Learning Objectives

After this presentation you will be able to:

- List the control stability criteria for open loop frequency response.
- Identify the gain and phase margins necessary for a stable control system.
- Use a Bode plot to determine if a control system is stable or unstable.
- Generate Bode plots of control systems the include dead-time delay and determine system stability.
Bode Plot Stability Criteria

- **Stable Control System**: Open loop gain of less than 1 (G<1 or G<0dB) at open loop phase angle of -180 degrees.
- **Oscillatory Control System Marginally Stable**: Open loop gain of exactly 1 (G=1 or G=0dB) at open loop phase angle of -180 degrees.
- **Unstable Control System**: Open loop gain of greater than 1 (G>1 or G>0dB) at open loop phase angle of -180 degrees.

Phase and Gain Margins

Inherent error and inaccuracies require ranges of phase shift and gain to insure stability.

- **Gain Margin** – Safe level below 1 required for stability. Minimum level: G=0.5 or -6 dB at phase shift of 180 degrees.
- **Phase Margin** – Safe level above -180 degrees required for stability. Minimum level: $\phi=40$ degree or $-180+40=-140$ degrees at gain level of 0.5 or 0 dB.
Determining Phase and Gain Margins

Define two frequencies: \( \omega_{\text{odB}} \) = frequency of 0 dB gain
\( \omega_{180} \) = frequency of -180 degree phase shift

1. Draw vertical lines through 0 dB on gain and -180 on phase plots.
2. Draw horizontal lines through 0 dB and -180 so that they intersect with the vertical lines.
3. Draw two more horizontal lines that intersect the -180 line on the gain plot and the 0 dB line on the phase plot.

Read Phase Margin here \(-180 + \beta_{\text{odB}}\)

Read Gain Margin \(-m_{180}\) dB
Stability Analysis Using Bode Plots

Bode plot stability analysis is idea for systems with dead-time delay. Delay represented by phase shift that increases with frequency.

Example 22-1: A first order lag process has a dead-time delay of 2 seconds and is controlled by a proportional controller. The open loop transfer function is:

\[ GH(s) = 40 \cdot \left[ \frac{1}{1 + 100s} \right] \cdot e^{-2s} \]

1) Find the magnitude and phase angle of the transfer function at the following frequencies: \( \omega = 0.001, 0.01, 0.1 \) and 1 radian/sec using hand calculations.

2) Use MatLAB and construct the Bode plots of the system and then determine the gain and phase margin of the system.

Example 22-1 Solution (1)

Substitute \( j\omega = s \)

\[ GH(j\omega) = 40 \cdot \left[ \frac{1}{1 + 100j\omega} \right] \cdot e^{-2j\omega} \]

Where \( e^{-2j\omega} = \begin{cases} G = 1 \text{ for all } \omega \\ \phi = -2 \cdot (57.6) \cdot \omega \end{cases} \)

For \( \omega = j0.001 \)

\[ \left[ \frac{1}{1 + 100j0.001} \right] = \left[ \frac{1}{1 + j0.1} \right] = \frac{1}{1.005 \angle -5.71^\circ} = 0.995 \angle -5.71^\circ \]

\[ e^{-2j0.001} = \begin{cases} G = 1 \text{ for all } \omega \\ \phi = -2 \cdot (57.6) \cdot 0.001 = -0.115^\circ \end{cases} = 0.995 \angle -0.115^\circ \]

\[ |GH(j0.001)| = 40 \cdot 0.995 \cdot 1 = 39.8 \]

\[ \text{ang}[GH(j0.001)] = -5.71^\circ + (-0.115^\circ) = -5.825^\circ \]
Example 22-1 Solution (2)

For \(j\omega = j0.01\)

\[
GH(j0.01) = \left[ \frac{1}{1+100j0.01} \right] = \left[ \frac{1}{1+ j \cdot 1} \right] = \frac{1}{1.414\angle45^\circ} = 0.707\angle-45^\circ
\]

\[
e^{-2j0.01} = \begin{cases} 
G = 1 \text{ for all } \omega \\
\phi = -2 \cdot (57.6) \cdot 0.01 = 1\angle -1.15^\circ
\end{cases}
\]

\[
|GH(j0.01)| = 40 \cdot 0.707 \cdot 1 = 28.28 \\
\text{ang}[GH(j0.01)] = -45^\circ + (-1.15^\circ) = -46.15^\circ
\]

