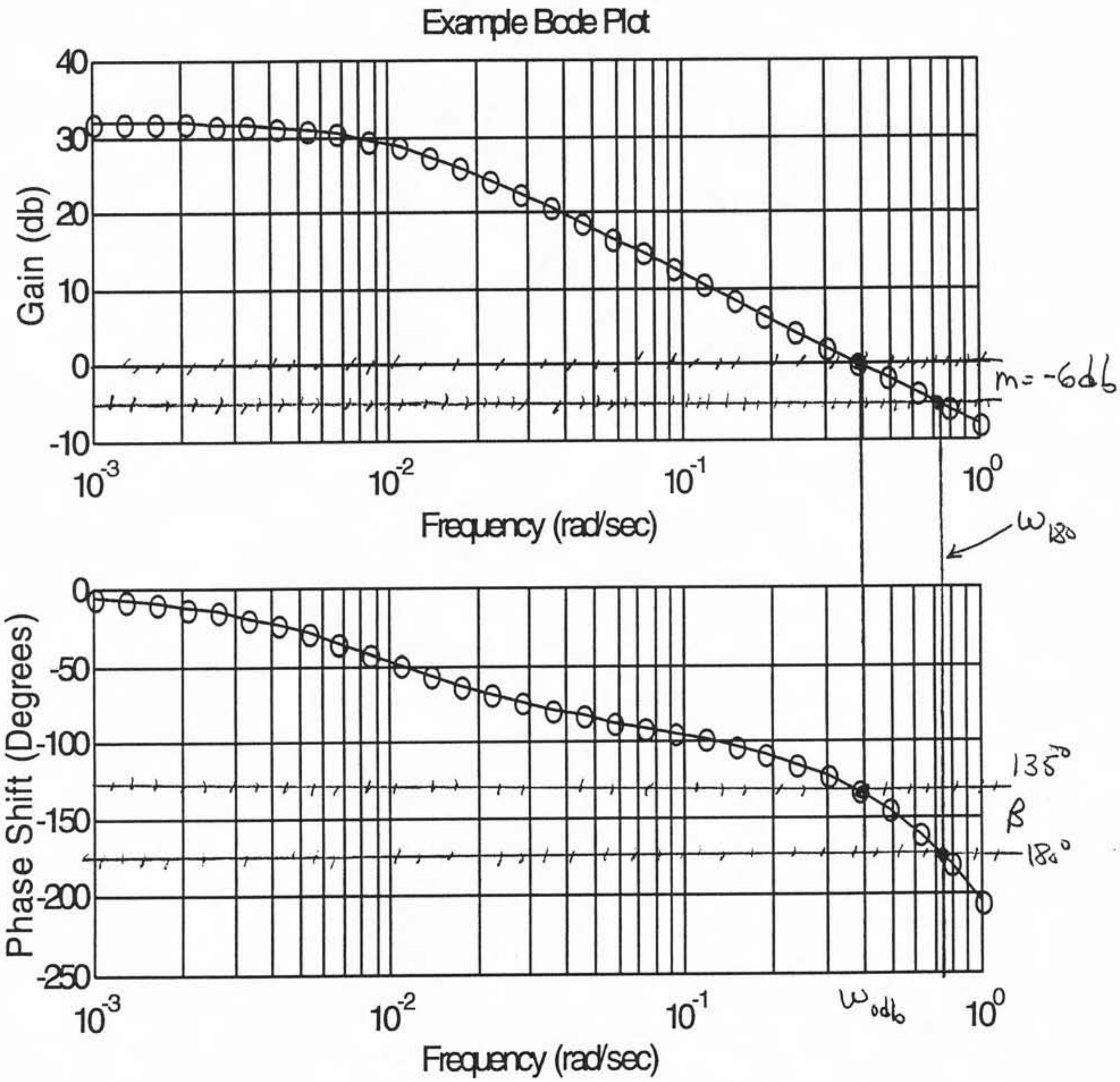


Phase Margin 45° Gain Margin -6db

$180 + (-135) = 45°$

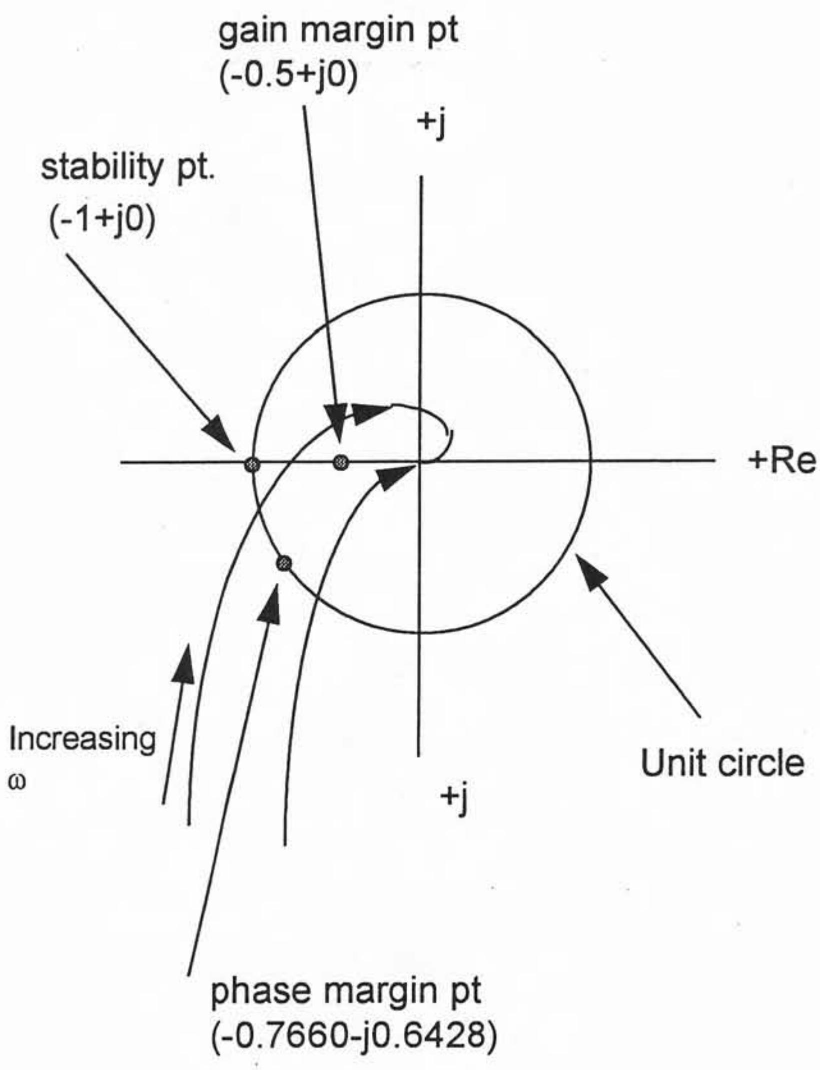
Bode plot for Example System

Nyquist Stability Criterion

Plot complex number that results when s=jω for a number of frequencies

Plot the real and imaginary parts on rectangular coordinates or make polar plots.

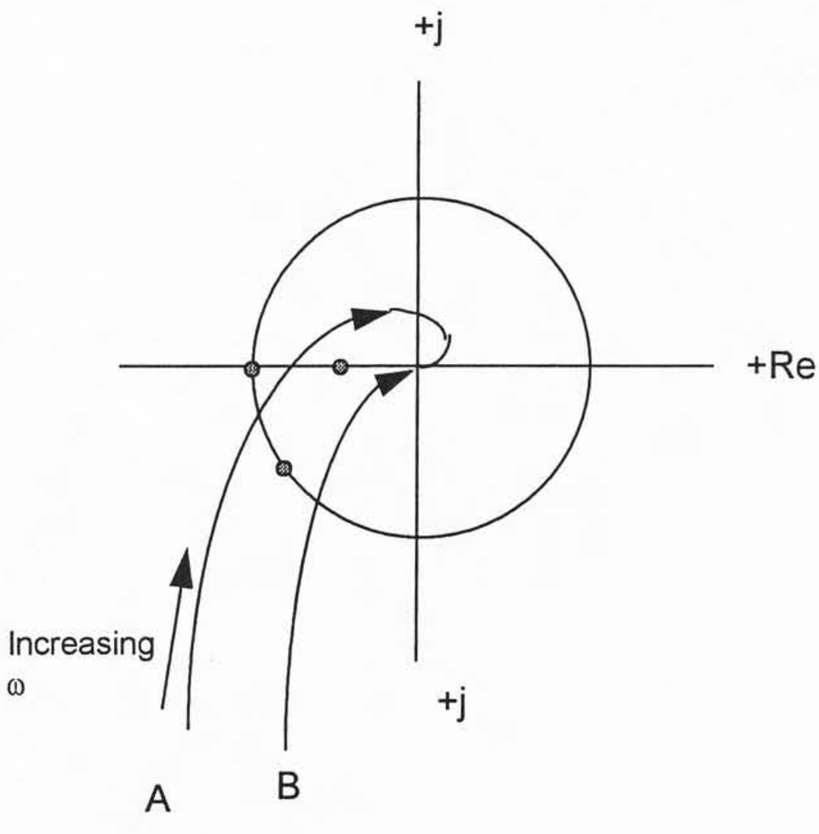
Method used to determine if characteristic equation, 1+G(s)H(s), has positive real parts.



Nyquist Stability Criterion

1.) system is unstable if the plot encircles the stability point (-1+j0)
2.) system is marginally stable if the stability point is not encircled but does encircle the gain point (0.5+j0) or the phase point (-0.7660-j0.6228), or both of these points.
3.) The system is stable if the plot does not encircle stability point, the gain margin point, and the phase margin point.

Example: System A is marginally unstable, while system B is stable.



