PART 1b

Logarithms

If $X = \log_{10} N$,

then "$\log_{10} N$" really means "What power does 10 need to be "raised to" to equal N?"

ex/ $\log_{10} 100 = 2$

ex/ $\log_{10} 1000 = 3$

ex/ $\log_{10} 10 = 1$

ex/ $\log_{10} 50 = 1.699$ ex/ $\log_{10} 1 = 0$

In the above examples 10 is the "base" of the log.

In general, $\boxed{\text{If } N = (b)^X, \text{ then } X = \log_b N}$

The base of natural logs is $e = 2.7183$.

The base of common logs is 10.

Notation:

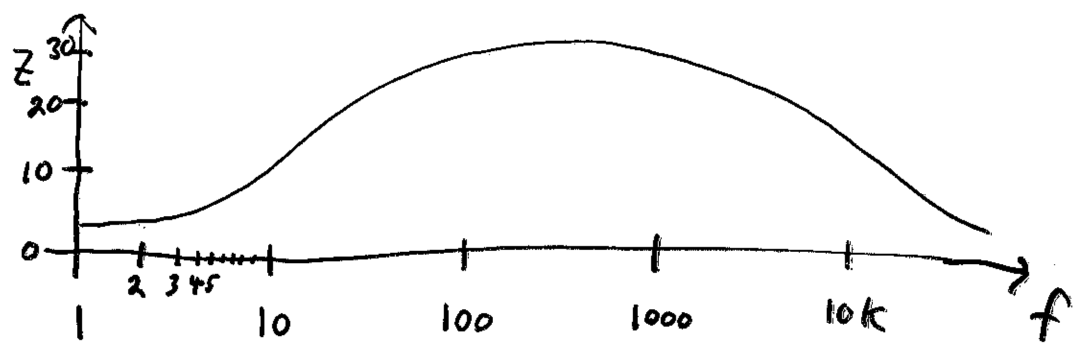
log ← means base 10 (if no subscript)

ln ← means base e (natural logs)

$$\boxed{\ln X = \log_e X = 2.3 \log_{10} X}$$

Graphs having frequency on the horizontal axis use a log type scale.

ex/



This allows low frequency rolloff to be seen on the same graph with high frequency rolloff.

If the vertical scale is linear, then the graph paper is called <u>semi-log paper</u>

Properties of Logarithms

$$\boxed{\log_{10} 1 = 0}$$

If $X < 1$, then $\log_{10} X$ is negative

ex/ $\log_{10} 0.5 = -0.3$

$$\boxed{\log_{10} ab = \log_{10} a + \log_{10} b}$$

$$\boxed{\log_{10} \frac{a}{b} = \log_{10} a - \log_{10} b}$$

$$\boxed{\log_{10} a^n = n \log_{10} a}$$

Decibels

Two levels of power can be compared using units of bels defined as:

$$B = \log_{10} \frac{P_2}{P_1} \qquad \text{(bels)}$$

A more commonly used unit is the decibel

$$1 \text{ bel} = 10 \text{ decibels} \qquad (dB)$$

The power ratio or Power Gain in dB is given by:

$$\boxed{db = 10 \log_{10} \frac{P_2}{P_1}}$$

ex/ If the power is doubled, i.e. $P_2 = 2P_1$

then $db = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} 2 = \boxed{3\ db}$

In communication systems a reference power level is defined as:

$$P_{ref} = 1 mW. \quad (\text{across a } 600\Omega \text{ load})$$

OR $\boxed{dB_m = 10 \log_{10} \frac{P}{1mW}\Big|_{600\Omega}}$

ex/ A power level of $P = 40 mW$ can be specified as: $dB_m = 10 \log_{10} \frac{40 mW}{1mW} = 10(1.6) = \boxed{16\ dB_m}$

Voltage Gain

If $P_2 = \dfrac{V_2^2}{R_2}$ and $P_1 = \dfrac{V_1^2}{R_1}$

then the gain in $dB = \log_{10} \dfrac{P_2}{P_1} = \log_{10} \dfrac{\frac{V_2^2}{R_2}}{\frac{V_1^2}{R_1}}$

$$dB = 10 \log_{10} \left(\frac{V_2}{V_1}\right)^2 - 10 \log_{10} \left(\frac{R_2}{R_1}\right)$$

$$dB = 20 \log_{10} \frac{V_2}{V_1} - 10 \log_{10} \frac{R_2}{R_1}$$

If $R_2 = R_1$, $\log_{10} 1 = 0$ Then,

$$\boxed{dB_v = 20 \log_{10} \frac{V_2}{V_1}} \quad (dB)$$

ex/ A 2 mV signal is applied to the input of an amplifier circuit and a output voltage of 1.2 V, results, what is the voltage gain of the amplifier?

solution:

$$dB_v = 20 \log_{10} \frac{V_o}{V_i} = 20 \log_{10} \frac{1.2V}{2mV}$$

$$dB_v = 20 \log_{10} 600 = \boxed{55.56 \ dB}$$

so $A_v = \dfrac{V_o}{V_i} = 600$ or $55.56 \ dB$

ex/ If $A_v = 100{,}000$, what is the gain in dB?

Solution: $dB = 20 \log_{10} 100{,}000 = 20(5) = \boxed{100 \ dB}$

ex) If a system has a voltage gain of 36 dB find the applied voltage if the output voltage is 6.8 V.

Solution:

$$dB_v = 20 \log_{10} \frac{V_o}{V_i}$$

so

$$36 = 20 \log_{10} \frac{V_o}{V_i}$$

solve for $\frac{V_o}{V_i}$:

$$1.8 = \log_{10} \frac{V_o}{V_i}$$

Take the antilog:

$$\log_{10}^{-1} 1.8 = \frac{V_o}{V_i}$$

$$\frac{V_o}{V_i} = 10^{1.8} = 63.096$$

Then $$V_i = \frac{V_o}{63.096} = \frac{6.8}{63.096} = \boxed{107.77 \text{ mV.}}$$

The Human Auditory Response

The human ear responds in a logarithmic fashion to changes in audio power levels.

To double the sound level received by the human ear, the power output of the acoustical source (in watts) must be increased by a factor of 10.

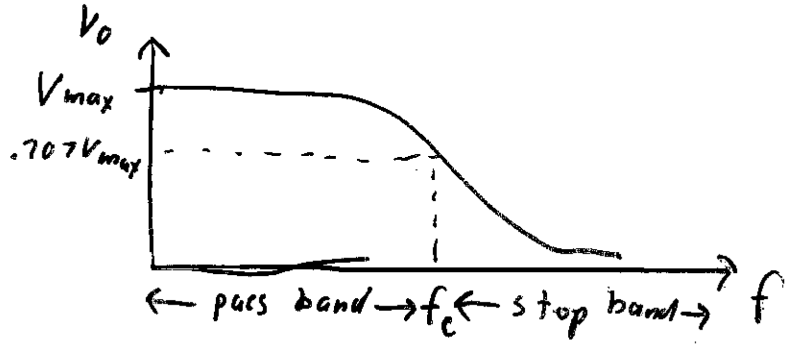
Filters

A filter uses RLC elements to pass a desired range (or band) of frequencies and reject unwanted frequency components.

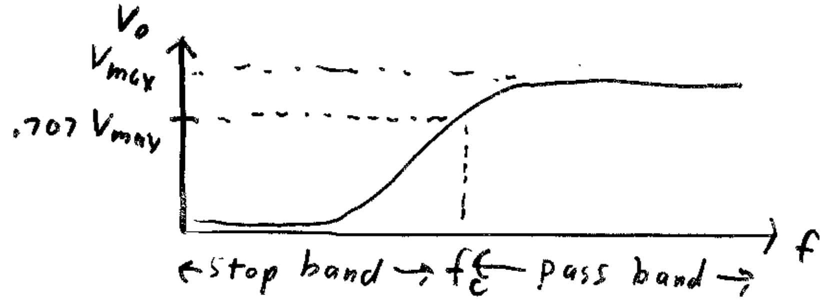
Passive filters consist of RLC elements.

Active filters employ transistors or operational amplifiers in combination with RLC elements.

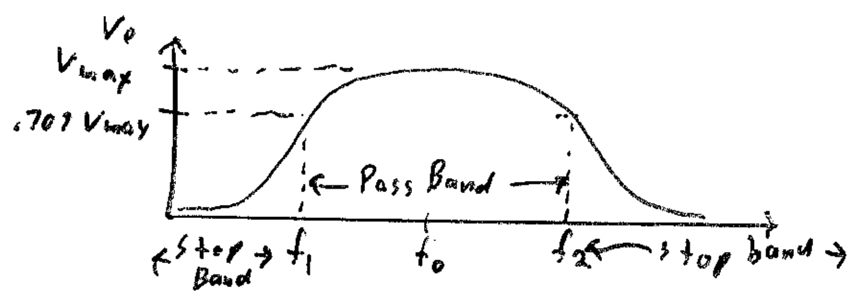
Low-pass Filter



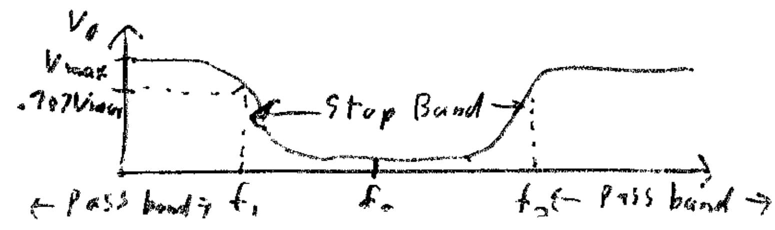
High-pass Filter



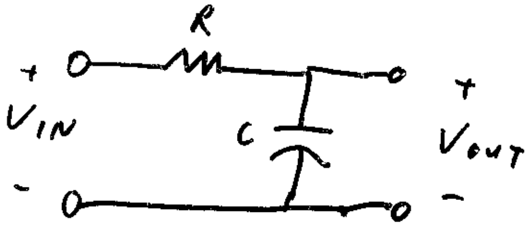
Band-pass filter



Band-stop or notch filter

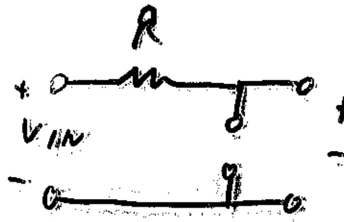


R-C Low Pass Filter



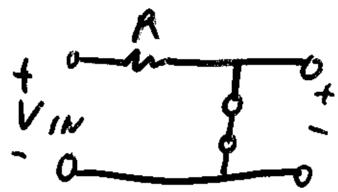
$$X_C = \frac{1}{2\pi f C}$$

At low freq. C looks like an open ckt. $\Rightarrow$



$V_O \simeq V_{IN}$

At high freq. C looks like a short ckt. $\Rightarrow$



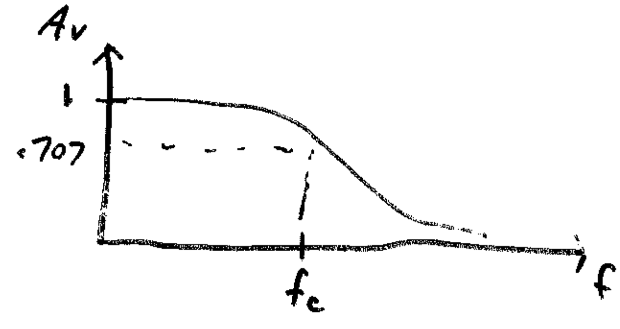
$V_o \approx 0$

By voltage divider rule:

$$V_o = V_{IN}\left(\frac{\overline{Z}_c}{\overline{Z}_R + \overline{Z}_c}\right) = V_{IN}\,\frac{X_c\angle -90^\circ}{R - jX_c}$$

