Lesson 2: Magnetic Circuits - Quantities and Definitions

ET 332a
Dc Motors, Generators and Energy Conversion Devices

Learning Objectives

After this presentation you will be able to:

- Explain how magnetic flux lines emanate from permanent magnets and produce force
- Define flux density, magnetomotive force, magnetic field intensity, permeability, and reluctance using mathematical equations.
- Identify the parts of a magnetization curve
- Perform calculations using magnetic quantities
Magnetic Fields and Polarity

Permanent Magnets

Direction convention: From N to S

Flux Density, \( B \)
Number of flux lines/ unit area

Mathematically \( B = \frac{\Phi}{A} \)

Where:
- \( B \) = flux density (Teslas, T or Wb/m\(^2\))
- \( A \) = area (m\(^2\))
- \( \Phi \) = flux (Wb)

Flux lines
Symbol: \( \Phi \)
Units: Webers (Wb)

Magnetic Forces

Magnetic field converted to mechanical force

Opposite Poles Attract

Like Poles Repel

No flux bunching

Flux bunching causes repulsion
Electromagnetism

Currents flowing in wires produce magnetic flux

Voltage Source, \( V \)

Current \( I \)

Direction of flux line (Conventional current)

Right-hand Rule

Thumb – current

Fingers – direction of flux

Magnetomotive Force of a Coil

Given a coil

Magnetomotive Force (MMF) \( \mathcal{F} \)

Mathematically \( \mathcal{F} = N \cdot I \)

Where \( \mathcal{F} = \text{MMF (A-t)} \)

\( N = \text{number of turns in coil (t)} \)

\( I = \text{current in coil (A)} \)
Magnetic Field Intensity of a Coil

Magnetic field intensity, \( H \) (MMF gradient of coil)
Amount of MMF dropped over length

Mathematically

\[
H = \frac{N \cdot I}{L} = \frac{\Phi}{L}
\]

Where: \( H \) = Magnetic field intensity (Oersteds, A·t/m)
\( L \) = path length (meters)

Reluctance of Magnetic Circuits

Reluctance – opposition to flux. Similar to R in dc electric circuit

Ferromagnetic core (iron)
Core has cross sectional area
\( A \) (m²) = LxW

Magnetic Circuit Relationship

\[
\Phi = \frac{\Phi}{R} = \frac{N \cdot I}{R}
\]

So

\[
R = \frac{\Phi}{\Phi}
\]

Where: \( \Phi \) = MMF (A·t)
\( R \) = Reluctance (A·t/Wb)
\( \Phi \) = flux (Wb)
\( N \) = coil turns (t)
\( I \) = coil current (A)
Reluctance of Magnetic Circuits

Coil reluctance related to core geometry and material

From previous math relationships

\[ \mathcal{F} = H \cdot L \]
\[ \Phi = B \cdot A \]

Substitute into equation from last slide and simplify

\[ R = \frac{L}{B} \cdot A \]

B and H depend on magnetic core material and relationship is usually non-linear

Electric analogy

H \(\rightarrow\) Electric field potential
B \(\rightarrow\) Current density

Magnetization Curves
(B-H curves)

Plot magnetic field intensity (H) Vs flux density (B) on semi-log plots

Magnetic circuit non-linear due to magnetic saturation

H (A-t/m or Oersteds) Logarithmic scale
Magnetization Curves

B-H curves different for each type of material

Free space B-H curve is linear, but permeability is very low (hard to magnetize)

Magnetic Permeability

**Permeability** - Amount of magnetic field intensity required to produce a given flux density for a given material.

Mathematically

$$\mu = \frac{B}{H}$$

Where:

- \(B\) = flux density (Wb/m²)
- \(H\) = magnetic field intensity (A·t/m)
- \(\mu\) = permeability (Wb/A·t·m)

**Characteristics:**

- similar to resistivity in conductors
- not a constant for a given material
- larger \(\mu\), less \(H\) required to produce given \(B\)
Relative Permeability

Relative permeability - Ratio of material’s permeability to that of free space

\[ \mu_r = \frac{\mu}{\mu_0} \]

Where:

- \( \mu_0 \) = permeability of free space
  
  \((4\pi \times 10^{-7} \text{Wb/A-t-m})\)

- \( \mu \) = permeability of material \((\text{Wb/A-t-m})\)

- \( \mu_r \) = relative permeability \((\text{dimensionless})\)

Permeability, \( \mu \), found from B-H plots. Experimental results of exciting magnetic material with current and measuring B.

Reluctance formulas In Terms of Permeability

\( R \) depends on:

- type of magnetic material \( \mu \)
- length of circuit, \( L \)
- cross-sectional area of circuit, \( A \)

\[ R = \frac{L}{\mu \cdot A} \]

\( r \rightarrow 0 \) suggests from previous relationship, so

\[ R = \frac{L}{\mu_0 \cdot \mu_r} \]
Calculation Examples

**Computing flux density:**
A magnetic flux of 0.0046 Wb passes through a core cross sectional dimensions of 10 cm x 17 cm. Find the flux density.

\[
\Phi = 0.0046 \text{ Wb} \\
L = 10 \text{ cm} \quad W = 17 \text{ cm} \\
L = 0.10 \text{ m} \quad W = 0.17 \text{ m} \\
A = 0.10 \text{ m} \times 0.17 \text{ m} = 0.017 \text{ m}^2 \\
B = \frac{\Phi}{A} = \frac{0.0046 \text{ Wb}}{0.017 \text{ m}^2} = 0.271 \text{ Wb/m}^2 \quad \text{(T)} \quad \text{Answe}
\]

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**Computing MMF and H:**
A coil for a solenoid is 20 cm long and made up of 200 turns of wire. This wire has a dc resistance of 2.25 Ω. The solenoid is connected to a 100 V dc source. Find the MMF the coil produces and the magnetic field intensity.

**Answer:**
\[
F = 200 \times 44.49 \text{ A} = 8888.9 \text{ A} \\
F = 8888.9 \text{ A} \cdot \text{t} \quad \text{Answe} = 49999 \text{ A} \cdot \text{t/m} \quad \text{Answe}
\]
Calculation Examples

Computing Reluctance and Permeability:
A magnetic core made of cast steel must carry a flux density of 1.0 T. It has a total length of 1.56 m and a cross-sectional area of .37 m². Find the permeability, the relative permeability and reluctance of the core.

Use B-H curve from text. For a B=1.0 T H = 800 A·t/m

\[
\mu_c = \frac{1.0 \text{ wb/m}^2}{800 \text{ A·t/m}} = 0.00125 \text{ wb/A·m} [\text{new}]
\]

\[
\mu_r = \frac{\mu_c}{\mu_0} = \frac{0.00125 \text{ wb/A·m}}{\frac{4\pi}{10^7} \text{ wb/A·m}} = 999.2 \text{ (dimensionless) [new]}
\]

\[
R = \frac{L}{\mu_r A} = \frac{L}{\mu_c \mu_0 A} \quad L = 1.56 \text{ m}
\]

Use Permeability of cast steel, \( \mu_c \)

\[
R = \frac{L}{\mu_c A} = \frac{1.56 \text{ m}}{0.00125 \text{ wb/A·m} \cdot 0.37 \text{ m}^2} = 3.28 \text{ A·t/wb [new]}
\]

End Lesson 2 - Magnetic Circuits - Quantities and Definitions

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