Example: Proportional controller used to control a 1st order lag process with dead time delay. The transfer functions are given as

$$G(s) = 5 \qquad H(s) = \frac{3.5}{1 + 0.7 \cdot s} \cdot e^{-0.08 \cdot s}$$

a.) find the open-loop transfer function
b.) use Matlab to create a Bode data table
c.) Use Matlab tocontruct an open loop Bode diagram and error ratio graph
d.) determine the gain and phase margins of the system and state whether or not the system is stable.
e.) find the maximum frequency limit
f.) Use Matlab to construct a Nyquist diagram of the system and state whether of not it meets the Nyquist stability criteria.

a.) Open-loop gain

$$G(s) \cdot H(s) = 5 \cdot \left(\frac{3.5}{1 + 0.7 \cdot s} \cdot e^{-0.08 \cdot s} \right) = \frac{17.5}{1 + 0.7 \cdot s} \cdot e^{-0.08 \cdot s}$$

b.) Matlab code to generate a bode table and plot the results

Enter the following code and use it to do homework. Test it with this example. The graphs should match.

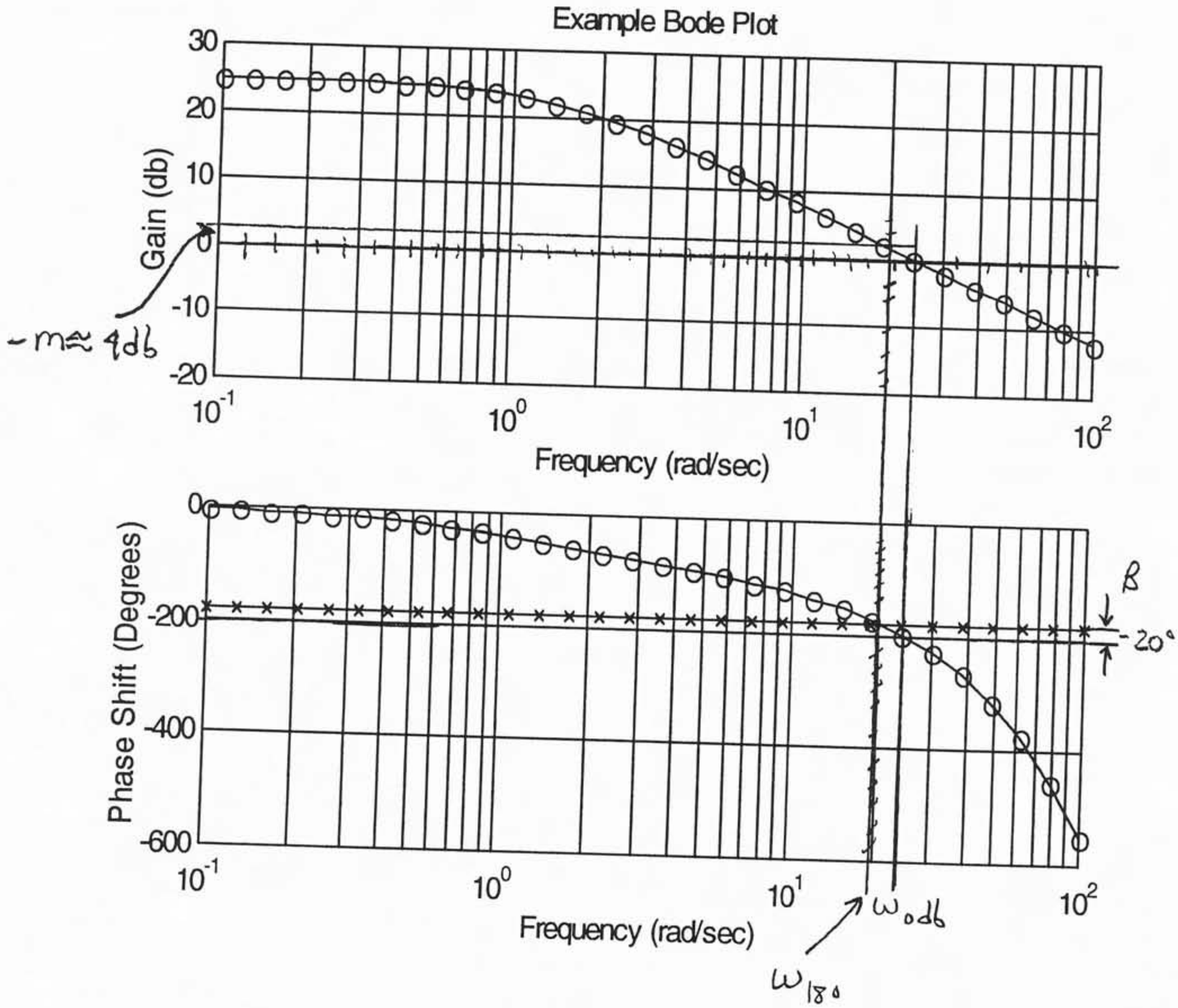
```
% Example bode calculations
clear all;
close all;
% define the forward gain numerator and denominator coefficients
numgh=[17.5];
demgh=[0.7 1];
% construct the transfer function
Gh=tf(numgh,demgh);
% Use the bode function with its arguments so that it returns the
% magnitude, m, the phase shift, p and the frequencies so that
% the effect of the dead time delay can be added to the system
[m p w]=bode(Gh,{0.1,100});
% now compute the values of phase shift for the time delay using the
% formula -2*w*57.6
pd=-0.08*w*57.6;
% Add the the phase shift of the transfer function to the deadtime delay
% take the phase shift out of the 3 column array [d p w]
phase=p(:);
pt=pd+phase;
% make a 180 deg marker line
% length function determines how many freq points exist;
i=length(w);
% use implicit looping to make array that holds -180 in every location
m180(1:i)=-180;
% compute the gain in db
db=20.*log10(m);
% create a figure window
figure;
% divide the plot area in two parts. The first part is for gain
subplot(2,1,1);
semilogx(w,db,'go-');
xlabel('Frequency (rad/sec)');
ylabel('Gain (db)');
title('Example Bode Plot');
grid on;
% second plot area for phase
subplot(2,1,2);
semilogx(w,pt,'go-',w,m180,'rx-');
xlabel('Frequency (rad/sec)');
ylabel('Phase Shift (Degrees)');
grid on;
% print the results to the screen using the fprintf command
% transpose (exchange columns to rows) to allow printing
w=w';
pt=pt';
% print header line
fprintf(1,'f (rad/sec)\tGain (db)\tPhase (deg)\n');
% print formatted data [w; db; pt] assembles data in n columns
% with w db and pt on each row
fprintf(1,'%8.2f\t%8.2f\t%8.2f\n',[w; db; pt]);
% print line return to reposition cursor
fprintf(1,'\n');
```

