ET 304b
Laboratory 3
Application of Ac Bridges-Inductance Measurements

Objectives: Construct a Maxwell ac bridge circuit and use it to measure inductors. Compare the results of the constructed bridge with the commercial bridge found in the lab. Compare the results of the Maxwell bridge at several frequencies to verify that the circuit is insensitive to frequency. Setup an oscilloscope to measure differential voltages and use it as the null detector of the bridge.

Theoretical Background

The ac bridge circuit has a number of measurement applications, particularly the measurement of unknown capacitance and inductance values. A properly designed ac bridge circuit can make very accurate measurements of these quantities because the bridge is based on a null indication. A standard value is compared to an unknown, and the bridge circuit is adjusted for a balance. When a bridge is balanced the accuracy of the measurement is related to the accuracy of the bridge components not the null detector itself. Highly accurate bridge components will produce highly accurate component measurements.

The general ac bridge circuit in Figure 1 is balanced when the detector branch current is zero which implies that $\mathbf{V}_{ba}=\mathbf{V}_{bc}$. All quantities are phasors. Since the current through the detector is zero the

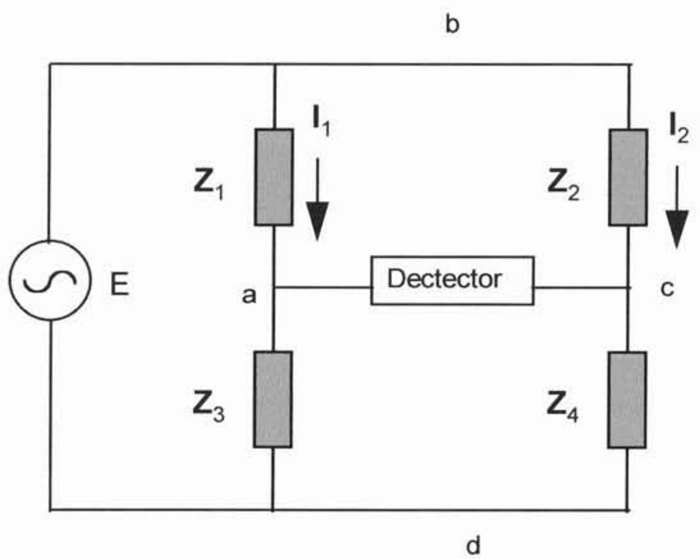

Figure 1. Generalized Ac Bridge Circuit

currents $\mathbf{I}_1$ and $\mathbf{I}_2$ can be expressed by solving two parallel circuit branches.

$$\mathbf{I}_1 = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_3} \tag{1}$$

$$\mathbf{I}_2 = \frac{\mathbf{E}}{\mathbf{Z}_2 + \mathbf{Z}_4} \tag{2}$$

Writing the voltages $\mathbf{V}_{ba}$ and $\mathbf{V}_{bc}$ in terms of the currents $\mathbf{I}_1$ and $\mathbf{I}_2$ gives:

$$\mathbf{I}_1 \cdot \mathbf{Z}_1 = \mathbf{I}_2 \cdot \mathbf{Z}_2 \tag{3}$$

Combining equations (1), (2), and (3) gives the phasor balance conditions for a generalized ac bridge.

$$\mathbf{Z}_1 \cdot \mathbf{Z}_4 = \mathbf{Z}_2 \cdot \mathbf{Z}_3 \tag{4}$$

Equation (4) states that for the ac bridge to be balanced the product of the pairs of impedances on opposite sides of the bridge must be equal. Figure 2 shows the grouping of the impedances for the balance equation.

