Lesson 9: Practical Transformer Model and Calculations

ET 332b Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Identify factors that contribute to non-ideal operation of power transformers
- Draw the schematic model of a non-ideal transformer and include all parameters
- Reflect impedances through a non-ideal transformer
- Identify step-up and step down transformer connections
- Compute transformer voltages and currents using the full circuit model.
Transformer Circuit Model

Equivalent Circuit Model for Leakage and Coil Resistance

Not all developed flux links primary to secondary. Leakage flux only links the turns of the coil that generates it.

Leakage flux creates leakage inductance that has leakage reactance. This causes additional voltage drops across the coils.

Leakage Reactance and Voltage Drop

Define inductive voltage drop

Where:
- $V_p$ = net voltage induced in primary
- $E_p$ = voltage induced due to mutual flux
- $E_{lp}$ = voltage induced in primary due to leakage
- $V_s$ = net voltage induced in secondary
- $E_s$ = voltage induced due to mutual flux
- $E_{ls}$ = voltage induced in the secondary due to leakage
Coil Resistance and Total Voltage Drop

Resistance of windings modeled by lumped resistance

Primary coil resistance

Secondary coil resistance

Ideal transformer purely turns ratio

Primary side

Secondary side

Circuit Model of Non-ideal Transformers

Use ideal transformer impedance formulas to get equivalent $Z_{in}$

$Z_{in}$ is called the reflected or referred impedance

From ideal transformers

$Z_{in} = a^2 \cdot \left( \frac{V_s}{I_s} \right)$

$a^2 = \left( \frac{N_p}{N_s} \right)^2$
Circuit Model of Non-ideal Transformers

From Ohm's Law

\[ \frac{V_a}{I_s} = (R_s + j \cdot X_{ls}) + Z_{\text{load}} \]

\[ Z_{\text{in}} = a^2 \cdot (R_s + j \cdot X_{ls}) + a^2 \cdot Z_{\text{load}} \]

\[ Z_{\text{in}} = a^2 \cdot R_s + j \cdot a^2 \cdot X_{ls} + a^2 \cdot Z_{\text{load}} \]

Where:
- \( a^2 \cdot R_s \) = secondary R \text{ referred to primary }
- \( a^2 \cdot X_{ls} \) = secondary \( X_{ls} \) \text{ referred to primary }
- \( a^2 \cdot Z_{\text{load}} \) = load impedance \text{ referred to primary }

Referring the secondary impedances to the primary side effectively removes the need for the ideal transformer. The turns ratio captures all the effects.

Circuit Model of Non-ideal Transformers

Transformer circuit with ideal turns ratio removed

Can solve this circuit to find anything about non-ideal transformer operation. Voltage drop, power losses, primary and secondary currents and voltages.
Another simplifying assumption: $I_o << I_{load}$. Combine primary and secondary impedances.

Equivalent series impedance referred to primary side

$$Z_{eq} = (R_p + a^2 \cdot R_s) + j \cdot (X_{sp} + a^2 \cdot X_b)$$

Simplified Model-Referred to Secondary

The impedance $Z_{eq}$ can be referred to the secondary side instead. Just divide the impedance components by $a^2$.

Equivalent impedance referred to the secondary side

$$Z_{eqs} = \frac{R_p}{a^2} + R_s + j \cdot \left( \frac{X_{sp}}{a^2} + X_b \right)$$
Series Impedance of Transformers

Power transformers are bi-directional devices. They can operate with a V source attached to either primary or secondary winding.

Step-down operation: load connected to low voltage coil

Referring Z's to high side

\[ Z_{eqHS} = \frac{a^2}{Z_{eqLS}} \]
\[ Z_{loadLS} = \frac{a^2}{Z_{loadLS}} \]

Step-up operation: load connected to the high voltage winding

Referring Z's to low side

\[ Z_{eqLS} = \frac{Z_{eqHS}}{a^2} \]
\[ Z_{loadLS} = \frac{Z_{loadHS}}{a^2} \]

Transformer Problems

**Example 9-1:** A 100 kVA, 7200 -480 V 60 Hz single phase transformer has the following parameters all given in ohms:

- \( R_{LS} = 0.00800 \)
- \( R_{HS} = 1.96 \)
- \( R_{feHS} = 53.2 \)
- \( X_{LS} = 0.01510 \)
- \( X_{HS} = 4.55 \)
- \( X_{MHS} = 7800 \)

This transformer is operated in the step-down mode and delivers 75% of its rated power to a load that has a power factor of 0.93 lagging. Find:

a) draw the equivalent circuit model of the transformer with the equivalent series Z's referred to the high voltage side
b) find the total \( Z_{in} \) of the transformer at the high side
c) input Z of the transformer with the load disconnected
d) input voltage at 75% load required to maintain rated load voltage
e) exciting current with the load disconnected.
Example 9-1 Solution (1)

Draw circuit model and find $Z_{eq}$

\[ Z_{eq} = Z_{in} + Z_{L} \]

\[ R_{eq} = 0.018 \, \Omega, \quad X_{eq} = 0.015 \, \Omega \]

Find Load Current

\[ \alpha = 15 \]

\[ V_p = 7200 \, V, \quad V = 960 \, V \]

\[ \frac{V_p}{V} = 15 \]

\[ Z_{LHS} = Z_{eq} \]

\[ S = 100 \, \text{kVA}, \quad (0.75) \]

\[ S_L = 75 \, \text{kVA} \]

\[ I_{LS} = 0.98 \, \text{kA}, \quad I_{LS} = 156.25 \, \text{A} \]

\[ \theta = 21.57^\circ \]

\[ 
\begin{align*}
\vec{Z}_{eq} &= 3.76 + j79.975 \, \Omega \\
\end{align*}
\]