Table of Results (cut and paste into MS Word)

f (rad/sec)	Gain (db)	Phase (deg)
0.10	24.84	-4.46
0.13	24.83	-5.66
0.16	24.81	-7.17
0.20	24.77	-9.08
0.26	24.72	-11.48
0.33	24.64	-14.49
0.42	24.50	-18.22
0.53	24.30	-22.79
0.67	23.99	-28.30
0.85	23.54	-34.78
1.08	22.89	-42.15
1.37	22.02	-50.21
1.74	20.90	-58.70
2.21	19.55	-67.34
2.81	17.99	-75.96
3.56	16.28	-84.56
4.52	14.44	-93.29
5.74	12.53	-102.45
7.28	10.55	-112.44
9.24	8.55	-123.77
11.72	6.52	-137.06
14.87	4.47	-153.05
18.87	2.42	-172.64
23.95	0.36	-196.95
30.39	-1.71	-227.35
38.57	-3.77	-265.59
48.94	-5.84	-313.84
62.10	-7.91	-374.85
78.80	-9.97	-452.09
100.00	-12.04	-549.98

Looked find on Computer screen

c. and d.) Contruct Bode plot and find phase and gain margins

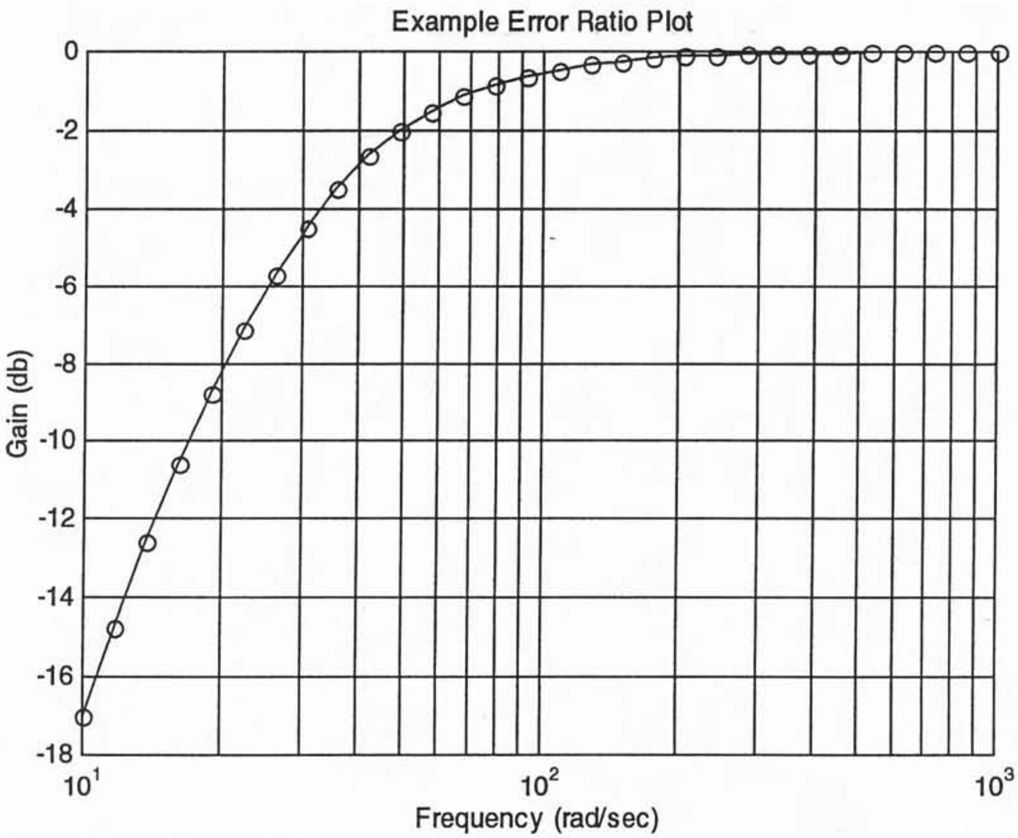


Draw lines and determine the margins from the graph.