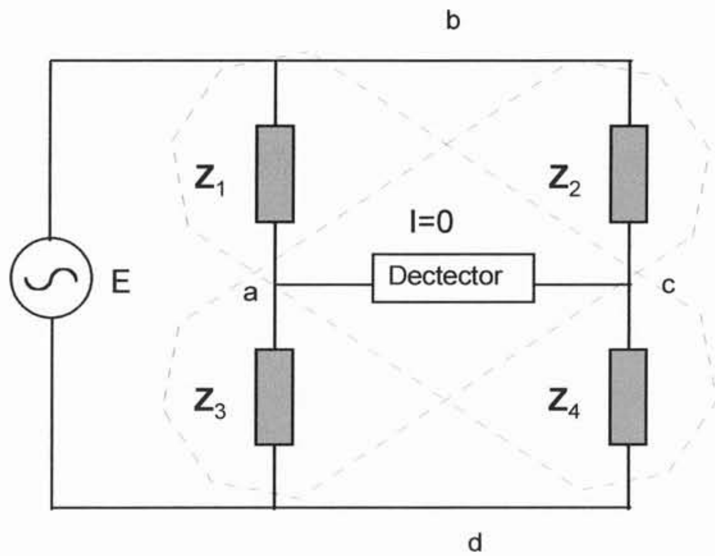

Figure 2. Impedance Groupings at Balance.

Equation (4) implies that the bridge impedances must meet two conditions simultaneously at balance:

1.) the magnitudes of the impedance products must be equal.
2.) the sum of the phase angles of the impedance products must be equal.

When the bridge impedances are in polar form, equation 4 decomposes into the following two equations:

$$Z_1 \cdot Z_4 = Z_2 \cdot Z_3 \tag{5a}$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3 \tag{5b}$$

In words, equation (5a) states that the product of magnitudes of the impedances on opposite arms must be equal at balance. Equation (5b) states the sum of the phase angles of the impedances on opposite arms must be equal at balance.

Two conditions must be met at balance, so two arms of the bridge must be adjustable for the criteria of equations (5a-b) to be obtained in practical circuits. Since the conditions must be met simultaneously, repeated adjustments of both variable quantities are usually necessary when balancing an ac bridge circuit.

The Maxwell bridge shown in Figure 3, compares an unknown inductor to a known value of capacitance. This bridge is limited to measurement of coils with medium Q ($1<Q<10$).

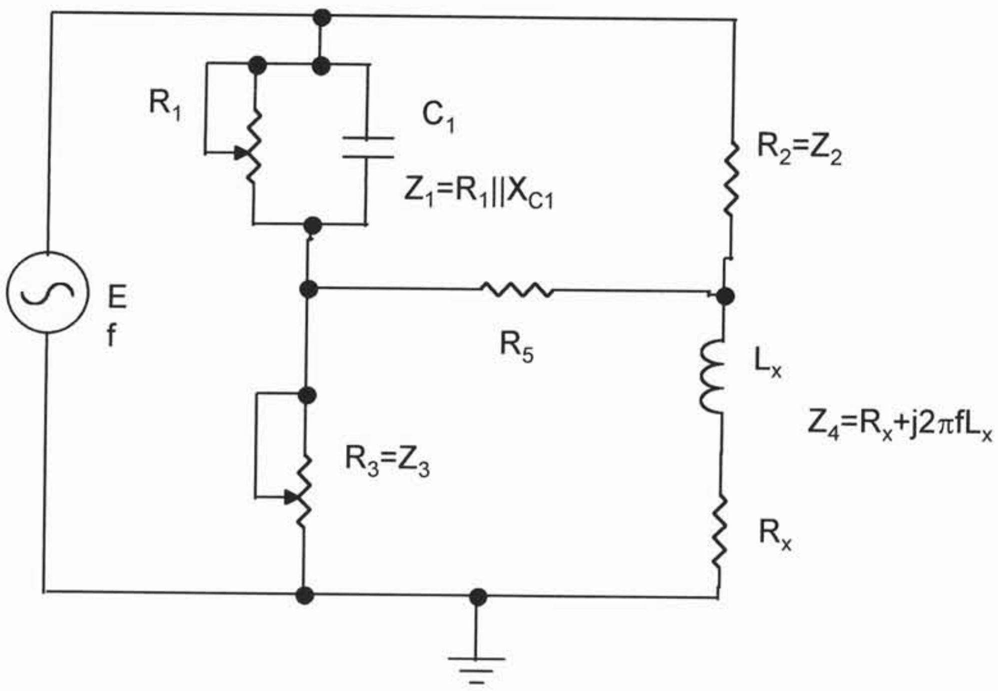

Figure 3. Maxwell Ac Bridge Circuit for Inductance Measurement.