Voltage Gain:
$$A_v = \frac{V_o}{V_{IN}} = \frac{X_c \angle -90^\circ}{\sqrt{R^2 + X_c^2}\;\angle -\tan^{-1}(X_c/R)}$$

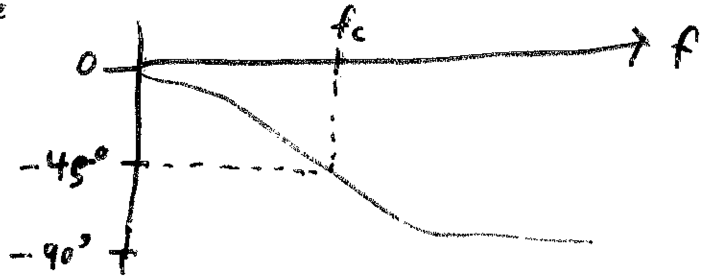
Magnitude plot



At f_c: $R = X_c$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

Phase angle plot A_v



V_o lags V_i

For R-C Low Pass Filter:

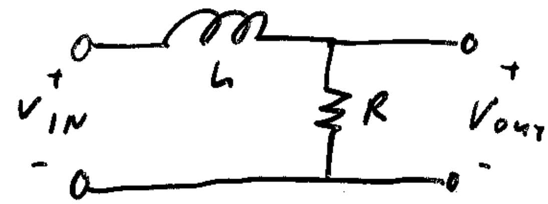
$$f_c = \frac{1}{2\pi RC}$$

For $f < f_c$ $\quad V_o > .707 V_{IN}$

For $f > f_c$ $\quad V_o < .707 V_{IN}$

At f_c $\quad \bar{V}_o$ lags $\bar{V}_{IN}$ by 45°

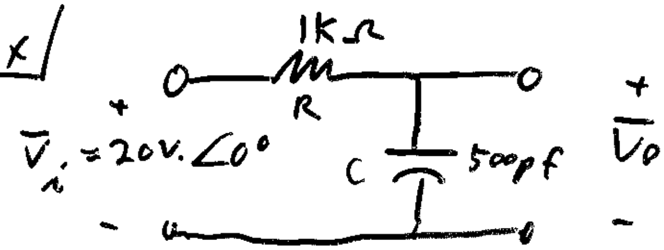
R - L Low Pass Filter



At f_c : $X_L = R$

$$f_c = \frac{R}{2\pi L}$$

ex/

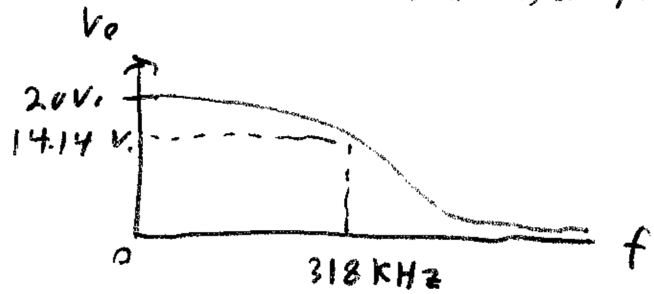


a.) Sketch V_o vs. freq.

Solution:

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1k)(500pf)} = 318 \text{ KHz}$$

$$(.707)(20) = 14.14V.$$



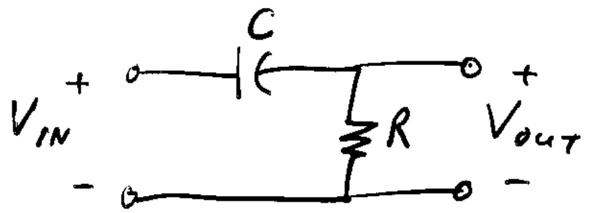
b.) Find V_o at 100 KHz and 1 MHz.

Solution: At 100 KHz: $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100k)(500pf)} = 3.18K$

$$V_o = V_i \frac{X_C}{\sqrt{X_C^2 + R^2}} = \boxed{19.08V.}$$

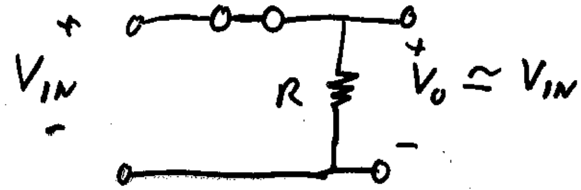
similarly at 1MHz: $V_o = \boxed{6.1V.}$

R-C High Pass Filter

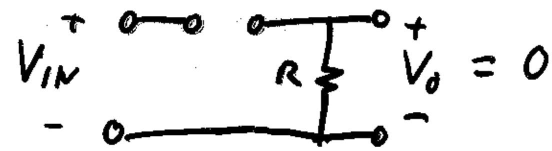


$$X_c = \frac{1}{2\pi f C}$$

At high freq. C looks like a short ckt. $\Rightarrow$



At low freq. C looks like an open ckt. $\Rightarrow$



By voltage divider rule:

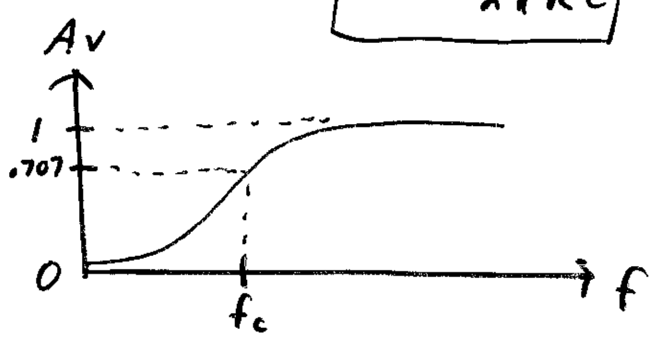
$$\bar{V}_o = \bar{V}_{IN}\left(\frac{\bar{Z}_R}{\bar{Z}_R+\bar{Z}_c}\right) = \bar{V}_{IN}\,\frac{R\angle 0^\circ}{R - jX_c}$$

Voltage Gain: $$A_v = \frac{V_o}{V_{IN}} = \frac{R\angle 0^\circ}{\sqrt{R^2+X^2}\ \angle -\tan^{-1}(X_c/R)}$$

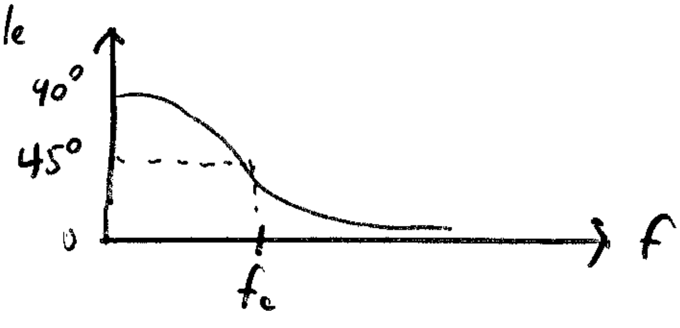
At f_c: $X_C = R$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

Magnitude plot



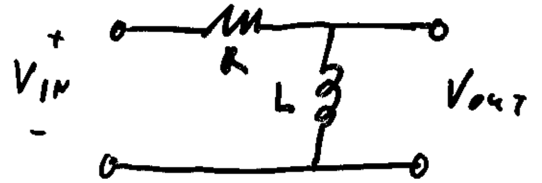
Phase angle plot



For R-C High Pass Filter:

$$\boxed{\begin{array}{lll} & f_c = \frac{1}{2\pi RC} & \\ \text{For} & f < f_c & V_o < .707\, V_{IN} \\ \text{For} & f > f_c & V_o > .707\, V_{IN} \\ \text{At} & f_c & \overline{V_o} \text{ leads } \overline{V_{IN}} \text{ by } 45^\circ \end{array}}$$

R-L High Pass Filter



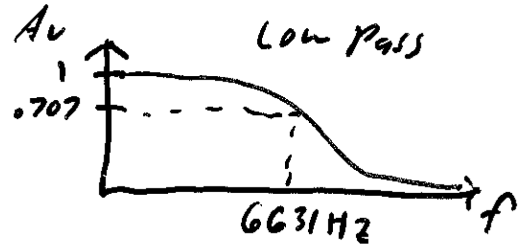
At f_c: $X_L = R$

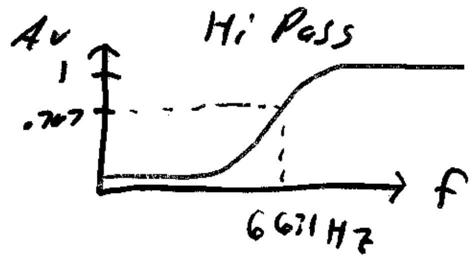
$$f_c = \frac{R}{2\pi L}$$

ex/ Given $R = 20\,k\Omega$ and $C = 1200\,pf$

a.) Sketch the normalized plot for a low pass and high pass RC filter

Solution: $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(20k)(1200pf)} = \boxed{6631\ Hz}$





b.) Determine the magnitude of A_v at $f = \frac{1}{2} f_c$

Solution: $f = \frac{1}{2} f_c = \frac{1}{2}(6631Hz) = 3315\,Hz$

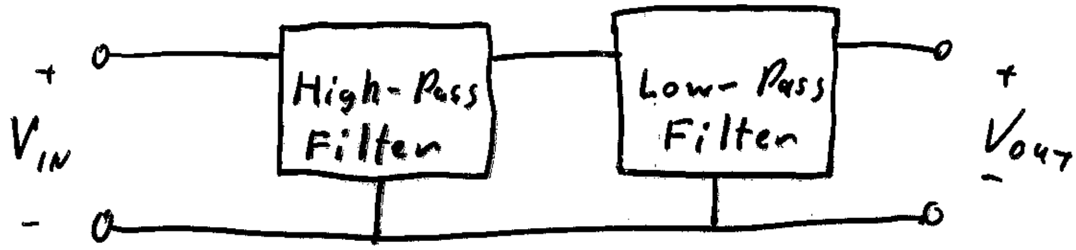
$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(3315)(1200pf)} = 40\,K\Omega$$