For \(j\omega = j0.1\)

\[
GH(j0.1) = \left[ \frac{1}{1+100j0.1} \right] = \left[ \frac{1}{1+ j \cdot 1} \right] = \frac{1}{10.05\angle84.3^\circ} = 0.0995\angle-84.3^\circ
\]

Example 22-1 Solution (3)

For \(j\omega = j0.1\) cont.

\[
e^{-2j0.1} = \begin{cases} 
G = 1 \text{ for all } \omega \\
\phi = -2 \cdot (57.6) \cdot 0.1 = 1\angle -11.52^\circ
\end{cases}
\]

\[
|GH(j0.1)| = 40 \cdot 0.0995 \cdot 1 = 3.98 \\
\text{ang}[GH(j0.1)] = -84.3^\circ + (-11.52^\circ) = -95.82^\circ
\]

For \(j\omega = j1\)

\[
GH(j1.0) = \left[ \frac{1}{1+100j1} \right] = \left[ \frac{1}{1+ j \cdot 100} \right] = \frac{1}{100\angle89.4^\circ} = 0.01\angle-89.4^\circ
\]

\[
e^{-2j0.1} = \begin{cases} 
G = 1 \text{ for all } \omega \\
\phi = -2 \cdot (57.6) \cdot 1 = 1\angle -115.2^\circ
\end{cases}
\]

\[
|GH(j0.1)| = 40 \cdot 0.01 \cdot 1 = 0.4 \\
\text{ang}[GH(j0.1)] = -89.4^\circ + (-115.2^\circ) = -204.6^\circ
\]
Example 22-1 Solution (4)

Calculation summary

Convert all magnitudes to decibels

\[ \text{GH}(j0.001)_{\text{db}} = 20 \log(39.8) = 32 \text{ dB} \]

\[ \text{GH}(j0.01)_{\text{db}} = 20 \log(28.28) = 29 \text{ dB} \]

\[ \text{GH}(j0.1)_{\text{db}} = 20 \log(3.98) = 12 \text{ dB} \]

\[ \text{GH}(j1)_{\text{db}} = 20 \log(0.4) = -8.0 \text{ dB} \]

<table>
<thead>
<tr>
<th>Frequency (rad/sec)</th>
<th>GH</th>
<th>GH (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>39.8\angle-5.83°</td>
<td>32\angle-5.83° dB</td>
</tr>
<tr>
<td>0.01</td>
<td>28.28\angle-46.15°</td>
<td>29\angle-46.15° dB</td>
</tr>
<tr>
<td>0.1</td>
<td>3.98\angle-95.82°</td>
<td>12\angle-95.82° dB</td>
</tr>
<tr>
<td>1.0</td>
<td>0.4\angle-204.6°</td>
<td>-8\angle-204.6° dB</td>
</tr>
</tbody>
</table>

Example 22-1 Solution (5)

Construct an open-loop Bode plot using MatLAB and find the gain and phase margins for the control system. Example code follows:

```matlab
clear all; close all;
numgh=[40]; % define the forward gain numerator and denominator coefficients
demgh=[100 1];
Gh=tf(numgh,demgh); % construct the transfer function
[m p w]=bode(Gh,{0.001,1}); % Use the bode function with its arguments so that it returns the % magnitude, m, the phase shift, p and the frequencies so that % the effect of the dead time delay can be added to the system % now compute the values of phase shift for the time delay using % the formula -2*w*57.6
pd=-2*w*57.6;
```
Example 22-1 Solution (6)

% Add the phase shift of the transfer function to the dead-time delay
% take the phase shift out of the 3 column array \([m\ p\ w]\)

```matlab
phase=p(:);
pt=pd+phase; % compute the gain in db

db=20.*log10(m);
figure; % create a figure window
subplot(2,1,1); % divide the plot area in two parts
semilogx(w,db,'go-'); %plot gain in dB on a semilog x-axis
xlabel('Frequency (rad/sec)'); % add labels and title. Turn on the grid.
ylabel('Gain (db)');
title('Example Bode Plot');
grid on;
subplot(2,1,2); % now do the same for the phase shift plot
semilogx(w,pt,'go-');
xlabel('Frequency (rad/sec)');
ylabel('Phase Shift (Degrees)');
grid on
```

Example 22-1 Solution (7)

```
180-135 =45
```

Gain Margin
-6 dB

Phase Margin
\(\beta=45^\circ\)
End Lesson 22: Determining Control Stability Using Bode Plots