Ans

Example 9-1 Solution (2)

b) Find the $Z_m$ as seen on the HV side

\[ \vec{Z} = \frac{\vec{V}_L}{\vec{I}_L} \]

\[ \vec{Z} = 400 \angle 45^\circ \]

\[ \vec{Z} = 156.25 \angle 24.25^\circ \]

\[ \vec{Z}_{eq} + \vec{Z}_{LH} = a \vec{Z}_{L} \]

\[ \vec{Z}_{LH} = (15) \, (3072/24.25^\circ) \]

\[ \vec{Z}_{LH} = 691/24.25^\circ \, \Omega \]

\[ \vec{Z}_{LH} = 642.8 + j259.5 \, \Omega \]

\[ \vec{Z}_m = \vec{Z}_{eq} + \vec{Z}_{LH} \]

\[ \vec{Z}_m = (2.76 + j7.975) + (642.8 + j259.5) \]

\[ \vec{Z}_m = 697.6 + j22.04 \, \Omega \]

Ans
Example 9-1 Solution (3)

c) $Z_{in}$ with the load disconnected

\[ Z_{in} = Z_{L} + Z_{eq} \]

Ans

\[ 7200 \text{ V} \]

d) Input voltage required at load to maintain $V_s = 480$ V

\[ V_{HS} = V_L \quad V_{HS} = 480 \, (15) \quad V_{HS} = 7200 \, V \]

Example 9-1 Solution (4)

e) Exciting current with load disconnected

With load disconnected only current is $I_o$

\[ I_o = \frac{E_{HS}}{Z_{in}} \]

Remember

\[ I_o = \frac{V_I E_{HS}}{Z_c} \]
Example 9-1 Solution (5)

\[ Y_T = \frac{1}{R_{feas}} + \frac{1}{X_{mHS}} \]
\[ Y_T = 53,280 \, \Omega + 78,800 \, \Omega \]
\[ Y_T = 1.8797 \times 10^4 \, \Omega + 1.282 \times 10^4 \, \Omega \]
\[ Y_T = 1.295 \times 10^4 \, \Omega / BL^2 \, \Omega \]
\[ I_0 = 0.9415 \, A / BL^2 \, A \]

Find \( I_o \) as percent of rate load \( I \)

\[ I_{PR} = \frac{\frac{S_{rated}}{V_{PR}}}{\frac{100,530 \, VA}{1285 \, V}} = 13.89 \, A \]
\[ I_o \% = \frac{I_o}{I_{PR}} = \frac{0.9415 \, A}{13.89 \, A} \times 100 \% \]
\[ I_o \% = 6.6 \% \]

Typical values: 3-5% of rated for large power transformers.

Transformer Voltage Drop and Impedance

**Example 9-2:** The equivalent resistance and reactance of a 50 kVA, 2400-480 V transformer’s windings are \( R = 2.80 \, \Omega \) and \( X = 6.00 \, \Omega \). (high side). A load of \( 10 \angle 20^\circ \) is connected to the low voltage side. Determine:

a) equivalent impedance of the transformer and load combined
b) primary current if rated voltage is applied to primary
c) voltage across the load.
Example 9-2 Solution (1)

Refer \( Z_L \) to primary side

\[
\begin{align*}
Z_{LHS} &= \alpha^2 Z_L \\
Z_{LHS} &= \bar{Z}_{ep} + \bar{Z}_{LHS} \\
Z_{LHS} &= (5)^2 \left(10 \angle 20^\circ\right) \\
Z_{LHS} &= 250 \angle 20^\circ
\end{align*}
\]

\[
\begin{align*}
\bar{Z}_{in} &= 2.8 + j4.6, 2.8 + j2.9, 2.9 \angle 20^\circ \\
\bar{Z}_{in} &= (2.8 + j4.6) + (2.9 + j2.9) \\
\bar{Z}_{in} &= 5.7 + j7.5 \angle 41.5^\circ \\
\bar{Z}_{in} &= 254.7 \angle 41.5^\circ \Omega
\end{align*}
\]

Ans

Example 9-2 Solution (2)

Find the current and voltage on primary

\[
\begin{align*}
I_p &= \frac{\bar{V}_p}{\bar{Z}_{in}} \\
I_p &= \frac{2900 \angle 20^\circ}{254.7 \angle 62.5^\circ} = 9.42 \angle 62.5^\circ \text{ A} \\
\bar{Z}_{eq} &= 6.62 \angle 62.5^\circ \Omega
\end{align*}
\]

\[
\begin{align*}
\bar{V}_{LHS} &= \bar{V}_p - \bar{Z}_{eq} I_p \\
\bar{V}_{LHS} &= 2900 \angle 20^\circ - (9.42 \angle 62.5^\circ)(6.62 \angle 62.5^\circ) \\
\bar{V}_{LHS} &= 2900 \angle 20^\circ - 62.4 \angle 125^\circ \\
\bar{V}_{LHS} &= 2355 \angle -93.2^\circ \approx 2355 \angle -93.2^\circ \text{ V}
\end{align*}
\]

Ans

Now refer to secondary side

\[
\begin{align*}
\frac{\bar{V}_{LHS}}{\alpha} &= \frac{2355 \angle -93.2^\circ}{5} \\
\bar{V}_{LV} &= 471 \angle 105^\circ \text{ V}
\end{align*}
\]

Ans
End Lesson 9: Practical Transformer Model and Calculations

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