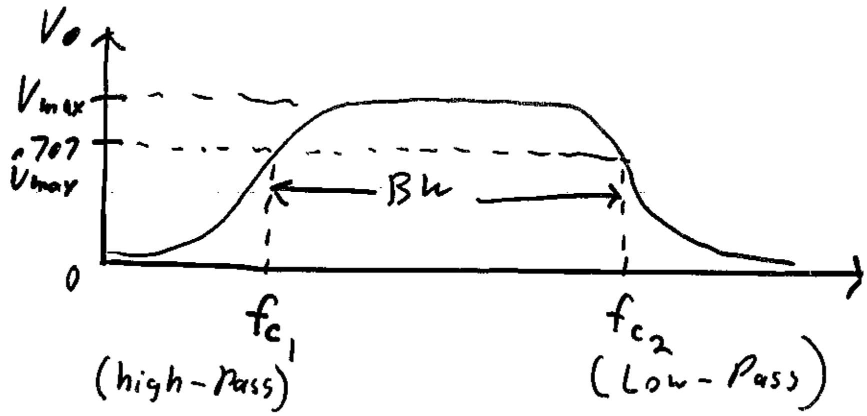
For the Low Pass: $A_v = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{40k}{\sqrt{20k^2 + 40k^2}} = \boxed{0.894}$

For the High Pass: $A_v = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{20k}{\sqrt{20k^2 + 40k^2}} = \boxed{0.447}$

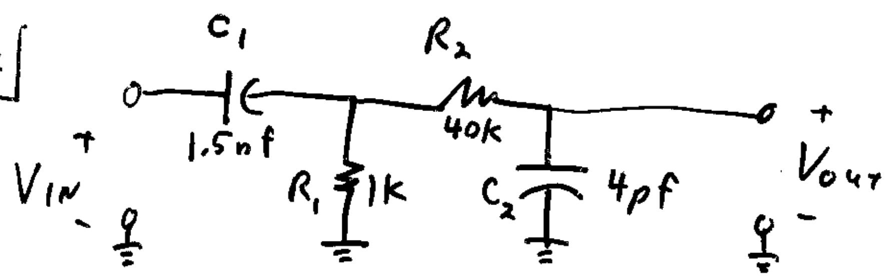
Band - Pass Filters

One method of making a Band-Pass filter is to cascade a high-pass filter with a low pass filter.





ex



Find the approximate cutoff freq.

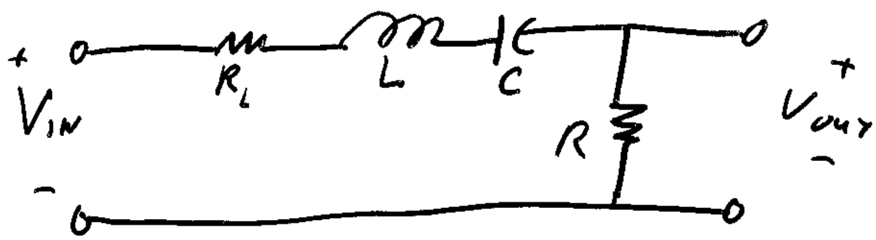
Solution:

$$f_{c_1} = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(1k)(1.5nF)} = \boxed{106\ KHZ}$$

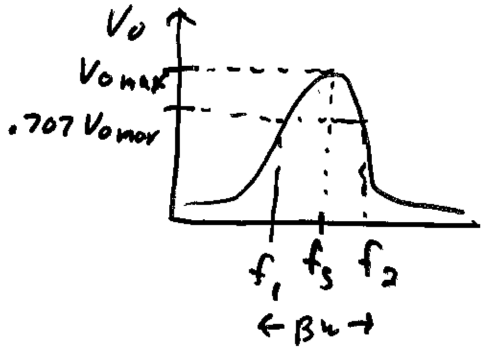
$$f_{c_2} = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(40k)(4pf)} = \boxed{994\ KHZ}$$

Note: The exact f_{c_1} will be slightly higher due to the impedance of R_2 and C_2 in parallel with R_1.

Series-Resonant Band Pass Filter



$$f_s = \frac{1}{2\pi\sqrt{LC}}$$



$$V_{o\,max} = \frac{R}{R+R_L}V_{IN}$$

$$Q_s = \frac{X_L}{R+R_L}$$

$$BW = \frac{f_s}{Q_s} = f_2 - f_1$$

ex/ Given $R_L = 2\Omega$, $L = 1mH$, $C = .01\mu f$, $R = 33\Omega$

a.) Determine the frequency response curve.

solution:

$$f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1mH)(.01\mu f)}} = \boxed{50{,}329\ Hz}$$

$$Q_s = \frac{X_L}{R+R_L} = \frac{2\pi(50{,}329)}{33\Omega + 2\Omega} = \boxed{9.04}$$

$$BW = \frac{f_s}{Q} = \frac{50{,}329\ Hz}{9.04} = 5.57\ KHz$$

b.) If $V_{IN} = 20mV$, determine V_{OUT} max.

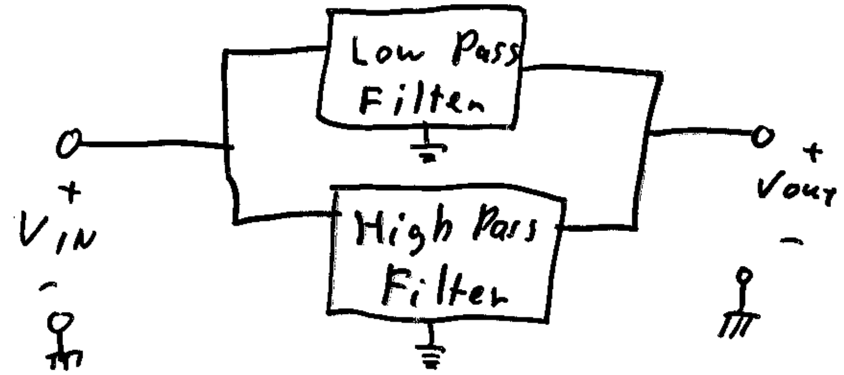
Solution:

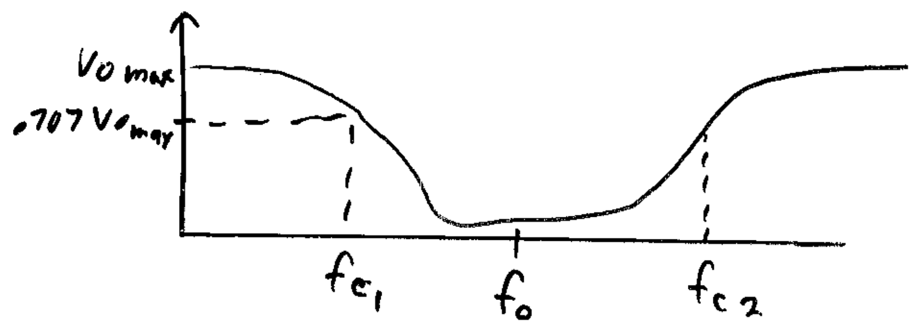
$$V_{o\,max} = \frac{R}{R+R_L}V_{IN} = \frac{33\Omega}{33\Omega+2\Omega}(20mV) = \boxed{18.86\ mV}$$

At the cutoff frequencies: $V_o = .707(18.86) = 13.34\,mV$

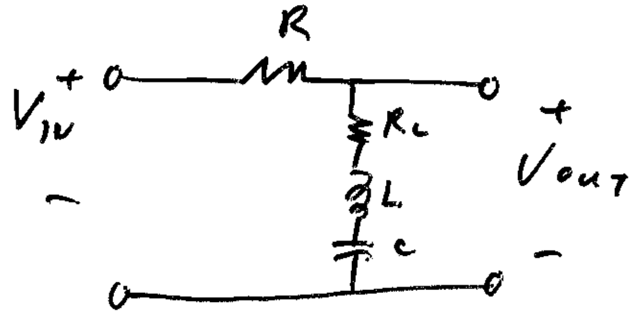
Band-Stop Filters

One method of making a Band-Stop Filter is to place a high-pass filter in parallel with a low-pass filter.





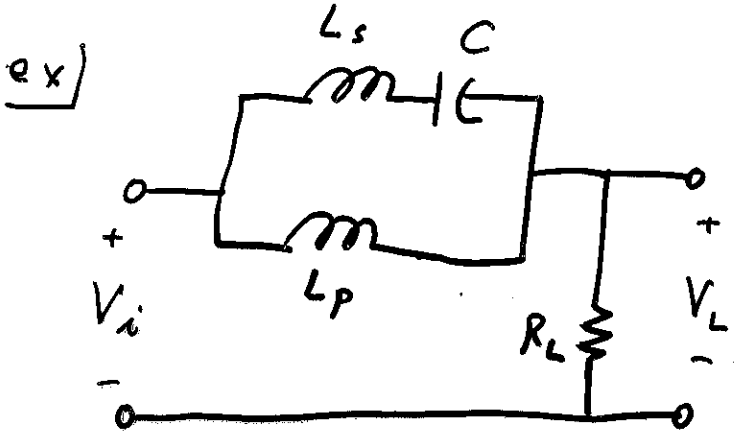
A resonant LC circuit can also be used to make a Band-Stop filter



$$V_{o\,min} = \left(\frac{R_L}{R_L + R}\right) V_{IN}$$

Double - Tuned Filter

ex)



The series $L_s C$ ckt will cause a pass band at its resonance.

At the stop-band the series $L_s C$ ckt. will be capacitive and will be parallel resonant with L_p.

Given $C = 500\,pf$, find L_s and L_p to reject 200 KHz and pass 600 KHz.