Solving the balance equation for the unknown impedance, $\mathbf{Z}_4$, of the inductor gives:

$$\mathbf{Z}_4 = \mathbf{Z}_2 \cdot \mathbf{Z}_3 \frac{1}{\mathbf{Z}_1}$$

where:
$$\frac{1}{\mathbf{Z}_1} = \mathbf{Y}_1 = \frac{1}{R_1} + j \cdot 2\pi \cdot f \cdot C_1$$

Substituting the bridge component parameters into the above equation relates the unknown quantities to the bridge component values.

$$R_x + j \cdot 2\pi \cdot f \cdot L_x = R_2 \cdot R_3 \left(\frac{1}{R_1} + j \cdot 2\pi \cdot f \cdot C_1 \right)$$

This is the rectangular form of the complex impedances and can be decomposed into two equations. Equating the real parts on both sides of the equation relates the unknown inductor resistance to the bridge arm resistances.

$$R_x = \frac{R_2 \cdot R_3}{R_1} \tag{6}$$

Equating the imaginary parts gives the unknown inductance in terms of the arm resistances and the known value of capacitance.

$$L_x = R_2 \cdot R_3 \cdot C_1 \tag{7}$$

Adjusting the quantities R_1 and R_3 will balance the bridge. Measuring these values at balance and applying equations (6) and (7) will give the parameters of the inductor.

Equations (6) and (7) are independent of the source voltage magnitude and the source frequency. This means that theoretically the accuracy of the measured values will only depend on the accuracy of the resistors and the capacitor. In practice, the source frequency and voltage magnitudes will have some effect on the accuracy of the measured values. This is due to the frequency response of the null detector and the stray capacitive effects when frequency becomes very high. It may also be difficult to balance the bridge as frequency changes which will effect the accuracy. This is due to the Q limits of this bridge.

The procedure for balancing the Maxwell bridge is:

1.) adjust R_3 for inductive balance (minimum null indication). This is the phase angle balance condition.
2.) adjust R_1 for resistive balance (minimum null indication). This is the magnitude balance condition.
3.) repeat 1 and 2 until the null indicator reads zero.

Step three is necessary because the value of R_3 appears in both equations. Adjusting one resistor value will upset the balance of the other condition. This process must be repeated and gives slow convergence to a final balance for coils that are outside the given Q range. For medium Q coils, the interaction is not pronounced and balance is achieved quickly.

If an adjustable capacitor is used for C_1 and R_3 is fixed, then it is not necessary to make repeated adjustments. Adjusting C_1 will give the inductive balance and adjusting R_1 will balance the resistance. This method is not always practical because an adjustable capacitor with sufficient range may not be available.

Procedure

1.) Construct the Maxwell bridge circuit show in Figure 4 with the values shown of the schematic. Use the inductance substitution box for the value of L_x.

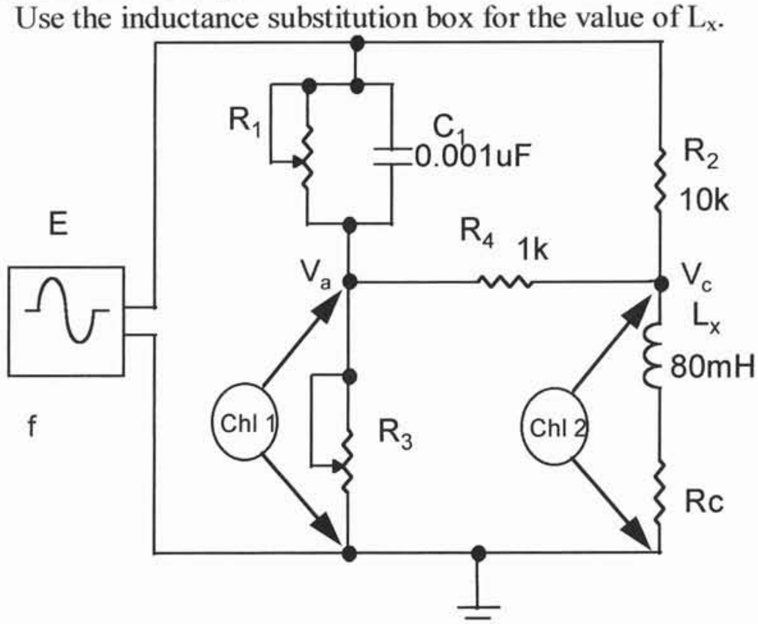

Figure 4. Maxwell Bridge Showing Values.