$m = -4$ db need -6db
$\beta = 180 + (-20) = 160°$ need $140°$
} unstable system

Example (cont.)

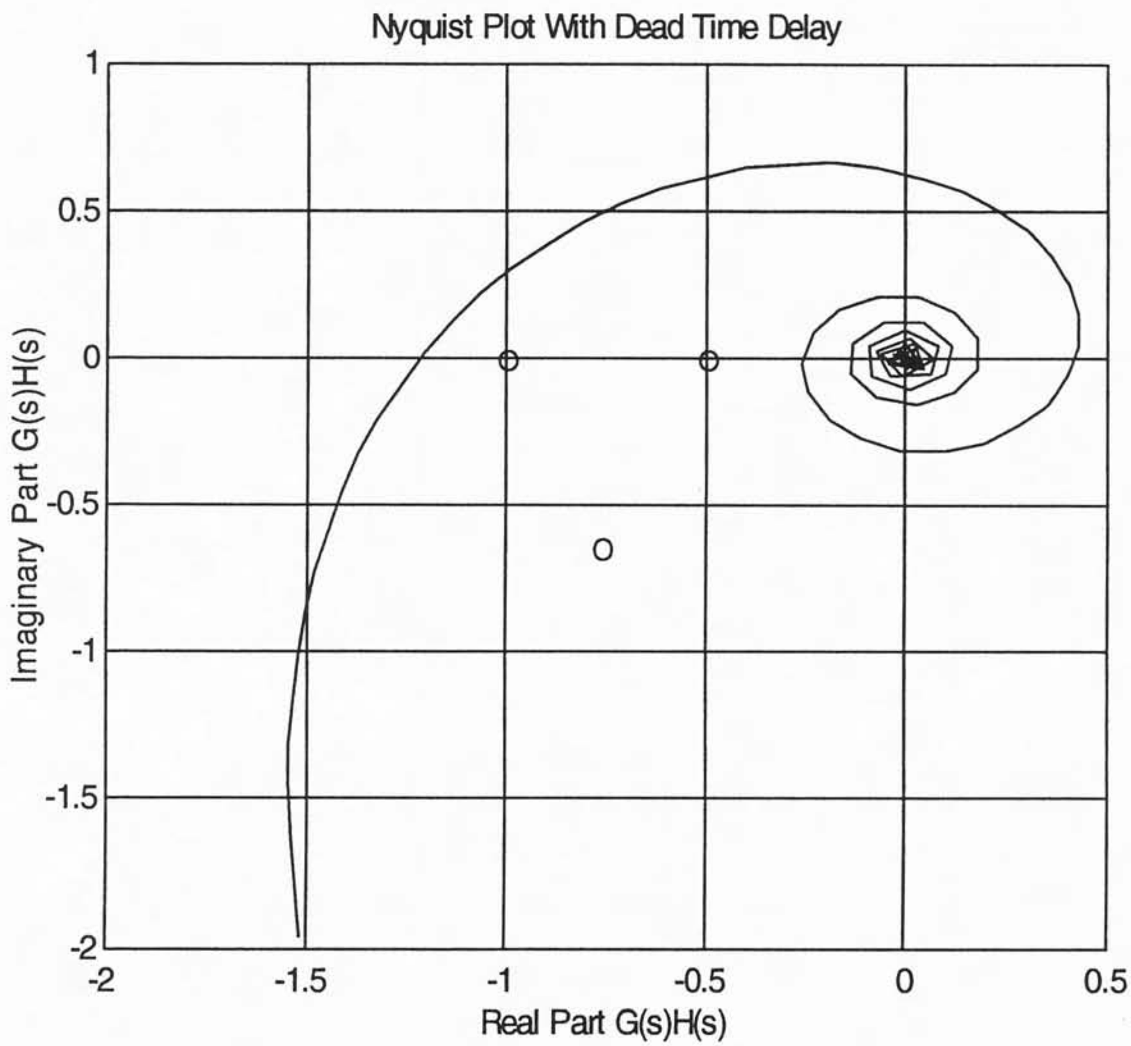
e.) find the frequency limit



The frequency limit is the point when the controller no longer has an effect, Error ratio 0 db
ω = 50 rad/sec

Note: there is no zone 2 in this error ratio plot.

f.) construct a Nyquist plot and determine if the system is stable.