Solution:

For series resonance: $f_s = \dfrac{1}{2\pi\sqrt{LC}}$

solving for L_s gives: $L_s = \dfrac{1}{4\pi^2 f_s^2 C} = \dfrac{1}{4\pi^2 (600\,KHz)^2 (500\,pf)} = \boxed{140.7\,\mu H}$

Now find the impedance of the series ckt at 200 KHz:

$$X_{L_s} = 2\pi f_s L_s = 2\pi(200\,KHz)(140.7\,\mu H) = 176.8\,\Omega$$

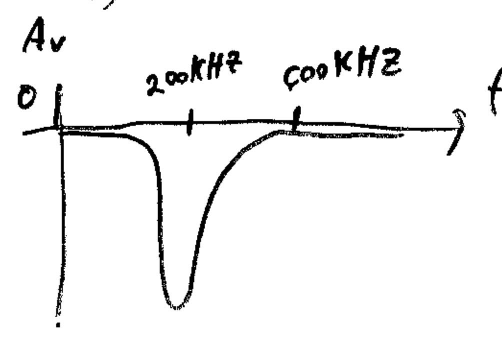
$$X_c = \frac{1}{2\pi f C} = \frac{1}{2\pi(200\,KHz)(500\,pf)} = 1591.5\,\Omega$$

$$Z_{series\,LC} = j(X_{L_s} - X_c) = j(176.8 - 1591.5) = -j\,1414.7\,\Omega = -j X_c'$$

For parallel resonance ($Q \geq 10$ assumed)

$$X_{L_p} = X_c'$$

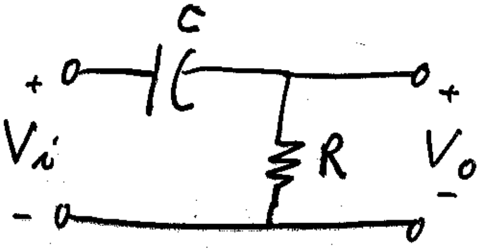
$$L_p = \frac{X_{L_p}}{\omega} = \frac{1414.7}{2\pi(200\,KHz)} = \boxed{1.13\,mH}$$



Bode Plots

Idealized Bode Plots use straight line segments to approximate frequency response curves.

ex/ High-Pass RC filter



$$\bar{A}_v = \frac{\bar{V}_o}{\bar{V}_i} = \frac{R}{R - jX_c} = \frac{1}{1 - j\frac{X_c}{R}} = \frac{1}{1 - j\frac{1}{2\pi f C R}}$$

$$\bar{A}_v = \frac{1}{1 - j\left(\frac{1}{2\pi RC}\right)\frac{1}{f}} \qquad \text{since } f_c = \frac{1}{2\pi RC}$$

Then $\boxed{\bar{A}_v = \frac{1}{1 - j\left(\frac{f_c}{f}\right)}}$

In Phasor form

$$\boxed{\bar{A}_v = \frac{\bar{V}_o}{\bar{V}_i} = A_v \angle \theta = \frac{1}{\sqrt{1 + \left(\frac{f_c}{f}\right)^2}} \angle \tan^{-1}\left(f_c/f\right)}$$

By substituting values for R, C and f, the above equation can be used to get an exact graph of the magnitude and phase angle of A_v.

However, a Bode Plot can also be obtained as follows:

The voltage gain can also be expressed in dB.

$$A_{V_{dB}} = 20 \log_{10} A_V$$

Substituting the magnitude of A_V for the Hi-Pass filter:

$$A_{V_{dB}} = 20 \log \frac{1}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}} = \cancel{20 \log 1}^{\,0} - 20 \log \sqrt{1+\left(\frac{f_c}{f}\right)^2}$$

Note: $\log \sqrt{x} = \frac{1}{2} \log X$

$$A_{V_{dB}} = -\frac{1}{2}(20) \log \left[1+\left(\frac{f_c}{f}\right)^2\right] = -10 \log \left[1+\left(\frac{f_c}{f}\right)^2\right]$$

Apply Bode approximation: for $\underline{f << f_c}$, $\left(\frac{f_c}{f}\right)^2 >> 1$

So: $1+\left(\frac{f_c}{f}\right)^2 \approx \left(\frac{f_c}{f}\right)^2$

Then: $A_{V_{dB}} = -10 \log \left[\left(\frac{f_c}{f}\right)^2\right] = -20 \log \frac{f_c}{f}$

Note: $-\log X = \log \frac{1}{X}$, here $x = \frac{f_c}{f}$

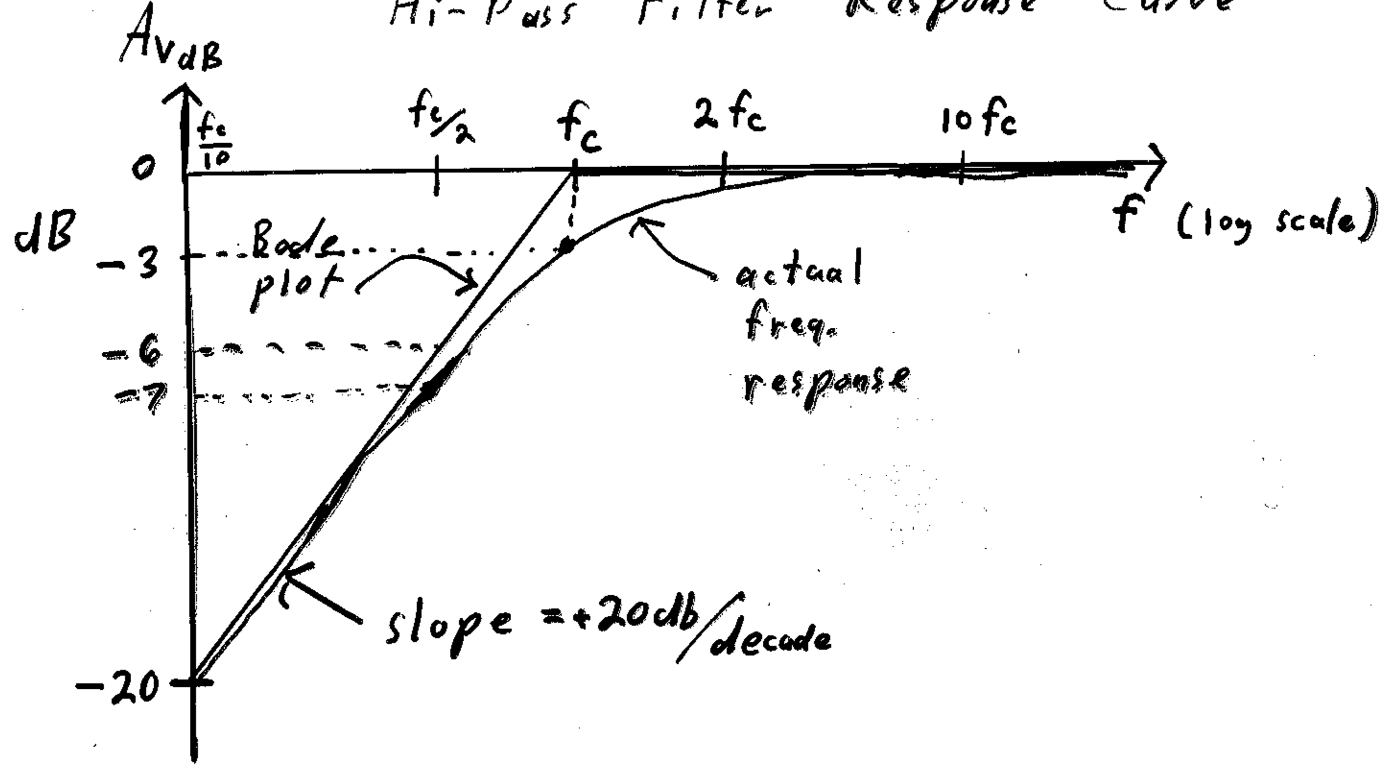
So: $\boxed{A_{V_{dB}} = +20 \log \frac{f}{f_c}}$ for $f << f_c$ for Hi Pass filter

An $\underline{\text{Octave}}$ is a factor of $\underline{2}$ change in frequency.

A $\underline{\text{Decade}}$ is a factor of $\underline{10}$ change in frequency

For the Bode plot the gain ($A_{V_{dB}}$) will roll-off at a rate of 20db per decade or 6 dB per octave change in frequency.

Hi-Pass Filter Response Curve

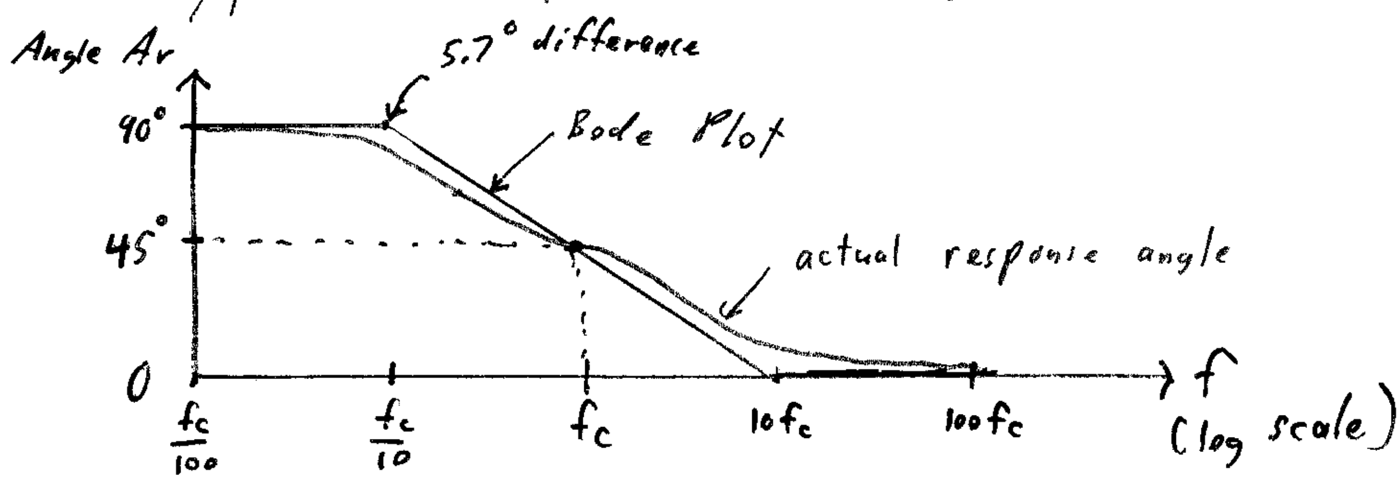


Bode plots are straight-line segments because dB change per decade or octave is constant.