Compute the values of R_1 and R_3 necessary to balance the bridge using equations (6) and (7). Assume that L_x=80 mH and Rc=200 ohms. Select potentiometers for R_1 and R_3 such that the computed resistance values at balance are between 40% and 60% of the full resistance of the potentiometer. Use 10-turn potentiometers if possible.

2.) With the potentiometers R_1 and R_3 out of the bridge, adjust them so that the resistance is approximately 50% of the full value. Install them in the bridge after making this rough adjustment.

3.) Set the function generator to produce a sine wave with an amplitude of 10V rms at 1000 Hz. Connect the generator to the bridge.

4.) The oscilloscope will act as the null indicator. It will measure the voltage difference between V_a and V_b. Connecting the channels of the scope as shown displays the voltages V_a and V_b referenced to ground. To measure the difference follow the following procedure:

    a.) make sure that both probes are in the 10x setting.
    b.) set the ranges of both channels to the same value.
    c.) find the add channel function switch on the scope face and activate it.
    c.) find the invert channel 2 function switch and activate it.

    The scope will now read the difference between the voltages. As the bridge is balanced, lower the range switches of both channels to observe the reduction in voltage. Continue to balance the bridge using the procedure given in the theory section until the scope reads zero.

5.) When the bridge is balanced, carefully remove the potentiometers from the circuit and measure the unshorted resistance of the potentiometers. Record these values in Table 1.

6.) Repeat the balancing and resistance measurements for the following values from the inductor substitution box: 100 mH, 40 mH, and 20 mH. Place the values of R_1 and R_3 in Table 1.

7.) Measure the same inductor values using the commercial RLC bridge located in the lab. Record the reading in Table 1 also.

8.) Replace the 80 mH inductor in the bridge and rebalance the circuit. Record the resistance values necessary to balance the bridge in Table 2. Compute the inductance and resistance values using the bridge formulas. Enter these values in Table 2.

9.) Change the source frequency to 2000 Hz and rebalance the bridge if necessary. If rebalancing is necessary record the values of R_1 and R_3 in Table 2 Compute the inductance and resistance values and enter them in Table 2.

10.) Repeat step 9 for f=5000 and 7500 Hz.

11.) Calculate the percentage error between the commercial bridge reading of the inductor and the Maxwell bridge values for each frequency in Table 2. Include these results in the report.

12.) Calculate the percentage error between the commercial bridge reading of the inductor and the Maxwell bridge values for every nominal value in Table 1. Include these results in the report.

Discussion Points

Was the theory from the class confirmed by these tests within the experimental error? From the results, is the Maxwell bridge actually frequency insensitive as the equations imply? Could the difference be attributed to other factors? How close to the commercial bridge were the Maxwell bridge measurements?

Experiment 3 Data
Maxwell Inductance Bridge Measurements

Table 1-Maxwell Bridge and Commercial Bridge

			Maxwell Bridge		Commercial Bridge	
Nominal Value	R_1 (kΩ)	R_3 (kΩ)	Rc (Ω)	L_x (mH)	Rc (Ω)	L_x (mH)
100 mH						
80 mH						
40 mH						
20 mH						

Table 2-Frequency Sensitivity of Maxwell Bridge

			Maxwell Bridge		Commercial Bridge	
Frequency	R_1 (kΩ)	R_3 (kΩ)	Rc (Ω)	L_x (mH)	Rc (Ω)	L_x (mH)
1000 Hz						
2000 Hz						
5000 Hz						
7500 Hz						

Nominal value of inductance L_x=80 mH for all frequencies

Error Formulas

$$\frac{L_{com} - L_{max}}{L_{com}} \cdot 100\% = \text{Lerror}$$

$$\frac{R_{com} - R_{max}}{R_{com}} \cdot 100\% = \text{Rerror}$$

Where L_{com}, R_{com} are the L and R value from the commercial bridge measurement
L_{max}, R_{max} are the L and R values from the Maxwell bridge measurement