System is not stable because the plot encircles the point (-1+j0) (point is circled).

See handout for complete Matlab program to generate these plots

Stability of Control Systems

For a closed-loop negative feedback system, the roots of the transfer function denominator indicate the systems stability.

All roots must have positive real parts for stability. Negative real parts indicate that transients decay out of system.

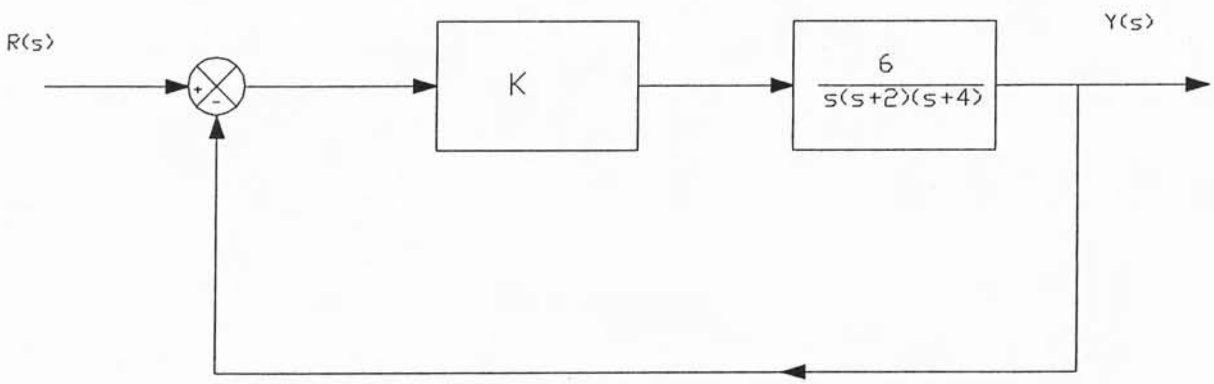
Example   Closed Loop Transfer Function

$$\frac{Y(s)}{R(s)} = \frac{15}{s\cdot\left(s^2 + 3\cdot s + 2\right)} = \frac{15}{s\cdot((s+2)\cdot(s+1))}$$

Roots of above   s = 0, s = -2, s = -1 all roots are negative.

Adding gain to system effects locations of roots

Example



Open loop system transfer function

$$\frac{Y(s)}{R(s)} = \frac{6 \cdot K}{s \cdot (s + 2) \cdot (s + 4)}$$

Poles of the open loop system

$s = 0 \quad s = -2 \quad s = -4$

System stable for any value of K

Find closed-loop transfer function with unity gain feedback

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$G(s) = \frac{6 \cdot K}{s \cdot (s + 2) \cdot (s + 4)}$$

$$\frac{Y(s)}{R(s)} = \frac{\dfrac{6 \cdot K}{s \cdot (s + 2) \cdot (s + 4)}}{1 + \dfrac{6 \cdot K}{s \cdot (s + 2) \cdot (s + 4)}}$$

Multiply top and bottom of expression by s(s+2)(s+4) and multiply factors

$$\frac{Y(s)}{R(s)} = \frac{6 \cdot K}{s^3 + 6 \cdot s^2 + 8 \cdot s + 6 \cdot K}$$

Roots of denominator of closed loop transfer function depend on the gain value

$$s^3 + 6 \cdot s^2 + 8 \cdot s + 6 \cdot K = 0$$

Denominator of transfer function call characteristic equation.

Roots of characteristic equation must all have negative real parts to have stable system.

Stability Testing
Routh-Hurwitz Criteria

Necessary condition for stability - all coefficients of the characteristic polynomial must be positive for all roots to have negative real parts. Not sufficient. Need another test.

Routh-Hurwitz Test - pencil and paper test that can find the number of roots in right-hand plane (positive real parts)

Array of coefficients from characteristic equation generate n rows using algorithm. n = highest power of characteristic equation.

Example $s^3 + 2 \cdot s^2 + 4 \cdot s + 10 = 0$ Characteristic equation

Create table

total number of
rows = number of coefficients

s^3		odd number coefficients a1 a3 etc
s^2		even number coefficients a0 a2 etc.
s^1		computed coefficients
s^0		computed coefficients

number of sign changes
in 1st column indicate number
of roots in RHP

Set up table for the example $s^3 + 2\cdot s^2 + 4\cdot s + 10 = 0$

general form $a_0\cdot s^n + a_1\cdot s^{n-1} + \ldots\ldots + a_n\cdot s + a_n$

pad with zeros

s^3	1	4	0		even number coefficients
s^2	2	10	0		odd number coefficients
s^1	b_0	b_2			computed values
s^0					

$$b_0 = -\frac{-1}{a_1}\cdot\det\cdot\begin{pmatrix} a_0 & a_2 \\ a_1 & a_3 \end{pmatrix}$$

$a_0 := 1 \quad a_2 := 4$

$a_1 := 2 \quad a_3 := 10$

$$b_2 = -\frac{-1}{a_1}\cdot\det\cdot\begin{pmatrix} a_0 & 0 \\ a_1 & 0 \end{pmatrix}$$