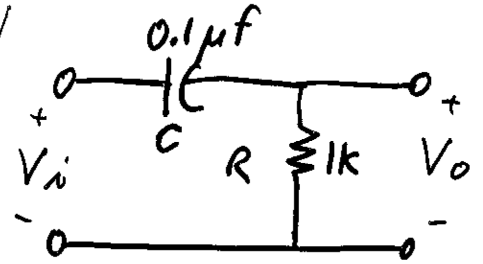
> At f_c (cutoff or corner freq.) the Bode plot differs from the actual freq. response by 3 dB.

At $f = 2f_c$ or $f = f_c/2$ the Bode plot differs from the actual freq. response by 1 dB.

Similarly, the Bode plot for the angle of A_V is:



ex/



a.) Sketch the freq. response of $A_{V_{dB}}$

b.) Find $A_{V_{dB}}$ at $f = 1\text{kHz}$

c.) sketch the phase response

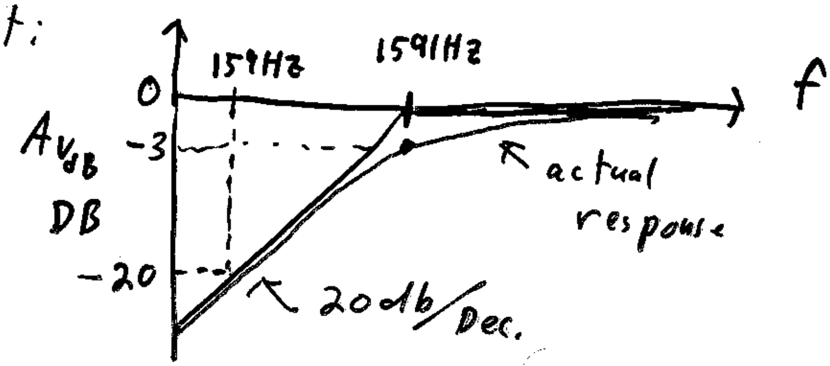
Solution:

a.) First find cutoff freq. $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1k)(.1\mu f)} = 1591\text{Hz}$

For very high freq. capacitor acts like a short ckt. making $V_o \approx V_i$ and $A_V = \frac{V_o}{V_i} \approx 1 = 0\text{dB}$

$\therefore$ it's a high Pass filter

Make Bode Plot:



b.) $A_{V_{dB}}$ at $f = 1\text{kHz}$. : estimate at $\frac{f_c}{2} = \frac{1591}{2} \cong 800\text{Hz}$

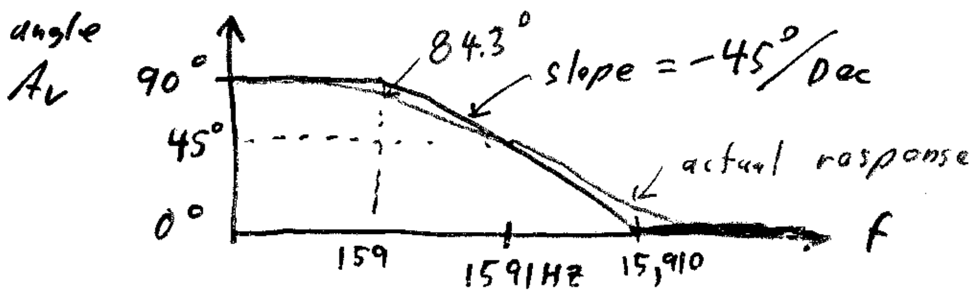
$A_V = -7\text{dB}$

so at $f = 1\text{kHz}$ $A_V \approx -5\text{dB}$

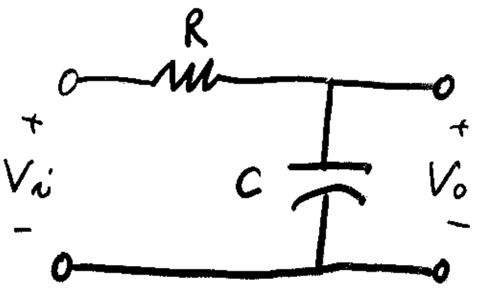
Exact Calculation:

$$|A_{V_{dB}}| = 20\log\left[\frac{1}{1+\left(\frac{f_c}{f}\right)^2}\right] = 20\log\left[\frac{1}{1+\left(\frac{1591}{1000}\right)^2}\right] = \boxed{-5.49\text{dB}}$$

c.) At high freq. $V_o \approx V_i$ $\therefore \theta = 0°$



ex



Low-Pass RC filter

(since C acts like a short ckt. at high freq. $V_o = 0$)

$$\bar{A}_v = \frac{\bar{V}_o}{\bar{V}_i} = \frac{-jX_c}{R - jX_c} = \frac{1}{\frac{R}{-jX_c} + 1} = \frac{1}{1 + j\frac{R}{X_c}}$$

$$\bar{A}_v = \frac{1}{1 + j\frac{R}{\frac{1}{2\pi f C}}} = \frac{1}{1 + j\frac{f}{\frac{1}{2\pi RC}}} \quad \text{since} \quad \boxed{f_c = \frac{1}{2\pi RC}}$$

$$\boxed{\bar{A}_v = \frac{1}{1 + j\left(\frac{f}{f_c}\right)}} = \frac{1}{\sqrt{1 + \left(\frac{f}{f_c}\right)^2}} \angle -\tan^{-1}\left(\frac{f}{f_2}\right)$$

Similar to the high Pass analysis earlier:

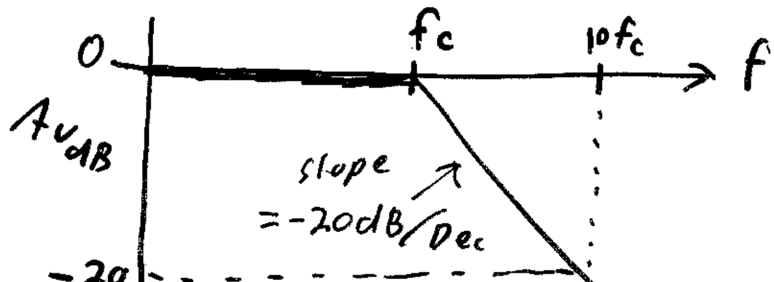
For $f >> f_c$ the gain roll-off is:

$$\boxed{A_{v_{dB}} = -20 \log \frac{f}{f_c}}$$

Sketch Bode Plot:

For $f \leq f_c$:

$A_{v_{dB}} = -20 \log 1 = 0$

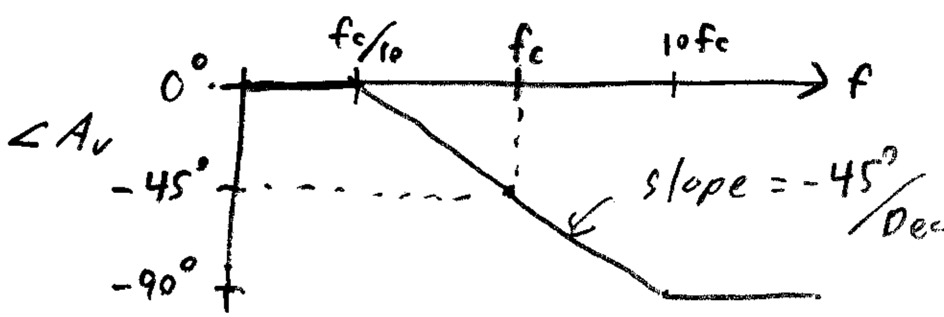


For $f << f_c$

$\angle A_v = -\tan^{-1} 0 = 0°$

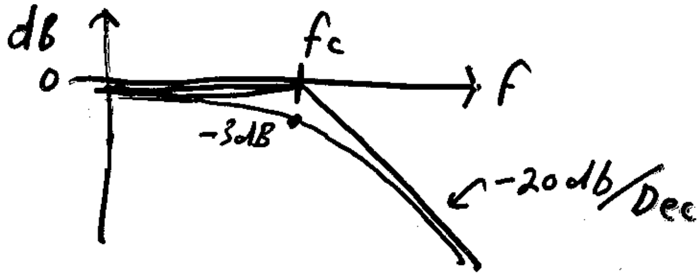
For $f >> f_c$

$\angle A_v = -\tan^{-1} \infty = -90°$

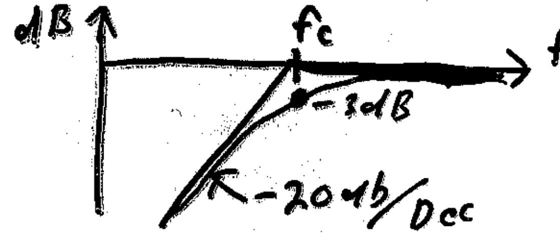


Sketching the Bode Response

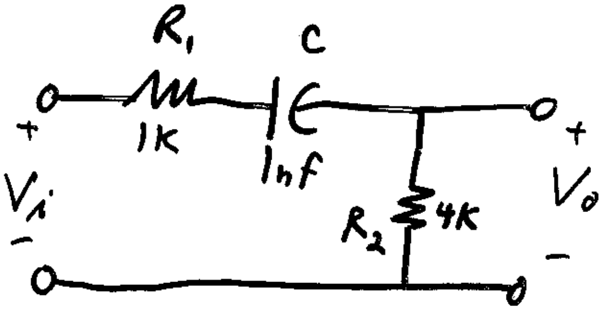
Low-pass: $\dfrac{1}{1+j\frac{f}{f_c}} \Rightarrow$



High-Pass: $\dfrac{1}{1+j\frac{f_c}{f}} \Rightarrow$



ex/

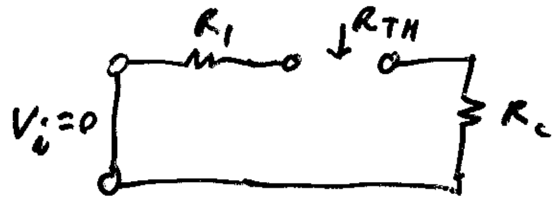


← Hi-Pass Filter with attenuated output.

At very high freq. C acts like a short ckt.,

then $V_o = \dfrac{R_2}{R_1+R_2} V_i = \dfrac{4k}{1k+4k} V_i = 0.8 V_i$

To find the cutoff freq. we must determine the Thevenin equivalent resistance "seen" by C.



