Z_7(f) := R_7 + 2 \cdot \pi \cdot f \cdot L_7 \cdot j + \frac{1}{2 \cdot \pi \cdot f \cdot C_7 \cdot j}$$ Seventh harmonic filter impedance

$$Z_9(f) := R_9 + 2 \cdot \pi \cdot f \cdot L_9 \cdot j + \frac{1}{2 \cdot \pi \cdot f \cdot C_9 \cdot j}$$ Ninth harmonic filter impedance

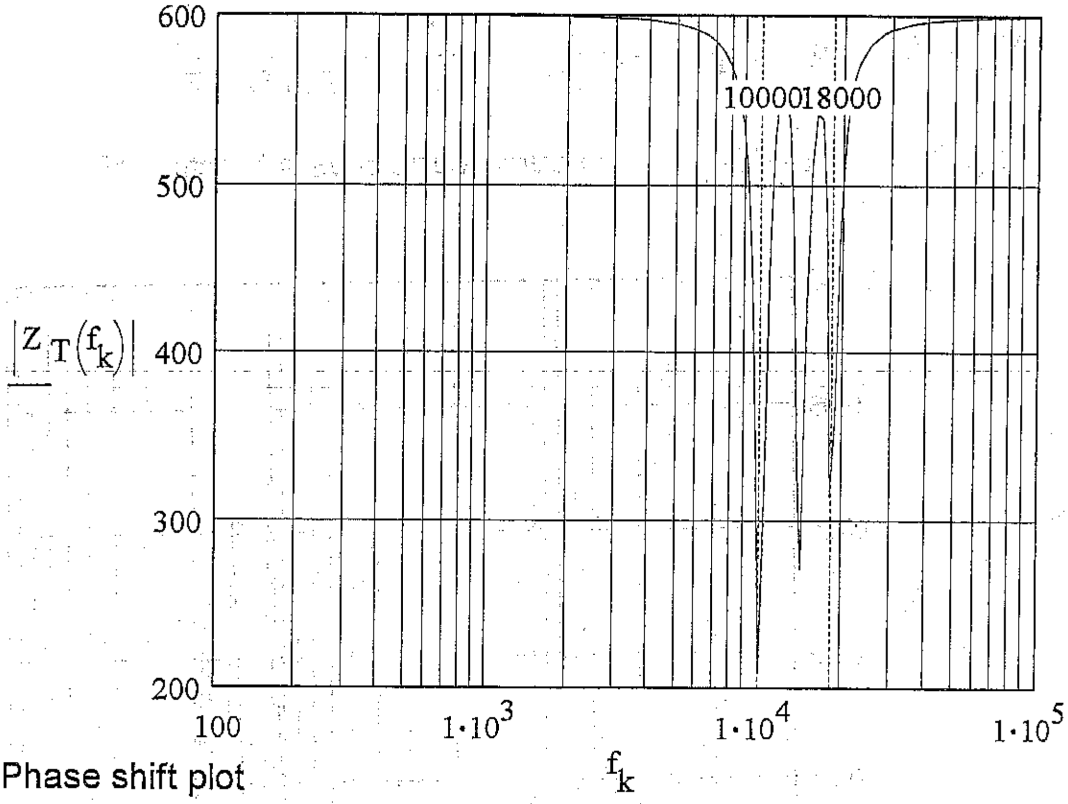
$Z_L := R_L$ Load resistance

Total impedance of the network including the load resistance

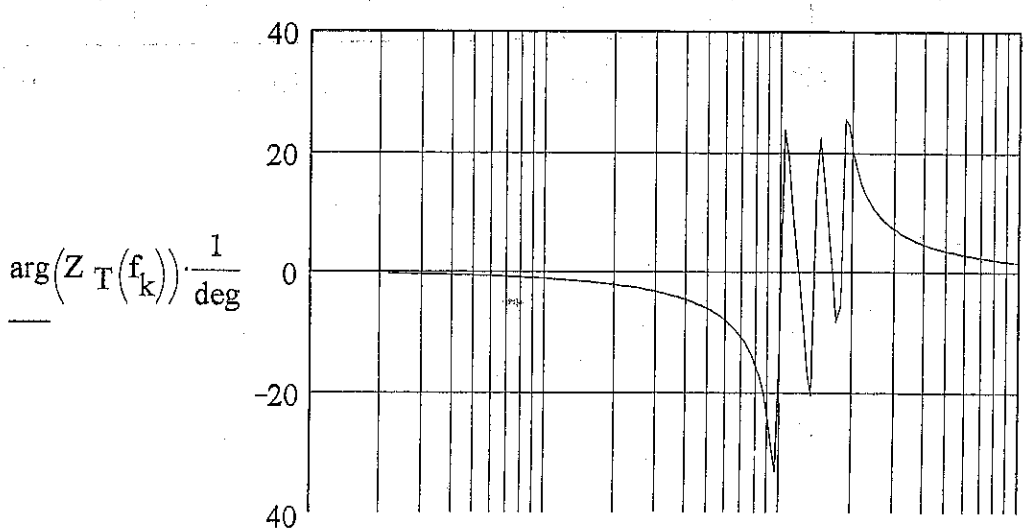
$$Z_T(f) := \frac{1}{\left(\frac{1}{Z_5(f)} + \frac{1}{Z_7(f)} + \frac{1}{Z_9(f)} + \frac{1}{Z_L}\right)}$$

Plot the magnitude and phase angle of the total impedance as the frequency changes from 1 Hz to 100 kHz frequency changes from 1 Hz to 100 kHz

Impedance plot for the filter circuit notice that the individual filters interact and reduce the impedance at frequencies beyond the resonance frequency.



Phase shift plot



−40

100 $1 \cdot 10^3$ $1 \cdot 10^4$ $1 \cdot 10^5$

f_k

Find the total current as a function of frequency

$$V_s := 7.07 + 0j \quad Y_T(f) := \frac{1}{Z_5(f)} + \frac{1}{Z_7(f)} + \frac{1}{Z_9(f)} + \frac{1}{Z_L}$$

$I_s(f) := V_s \cdot Y_T(f)$ The source current as a function of f