No a_4 and a_5. Use padding values

$$b_0 := \frac{-1}{a_1}\cdot\left(a_0\cdot a_3 - a_2\cdot a_1\right)$$

$b_0 = -1$

$$b_2 := \frac{-1}{a_1}\cdot\left(a_0\cdot 0 - a_2\cdot 0\right)$$

$b_2 = 0$

Compute final row. Use two most current rows to make calculation.

s^3	1	4	0	
s^2	2	10	0	
s^1	−1	0		
s^0	b_1			

Divisor element is next element in column 1

$b_0 := -1$ From previous calculation

$$b_1 := -\frac{1}{b_0}\cdot\left(a_1\cdot 0 - a_3\cdot b_0\right) \quad b_1 = 10$$

Table completed when last row is completed

To determine number of roots in Right hand Plane (RHP) count number of sign changes in column 1 of table

For the example

+2 to -1 1
-1 to 10 2 two changes indicate two roots in RHP

Example Determine the number of roots in RHP, if any, for the characteristic equation.

$$s^4 + 4\cdot s^3 + 6\cdot s^2 + 4\cdot s + 2 = 0$$

Example solution

$$s^4 + 4\cdot s^3 + 6\cdot s^2 + 4\cdot s + 2 = 0$$

s^4	1	6	2		
s^3	4	4	0		
s^2	b_0	b_2			
s^1	b_1	b_3			
s^0	C_0				

a_0, a_2, a_4

a_1, a_3 padding 0

$$b_0 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} = \frac{-1}{4}\left[1(4) - 4(6)\right]$$

$$b_0 = -\left[\frac{4-24}{4}\right] = 5$$

Computing S1 Row

s^4	1	6	2		
s^3	4	4	0		
s^2	5	2			
s^1	b_1	b_3			
s^0	C_0				

$$b_2 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_4 \\ a_1 & 0 \end{vmatrix} = -1/4\left[1(0) - 2(4)\right]$$

$$b_2 = -\left[\frac{-8}{4}\right] = 2$$

Use b_0 as divisor and Last two rows of table

$$b_1 = \frac{-1}{b_0}\begin{vmatrix} a_1 & a_3 \\ b_0 & b_2 \end{vmatrix} = -1/5\left[4(2) - 4(5)\right]$$

$$b_1 = -\left[\frac{8-20}{5}\right] = 12/5$$

Computing S^0 row

s^4	1	6	2		
s^3	4	4	0		
s^2	5	2	0		
s^1	12/5	0			
s^0	2				

$$b_3 = -1/b_0\begin{vmatrix} a_1 & 0 \\ b_0 & 0 \end{vmatrix} = -1/5\left[a_1(0) - 0(b_0)\right]$$

$$b_3 = 0$$

No sign changes in Column 1 so No R.H.P. roots (pos. real parts)

$$C_0 = -1/b_1\begin{vmatrix} b_0 & b_2 \\ b_1 & 0 \end{vmatrix} = \frac{-5}{12}\left[5(0) - 2\left(\frac{12}{5}\right)\right]$$

$$C_0 = 2$$

Special Cases for Routh-Hurwitz test

A row of all zeros

Meaning: Pair of roots are on the imaginary axis. System will oscillate when stimulated.

How to compute: Take the coefficients for the row prior to zero row to form an auxiliary equation. polynomial starts with power of row and descends. Solve resulting equation for s.

How to continue: Take derivative of auxiliary equation with respect to s and substitute these coefficients in for zero row.

Example Check the stability of system with the following characteristic equation

$$\underset{a_0}{s^3} + \underset{a_1}{s^2} + \underset{a_2}{2\cdot s} + \underset{a_3}{2} = 0$$

Solution

s^3	1	2	0	
s^2	1	2	0	
s^1	b_0	b_2		
s^0	b_1			

a_0, a_2 pad 0

a_1, a_3 pad 0

$$b_0 = \frac{-1}{a_1}\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix} = \frac{-1}{1}\left[1(2) - 2(1)\right] = 0$$

$$b_2 = \frac{-1}{1}\begin{vmatrix} a_0 & 0 \\ a_1 & 0 \end{vmatrix} = \frac{-1}{1}\left[a_0(0) - a_1(0)\right] = 0$$

Solution Handling Row of Zeros

s^3	1	2	0	
s^2	1	2	0	
s^1	0	0		
s^0				

can not continue. For Auxillary equation from s^2 row
$s^2+2=0$ solve to find pure imaginary roots
$s^2=-2 \Rightarrow s=\pm\sqrt{-2}=\pm j\sqrt{2}$

Take Derivative of Auxillary Equation to continue

s^3	1	2	0	
s^2	1	2	0	
s^1	2	0		
s^0	b_1			

$\frac{d}{ds}(s^2+2)=2s+0$
coefficients of s1 row (2, 0)

