Lesson 9: Proportional Control Action

After this presentation you will be able to:

- Identify the components of a proportional feedback control system.
- Write a mathematical model for a proportional controller.
- Compute the proportional bandwidth of a proportional controller.
- Explain the relationship between steady-state error and proportional gain.
- Perform lab experiment 2 more effectively.
Proportional Control Action

Simplest form of controller - amplify error and apply signal to the process through final control element.

Basic control loop

Example: DC Motor Speed Control
Proportional Control Math Relationship

Amplify error and send to final control element.

\[ C_o = K_p e + C_b \]

Where
- \( C_o \) = the controller output
- \( C_b \) = the controller output with \( e = 0 \)
- \( K_p \) = the proportional gain
- \( e \) = the control error

\[ e = SP - \text{Measurement}, \]

\( e \) = error signal
\( SP \) = setpoint value
\( \text{Measurement} \) = sensor measurement

Proportional Control Action

Measurement > SP  \( e \) negative - output decrease

\[ C_o = K_p e + C_b \]

Measurement < SP  \( e \) positive - output increase

\[ C_o = K_p e + C_b \]
Practical controller output devices have limits on \( C_o \) and final control element. Examples: flow value position, motor power supply.

Controller Output Limits

At higher \( K_p \), \( e \) produces more correction but reduces the range between controller limits.

\[
K_1 > K_2 > K_3
\]

Band increases as gain decreases

Proportional Band
Computing Proportional Band and $K_p$

Proportional gain - change in output / change in error

$$K_p = \frac{\Delta C_o}{\Delta e} = \frac{\Delta \text{Output}}{\Delta (\text{SP} - \text{Measurement})}$$

Increasing proportional gain decreases proportional band (PB). In terms of percent:

$$\% \text{PB} = \left( \frac{1}{K_p} \right) \cdot 100\%$$

$\% \text{PB}$ inversely proportional to $K_p$

Proportional Control Action

**Example 9-1:** A proportional controller uses an OP AMP with $\pm 15 \text{ Vdc}$ output limits when the error input is $\pm 3 \text{ Vdc}$. Saturation of the OP AMP sets these limits. Find the proportional gain and the percent proportional bandwidth from this information. Determine the proportional band and max output if $K_p = 2$.

Find $K_p$ by using

$$K_p = \frac{\Delta C_o}{\Delta e}$$

Find the $\% \text{PB}$

$$0\% \text{ PB} = \left( \frac{1}{K_p} \right)\infty\%$$

$$\% \text{ PB} = \left( \frac{1}{\xi} \right) (100\%) = 20\%$$

20% change in error gives maximum change in output
Example 9-1 Solution

Now set the value of proportional gain to 2 and compute the %PB and output

\[
\%PB = \frac{1}{K_p} (100\%) \\
\%PB = \left(\frac{1}{2}\right) (100\%) = 50\%
\]

\[
K_p = \frac{\Delta \text{output}}{\Delta e} \Rightarrow \Delta \text{output} = \Delta e K_p
\]

\[
\Delta e = 3 - (-3) = 6 \Rightarrow \Delta \text{output} = 6 \cdot (2) = \pm 12 \text{Vdc}
\]

Lower gain allows the OP AMP to handle more error before saturation.

Characteristics of Proportional Control

Control formula \( V_o = K_p e + V_b \)

There is a band of steady-state error about 0 with magnitude of PB% where the output is not saturated.

If \( e = 0 \), then \( V_o = V_b \)

If \( e \neq 0 \), then every unit of error produces \( K_p(e) \) units of correction that is added/subtracted to \( V_o \).
Control Offset and Residual Error

Proportional control always produces a steady-state or residual error when a change in the process load occurs. (When \( e \neq 0 \))

Increasing \( K_p \) decreases steady-state error but cannot eliminate it.

Permanent load changes produce permanent error. For transient load changes, the error returns to bias value, 0.

Increased Gain & Residual Error

High gain reduces steady-state error but increases chances of instability

**Example 9-2:** A proportional controller with a gain of 2 has an output range of 0-15 Vdc for an error input range of 0 to \( \pm 1 \) Vdc. The output has a balance value of 7.5 Vdc. Determine: a.) controller output when error is zero, b.) residual error when a process load change produces an error voltage of -0.25 Vdc, c.) residual error when control gain increases to 5.
Example 9-2 Solution (1)

For $K_p=2$, $V_o=C_o=0$, $V_{dc}=15$ Vdc, and $V_e=e=0$ to $\pm$1 Vdc. Also $V_b=C_b=7.5$ Vdc.

- $C_o = K_p V_e + C_b$
- $V_o = K_p V_e + V_b$
- $C_o = 2V_e + 7.5$

At $V_e=0$

- $V_o = 2(0) + 7.5 V$
- $V_o = 7.5$

No residual error (Steady-state error)

Process load change produces... Find residual error....

- $V_e = -0.25 V$
- $V_o = 2(-0.25V) + 7.5$
- $V_o = 7.0$

Ans

Ans

Example 9-2 Solution (2)

For residual (steady-state) error to reach 0, $K_p$ must increase to infinity.

Set $K_p=5$ and compute new residual error

$$\frac{V_o}{K_p} = \frac{2.0}{5} = 0.4 V_{dc}$$

Residual or steady-state error decreases inversely as the proportional gain increases
Computing residual error

\[ \% \text{ error} = \frac{SS_d - SS_a}{SS_d} \]

Where:
- \( SS_d \) = desired steady-state output (1 for unit step)
- \( SS_a \) = actual output at steady-state
- \( \% \text{ error} \) = percentage of residual error based on desired value.

Plot the change in residual error as the proportional gain increases using a typical control system.

Low values of \( K_p \) produce high residual error.
Model of Proportional System (Bias = 0)

Block Diagram:

\[ x(t) = \text{input function} \]
\[ K_p = \text{Proportional controller gain} \]
\[ \text{Plant is modeled using RC circuit} \]
\[ r(t) = \text{output response of the control system} \]
\[ K_d = \text{feedback gain (voltage divider in lab)} \]

Proportional System Time Response

**Method of solution:** Signal flow algebra and Laplace transforms

Final solution to step change input:

\[
r(t) = \frac{K_p}{1 + K_p \cdot K_d} \left[ e^{-\frac{t}{R C}} - \frac{1}{e^{\frac{-t}{R C}}} \right]
\]

*\(K_p\) affects response speed

*\(K_p\) affects final value

Plot the response of this system to a step input (0-1) for several values of proportional gain and compare them based on response speed and residual error.
Proportional System Time Response

Comparison of response time and residual errors

End Lesson 9: Proportional Control Action