$R_{TH} = R_1 + R_2 = 1K + 4k = 5K$

Then $f_c = \dfrac{1}{2\pi R_{TH} C} = \dfrac{1}{2\pi(5K)(1nf)} = 31.83kHz$

In general,

$$A_v = \frac{V_o}{V_i} = \frac{R_2}{R_2 + R_1 - jX_C} = \frac{R_2}{R_1+R_2}\left[\frac{1}{1-j\frac{X_C}{R_1+R_2}}\right]$$

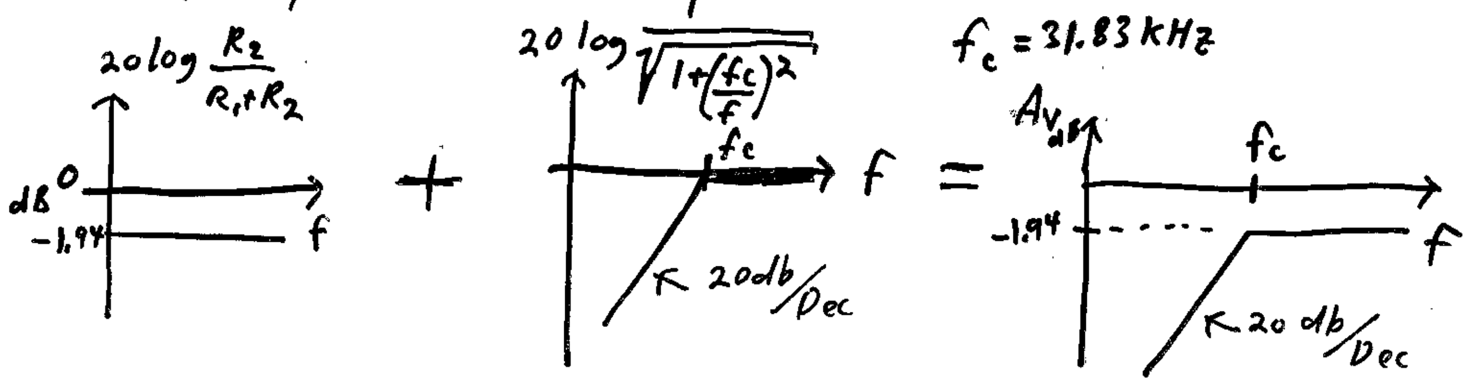
In terms of f and f_c:

$$A_v = \frac{R_2}{R_1+R_2}\left[\frac{1}{1-j\left(\frac{f_c}{f}\right)}\right]$$

$$A_{v_{dB}} = 20\log\left[\frac{R_2}{R_1+R_2}\left(\frac{1}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}\right)\right] \leftarrow$$ when two "gains" are multiplied together, there "dBs" can be added.

$$A_{v_{dB}} = 20\log\frac{R_2}{R_1+R_2} + 20\log\left(\frac{1}{\sqrt{1+\left(\frac{f_c}{f}\right)^2}}\right)$$

Graphicly the Bode plots can be combined:



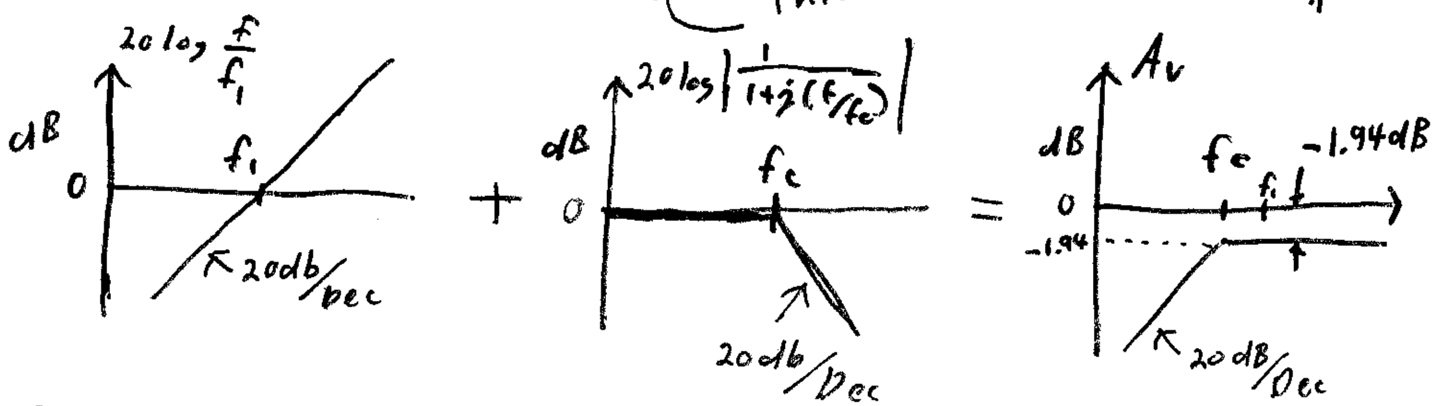
In terms of f_1 and f_c:

$$A_v = \frac{j\left(f/f_1\right)}{1+j\left(f/f_c\right)} \qquad \text{where } f_1 = \frac{1}{2\pi R_2 C} = 39.79\,Hz$$

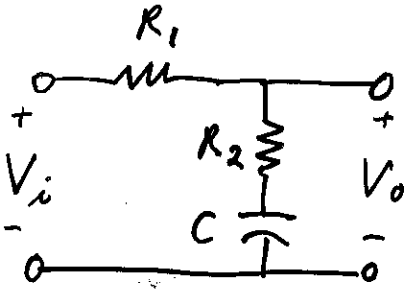
$$A_v = \left[j\left(\frac{f}{f_1}\right)\right]\left[\frac{1}{1+j\left(f/f_c\right)}\right]$$

this is a standard low pass.

$20\log\frac{f_c}{f_1} = -1.94dB$



ex/



← Low-Pass filter with limited attenuated output

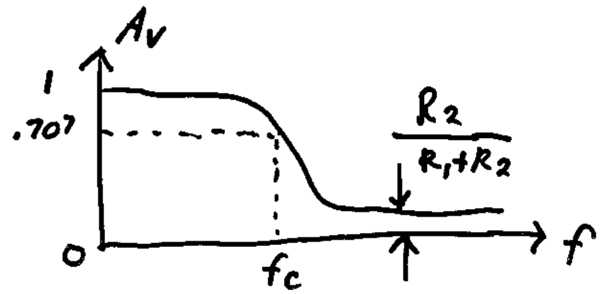
At very high freq. C acts like a short ckt.

then $V_o = V_i \dfrac{R_2}{R_1+R_2} \rightarrow A_v = \dfrac{R_2}{R_1+R_2}$

At very low freq. C acts like an open ckt.

then $V_o = V_i \quad \rightarrow \quad A_V = 1$

$$f_c = \frac{1}{2\pi(R_1+R_2)C}$$



In general,

$$A_V = \frac{(R_2 - jX_c)}{R_1+R_2-jX_c} = \frac{R_2/X_c - j}{(R_1+R_2)/X_c - j} = \frac{1+j2\pi f R_2 C}{1+j2\pi f(R_1+R_2)C}$$

$$A_V = \frac{1+j(f/f_1)}{1+j(f/f_c)} \qquad \text{where} \quad f_1 = \frac{1}{2\pi R_2 C}$$

$$A_V = \left[1+j(f/f_1)\right]\left[\frac{1}{1+j(f/f_c)}\right]$$

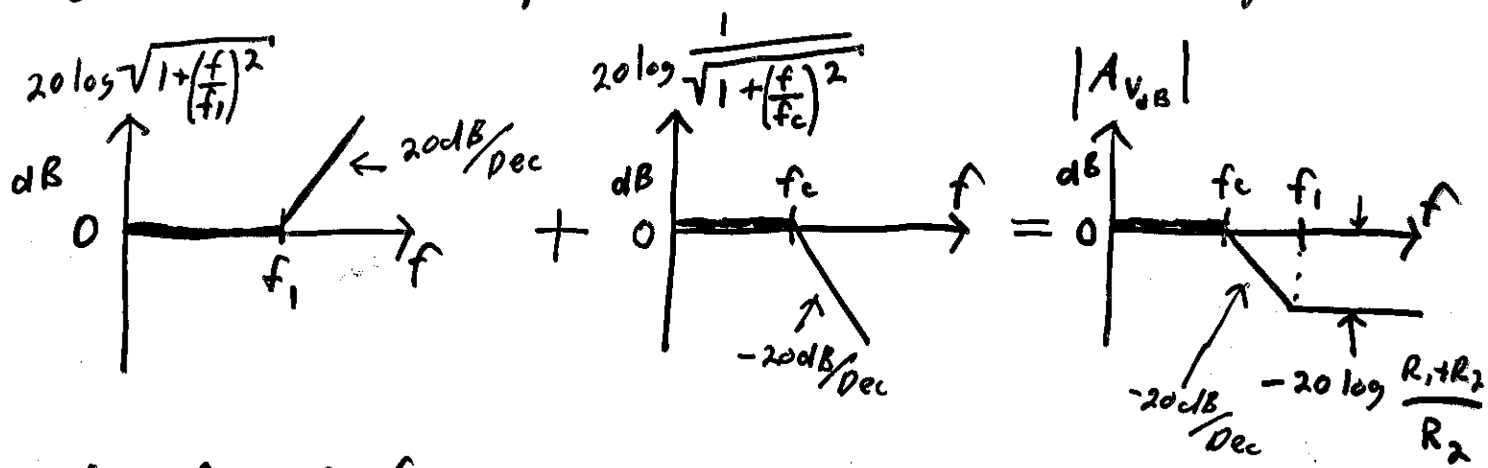
This is the standard low-pass response term.