Find b_1 using normal Technique

$$b_1=\frac{-1}{b_0}\begin{vmatrix} a_1 & a_3 \\ b_0 & b_2 \end{vmatrix}=\frac{-1}{2}(1(0)-2(2))$$

$b_1=2$

s^3	1	2	0	
s^2	1	2	0	
s^1	2	0		
s^0	2			

No sign changes in column 1
So No roots in RHP

Roots on Imaginary axis indicate bounded but Oscillatory response

Special Cases for Routh-Hurwitz test

Column 1 has a zero value. Division by zero when calculating succeeding values

How to handle it: replace 0 by variable, say ε and continue calculation normally. When table completed let ε go to zero and see if there are any sign changes

Example $s^4 + 3 \cdot s^3 + 4 \cdot s^2 + 12 \cdot s + 12 = 0$

$\uparrow a_0 \quad \uparrow a_1 \quad \uparrow a_2 \quad \uparrow a_3 \quad \uparrow a_4$

s^4	1	4	12		a_0, a_2, a_4
s^3	3	12	0		a_1, a_3
s^2	b_0	b_2			
s^1	b_1	b_3			
s^0	c_0				

$$b_0 = \frac{-1}{3}\begin{vmatrix} 1 & 4 \\ 3 & 12 \end{vmatrix} = \frac{-1}{3}\left[12 - 3(4)\right] = 0$$

$$b_2 = \frac{-1}{3}\begin{vmatrix} 1 & 12 \\ 3 & 0 \end{vmatrix} = \frac{-1}{3}\left[1(0) - 3(12)\right]$$

$$b_2 = 12$$

Substitute for 0

s^4	1	4	12	
s^3	3	12	0	
s^2	$\epsilon \to 0$	12	0	
s^1	b_1	b_3	0	
s^0	C_0			

$$b_1 = \frac{-1}{\epsilon}\left[3(12) - 12\epsilon\right]$$

$$b_1 = \frac{-36 + 12\epsilon}{\epsilon}$$

$$b_3 = \frac{-1}{\epsilon}\left[3(0) - 0(\epsilon)\right] = 0$$

$$C_0 = -\left[\frac{\epsilon}{-36 + 12\epsilon}\right]\begin{vmatrix} \epsilon & 12 \\ \frac{-36+12\epsilon}{\epsilon} & 0 \end{vmatrix}$$

$$C_0 = \left[\frac{-\epsilon}{(-36 + 12\epsilon)}\right]\left[12\left[\frac{-36+12\epsilon}{\epsilon}\right] - 0\right]$$

$$C_0 = 12$$

Evaluate the first column as ε goes to zero

s^4	1	4	12
s^3	3	12	0
s^2	$\epsilon \to 0$	12	0
s^1	b_1	0	
s^0	12		

ϵ positive

$b_1 = \dfrac{-36+12\epsilon}{\epsilon}$ $\epsilon \to 0$ $b_1 = \dfrac{-36}{\epsilon}$ $\epsilon \to 0$

$b_1 \to -\infty$

Sign changes from

0 to $-\infty$ } 2 roots in
$-\infty$ to 12 } R. H. P.

Routh-Hurwitz test useful in finding values of gain parameters that cause system to become marginally stable.

Example

$$s^3 + 6 \cdot s^2 + 8 \cdot s + 6 \cdot K = 0$$

Check to see what value of K cause this characteristic equation to oscillate.

$a_0 = 1$ $a_1 = 6$ $a_2 = 8$ $a_3 = 6K$

s^3	1	8	0
s^2	6	6K	0
s^1	b_0	b_2	
s^0	b_1		

a_0, a_2 Pad 0

a_1, a_3 Pad 0

$b_0 = \dfrac{-1}{6}\begin{vmatrix} 1 & 8 \\ 6 & 6K \end{vmatrix} = \dfrac{-1}{6}\left[6K-48\right]$

$b_0 = \dfrac{48-6K}{6}$

$b_2 = -1/6\begin{vmatrix} 1 & 0 \\ 6 & 0 \end{vmatrix} = 0$

Parameter Solution

s^3	1	8	0	
s^2	6	6K	0	
s^1	b_0	0		
s^0	b_1			

$$b_0 = \frac{48 - 6 \cdot K}{6}$$

$$b_1 = -\frac{1}{b_0} \cdot \det \cdot \begin{pmatrix} 6 & 6 \cdot K \\ b_0 & 0 \end{pmatrix}$$

$$b_1 = -\frac{1}{b_0} \cdot \left(6 \cdot 0 - 6 \cdot K \cdot b_0\right)$$

$$b_1 = \frac{6 \cdot K \cdot b_0}{b_0} = 6 \cdot K$$

s^3	1	8	0	
s^2	6	6K	0	
s^1	b_0	0		
s^0	6K			

For oscillatory Response $b_0=0$ Find K for this case

$$\frac{48 - 6 \cdot K}{6} = 0$$

$$K = -\frac{48}{-6} = 8$$

When K=8 roots only have imaginary parts.

Sine response
For K>8 system unstable
Real part of roots are positive