To get Bode plot for $1+j\left(\frac{f}{f_1}\right)$ term:

Consider: for $f \ll f_1 \rightarrow 1+j\left(\frac{f}{f_1}\right) \approx 1 = 0\,dB$

for $f \gg f_1 \rightarrow 1+j\left(\frac{f}{f_1}\right) \approx j\left(\frac{f}{f_1}\right) \rightarrow \left|j\left(\frac{f}{f_1}\right)\right| = +20\,dB/Dec$ slope

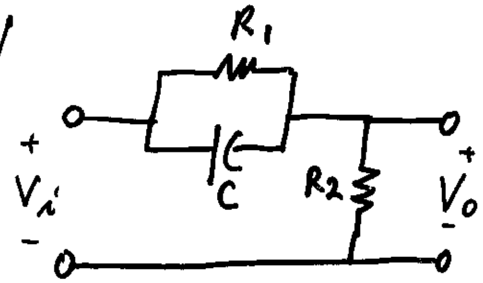
Combine 2 Bode plots to get total Bode plot



A_v after $f > f_1$:

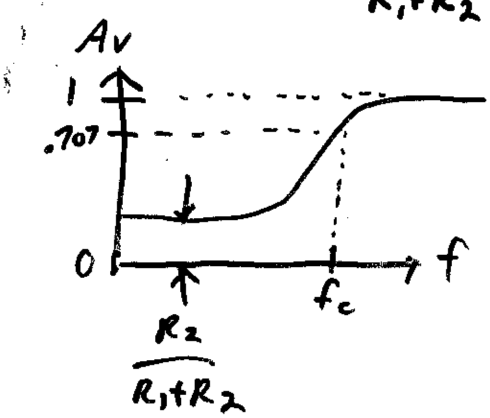
$$A_v = 20\log\frac{R_2}{R_1+R_2} = -20\log\frac{R_1+R_2}{R_2}$$

ex/



$\leftarrow$ High-Pass filter with Limited Attenuation

At very high freq. C acts like a short ckt,
then $V_o = V_i \rightarrow A_v = 1 = 0\,dB$
At very low freq. C acts like an open ckt,
then $V_o = V_i\frac{R_2}{R_1+R_2} \rightarrow A_v = \frac{R_2}{R_1+R_2}$



To find f_c, determine R_{TH} "seen by" C. (Kill V_i source i.e. replace V_i with short ckt.)
Then $R_{TH} = R_1 \,\|\, R_2$

So, $f_c = \frac{1}{2\pi(R_1\|R_2)C}$

In general,

$$A_v = \frac{R_2}{R_2 + R_1\|(-jX_c)} = \cdots = \frac{1 - j\left(f_1/f\right)}{1 - j\left(f_c/f\right)}$$ where $f_1 = \frac{1}{2\pi R_1 C}$

$$A_v = \left[1 - j\left(\frac{f_1}{f}\right)\right]\left[\frac{1}{1 - j\left(\frac{f_c}{f}\right)}\right] \leftarrow$$ get Bode plots for each term and then Combine $A_v = A_{v_1} \cdot A_{v_2}$

$A_{v_{dB}} = A_{v_{1\,dB}} + A_{v_{2\,dB}}$

Consider $\left[1 - j\frac{f_1}{f}\right] = A_{v_1}$

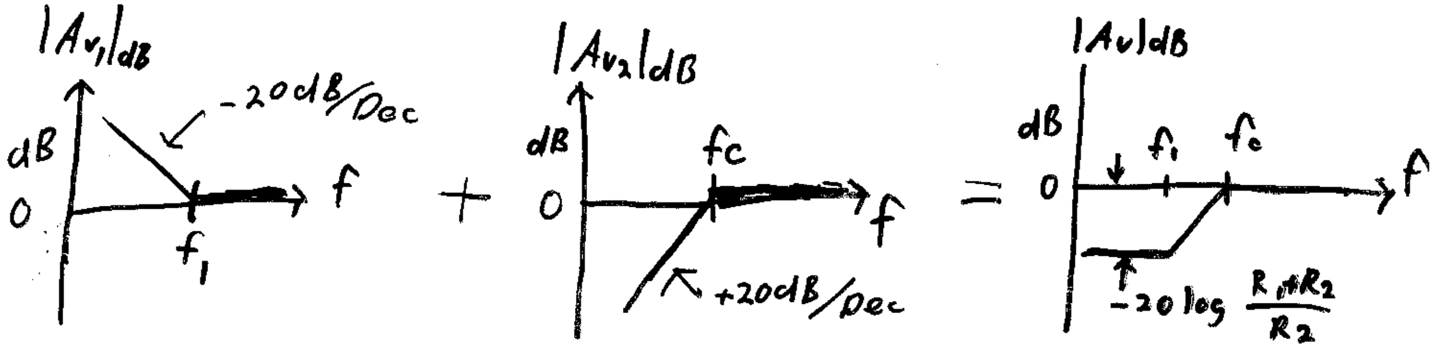
For $f \ll f_1 \rightarrow A_{v_1} \approx -j\frac{f_1}{f} \rightarrow |A_{v_1}|_{dB} = 20\log\frac{f_1}{f} \rightarrow -20\,dB/Dec$ slope

For $f \gg f_1 \rightarrow A_{v_1} \approx 1 \rightarrow |A_{v_1}|_{dB} = 0\,dB$

Consider $\left[\frac{1}{1 - j\left(\frac{f_c}{f}\right)}\right] = A_{v_2}$

For $f \ll f_c \rightarrow A_{v_2} \approx \frac{1}{-j\left(\frac{f_c}{f}\right)} = j\left(\frac{f}{f_c}\right) \rightarrow |A_{v_2}|_{db} \rightarrow +20\,dB/Dec$ slope

For $f \gg f \rightarrow A_{v_2} \approx \frac{1}{1} = 1 \rightarrow |A_{v_2}|_{dB} = 0\,dB$



Note: Since $f_c = \frac{1}{2\pi(R_1 \| R_2)C}$ and $f_1 = \frac{1}{2\pi R_1 C}$, then $f_c > f_1$

ex/ Consider above example with: $R_1 = 9.1k$, $C = 0.47\mu f$
$R_2 = 1k$

Then $f_1 = \frac{1}{2\pi R_1 C} = \frac{1}{2\pi(9.1k)(.47\mu f)} = 37.2\,HZ$

$f_c = \frac{1}{2\pi\left(\frac{R_1 R_2}{R_1 + R_2}\right)C} = \frac{1}{2\pi(0.9k)(.47\mu f)} = 376\,HZ$

Maximum low-level attenuation $= -20\log\frac{R_1+R_2}{R_2} = -20\log 10.1 = -20.09\,db$

ex/ A transistor amplifier has the following gain:

$$A_v = \frac{100}{\left(1 - j\frac{50}{f}\right)\left(1 - j\frac{200}{f}\right)\left(1 + j\frac{f}{10K}\right)\left(1 + j\frac{f}{20K}\right)}$$

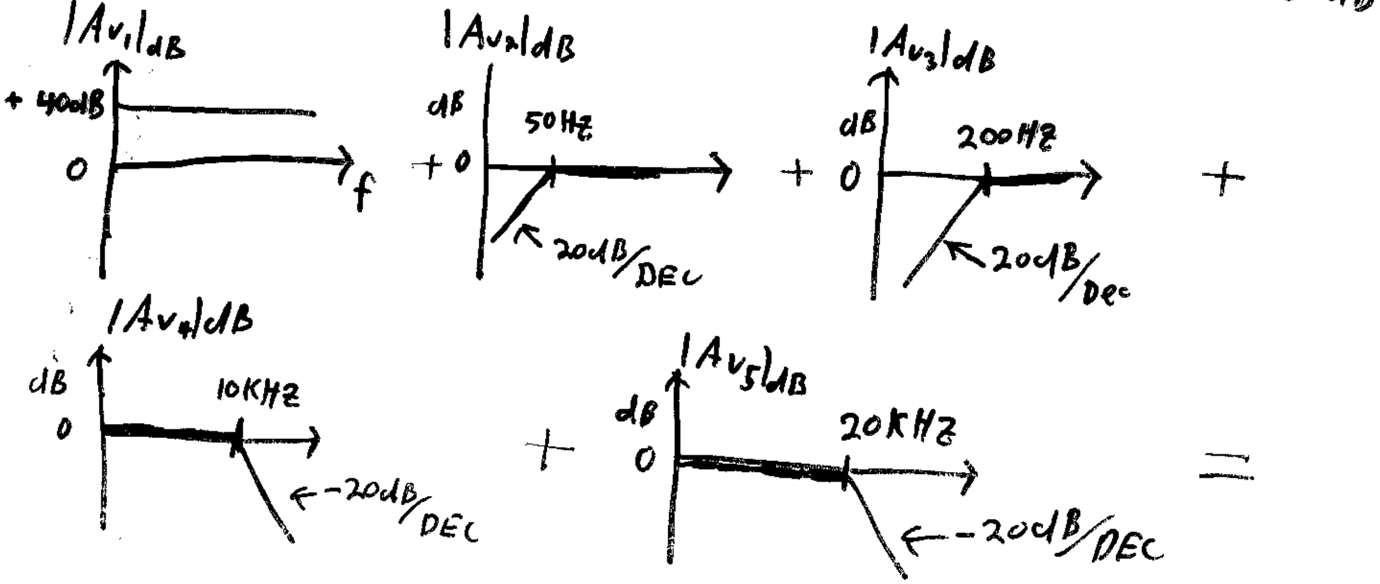
sketch Bode plot of gain.

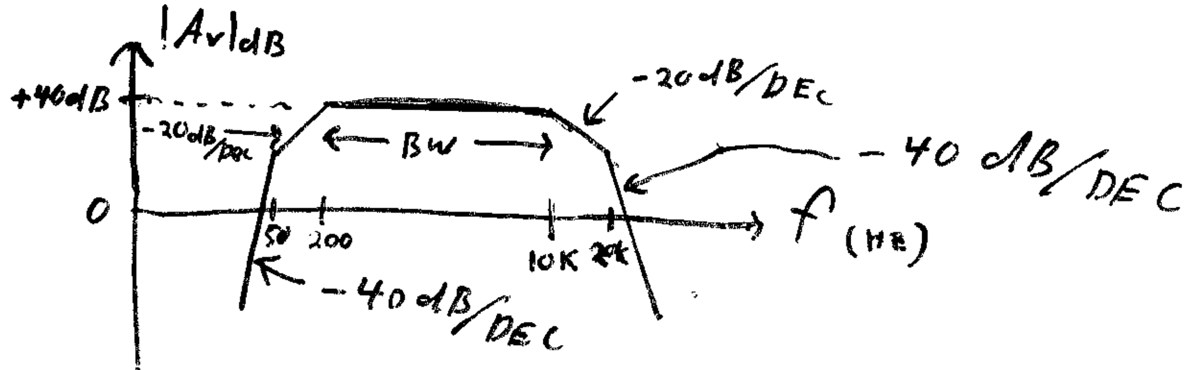
Solution: Break up gain expression into parts.

$$A_v = [100]\underbrace{\left[\frac{1}{1 - j\frac{50}{f}}\right]\left[\frac{1}{1 - j\frac{200}{f}}\right]}_{\text{High-Pass}}\underbrace{\left[\frac{1}{1 + j\frac{f}{10K}}\right]\left[\frac{1}{1 + j\frac{f}{20K}}\right]}_{\text{Low-Pass}}$$

$$A_v = A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdot A_{v_4} \cdot A_{v_5}$$

$$|A_v|_{dB} = |A_{v_1}|_{dB} + |A_{v_2}|_{dB} + |A_{v_3}|_{dB} + |A_{v_4}|_{dB} + |A_{v_5}|_{dB}$$

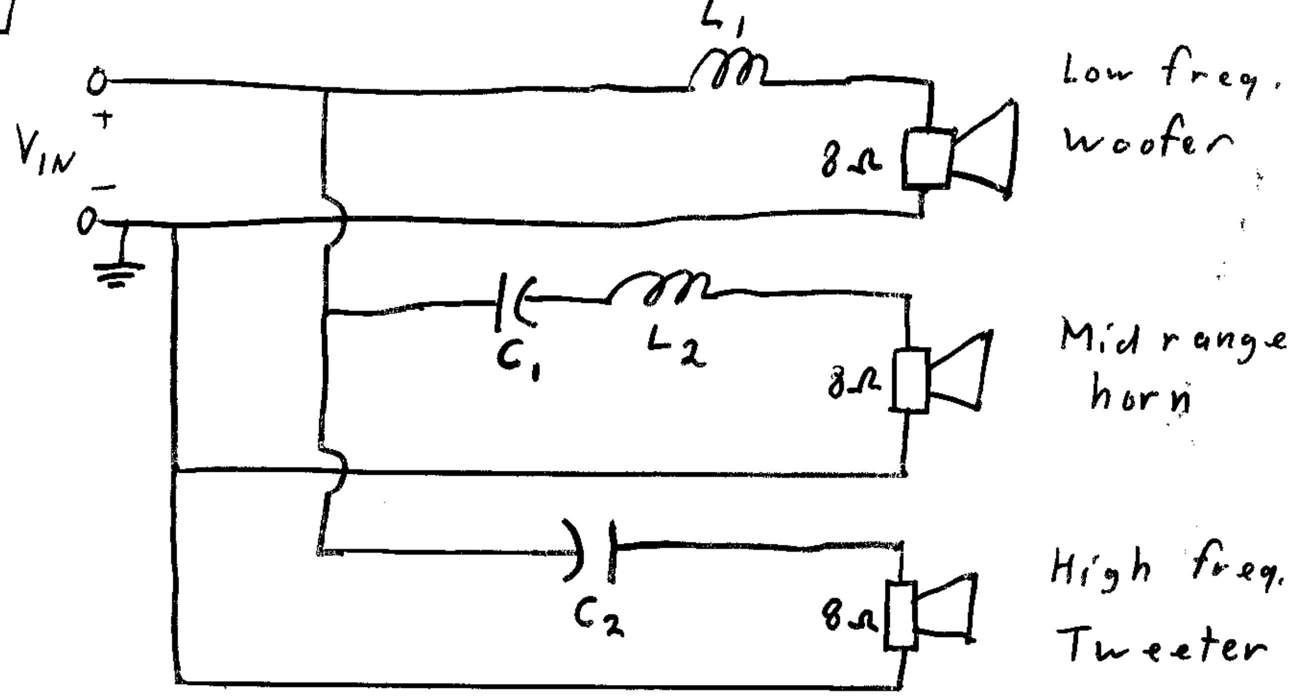




Crossover Network Design

A crossover network consists of filters used to direct frequency components of an audio signal to the optimal speaker.

ex/



Each speaker acts like an 8Ω Resistor in the RC and RL filters.

Design Low Pass filter with $f_{c_1} = 400\,Hz$ for woofer:

$$L_1 = \frac{R}{2\pi f_{c_1}} = \frac{8\Omega}{2\pi(400\,Hz)} = 3.183\,mH \rightarrow \boxed{3.3\,mH} \leftarrow \text{standard value}$$

Design High Pass filter with $f_{c_1} = 400\,Hz$ for midrange:

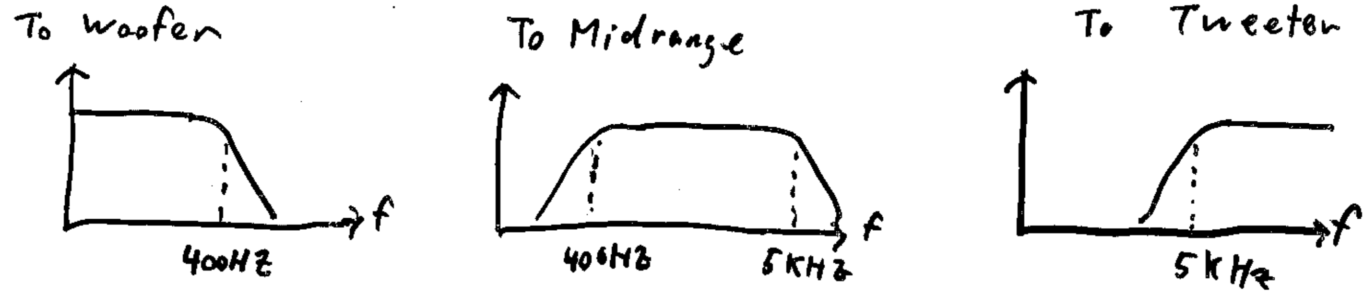
$$C_1 = \frac{1}{2\pi f_{c_1} R} = \frac{1}{2\pi(400\,Hz)\,8\Omega} = 49.7\,\mu f \rightarrow \boxed{47\,\mu f} \leftarrow \text{standard value}$$

Design Low Pass filter with $f_{c_2} = 5\,Kz$ for midrange

$$L_2 = \frac{R}{2\pi f_{c_2}} = \frac{8\Omega}{2\pi(5\,KHz)} = 254.6\,\mu H \rightarrow \boxed{270\,\mu H} \leftarrow \text{standard value}$$

Design High Pass filter with $f_{c_2} = 5KHz$ for tweeter

$$C_2 = \frac{1}{2\pi f_{c_2} R} = \frac{1}{2\pi(5KHz)\,8\Omega} = 3.979\mu f \rightarrow \boxed{3.9\mu F} \leftarrow \text{standard Value}$$



Consider $V_{IN} = 4$ Volts $_{RMS}$ @ 1 KHz

At $f = 1$ KHz:

$$X_{L_1} = 2\pi f L_1 = 2\pi(1k)(3.3mH) = 20.74\Omega$$

$$V_{o_1} = \frac{(Z_R \angle 0)(V_i \angle 0)}{Z_T} = \frac{(8\Omega\angle 0)(4V\angle 0)}{8 + j20.74} = 1.44V.\angle -68.4^\circ$$

$$X_{L_2} = 2\pi f L_2 = 2\pi(1k)(270\mu H) = 1.696\Omega$$

$$X_{C_1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(1k)47\mu f} = 3.386\Omega$$

$$V_{o_2} = \frac{(8\Omega\angle 0)(4V\angle 0)}{8 + j1.696 - j3.386} = 3.94V.\angle 11.93^\circ$$

$$X_{C_2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(1k)3.9\mu f} = 40.81\Omega$$

$$V_{o_3} = \frac{(8\Omega\angle 0)(4V\angle 0)}{8 - j40.81} = 0.77V.\angle 78.91^\circ$$

Power to Woofer $= P_1 = \frac{V_{o_1}^2}{R} = \frac{(1.44V.)^2}{8\Omega} = \boxed{0.259\ W.}$

Power to Midrange $= P_2 = \frac{V_{o_2}^2}{R} = \frac{(3.94V.)^2}{8\Omega} = \boxed{1.94\ W.}$

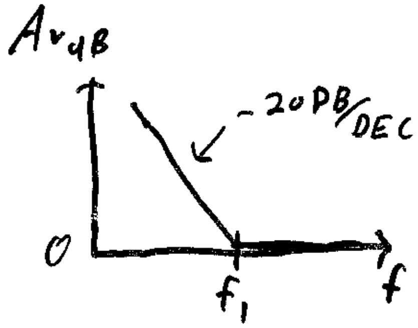
Power to tweeter $= P_3 = \frac{V_{o_3}^2}{R} = \frac{(0.77)^2}{8\Omega} = \boxed{0.074\ W.}$

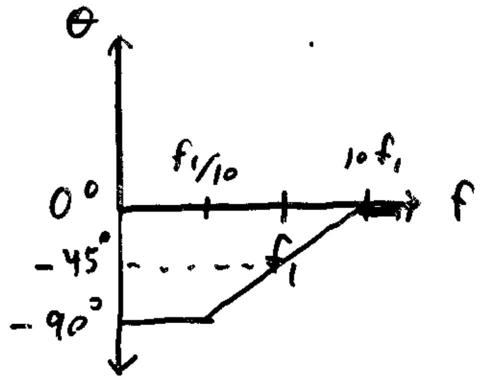
Bode Plot Summary

ex/ $A_v = 1 - j\frac{f_1}{f}$

at $f << f_1$: $A_v \approx -j\frac{f_1}{f}$ → 20 dB/Dec decrease with freq.
→ $\theta = -90^\circ$

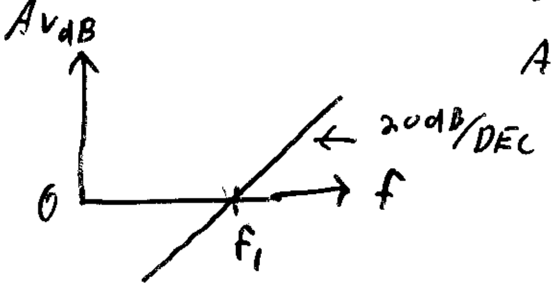
at $f >> f_1$: $A_v \approx 1$ → 0 dB gain
→ $\theta = 0^\circ$





ex/ $A_v = j\frac{f}{f_1}$ → 20 dB/DEC increase with freq
→ $\theta = +90^\circ$

$A_v = 1 = 0$ dB at $f = f_1$



ex/ $A_v = \frac{1}{1 + j\frac{f}{f_c}}$

at $f << f_c$: $A_v \approx \frac{1}{1} = 1$ → 0 dB gain, $\theta = 0^\circ$

At $f >> f_c$: $A_v \approx \frac{1}{j\frac{f}{f_c}} = -j\frac{f_c}{f}$ → 20 dB/DEC decrease with f
→ $\theta = -90^\circ$