# Lesson 19: Non-Linear Operation of Dc Motors

ET 332a
Dc Motors, Generators and Energy Conversion Devices



## Learning Objectives

- Explain how saturation can affect the performance of a dc motor
- Explain a piece-wise linear approximation can represent a non-linear magnetization curve
- Use mathematical relationships to represent the magnetomotive force in dc motors
- Solve non-linear dc motor problems given a magnetization curve.

# Non-linear Dc Motor Model

Non-linear operation includes effects of magnetic saturation.

1.) Must have magnetization curves of machine.

2.) Gives more accurate analysis for operation of motors/generation at extreme operating points and during transients.



# Non-Linear Dc Motor Model

Need to represent the motor's net magnetomotive force

For compound motors, include both shunt and series fields

$$\mathfrak{F}_{net} = \mathfrak{F}_f + \mathfrak{F}_s - \mathfrak{F}_d$$

Where:

$\mathfrak{F}_f$ = mmf of shunt field (A-t/pole) $N_f(I_f)$

$\mathfrak{F}_s$ = mmf of series field (A-t/pole) $N_s(I_a)$

$\mathfrak{F}_d$ = equivalent demagnetizing due to armature reaction (A-t/pole)

With compensating winding $\mathfrak{F}_d = 0$

# Non-Linear Dc Motor Model

Magnetic saturations affects both motor speed and torque

Remember

$$n = \frac{V_T - I_a \cdot (R_{acir})}{k_G \cdot \Phi_p} \qquad T_D = k_m \cdot \Phi_p \cdot I_a$$

Relate flux to motor parameters

$$\Phi_p = \frac{\mathfrak{F}}{\mathfrak{R}} \qquad \mathfrak{F} = N \cdot I_f \qquad \Phi_p = \frac{B}{A} \Rightarrow \Phi_p \cdot A = B$$

Use B vs $\mathfrak{F}$ curves to determine performance with saturation

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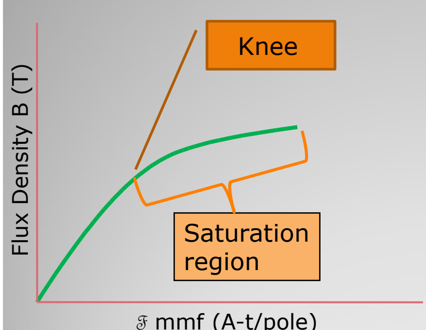
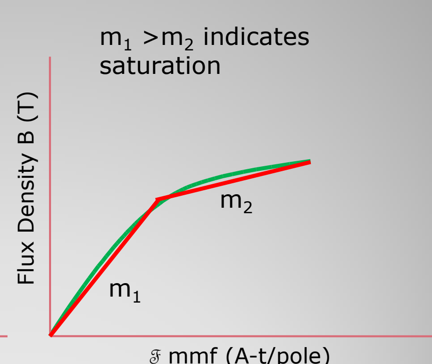



Typical magnetizing curve




Can use piece-wise linear approximation of curve

# Magnetizing Curves

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# Motor Performance and Saturation

**Example 19-1**: A 1200 rpm, 75 HP, 240 Vdc series motor draws an armature current of 255 A when delivering rated load. The total armature and series field resistance is 0.029 Ω. Assume brush drop is negligible. The series field has 10 turns/pole. The total armature reaction is 8% of the total series field strength.

**Find:**

**a.)** The torque that the motor develops at rated speed and power output.

**b.)** If the motor is operating in the linear region of its magnetizing curve. If it is in the linear region, compute the torque constant.

**c.)** If the armature current increases to 250% of rated on startup, determine if the motor is in saturation and what the developed torque would be.



Assume a magnetizing curve similar to Fig. 11-8 in textbook.

Define the variables

$R_a = 0.029 \cdot \Omega$ $V_T = 240 \cdot V$ $I_a = 255 \cdot A$ $n = 1200 \cdot rpm$

$P_{mech} = 75 \cdot hp$ Number of turns/pole $N_S = 10 \cdot T$

The mmf developed by the series field poles is defined as....

$$mf = N_S \cdot I_a \qquad mf = 2550 A \cdot T$$

10 T

255 A

## Example 19-1 Solution (1)

Include the 8% reduction due to armature reaction

$$mf_{net} = mf - 0.08 \cdot mf$$

2550 AT

$$mf_{net} = 2346\,A \cdot T$$

Convert the speed to rad/s

$$n = 1200\,rpm \qquad \omega = n \cdot \frac{2 \cdot \pi \cdot rad}{60 \cdot sec} \qquad \omega = 125.7\,\frac{rad}{sec}$$

1200 rpm

## Example 19-1 Solution (2)

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Compute the value of $E_a$ so the power developed in the armature can be computed.

$$E_a = V_T - I_a \cdot R_a$$

240 V

255 A

0.029 Ω

$$E_a = 232.6\,V$$

The electric power developed in the armature equals the electromechanical power developed so.......

$$P_{em} = E_a \cdot I_a \qquad P_{em} = 59314\,W$$

232.6 V

255 A

## Example 19-1 Solution (3)

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Now compute motor torque from the developed power

59314 W

125.7 rad/s

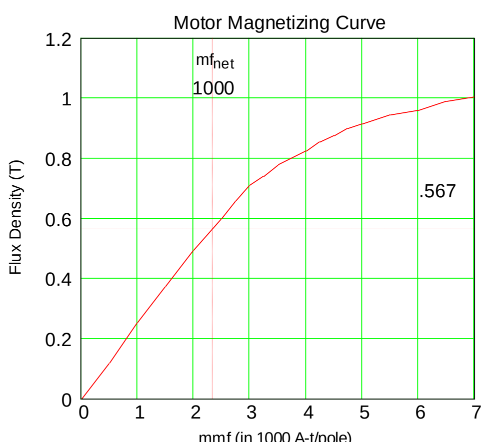
$$T_{D1} := \frac{P_{em}}{\omega}$$

$T_{D1} = 472\,N\cdot m$ — Answer Part a

**Part b:** Examine the motor magnetization curve to see if the motor is operating in the linear region

## Example 19-1 Solution (4)






Motor appears to be **operating in the linear region** of the curve. Compute the torque constant for the motor.

General torque equation

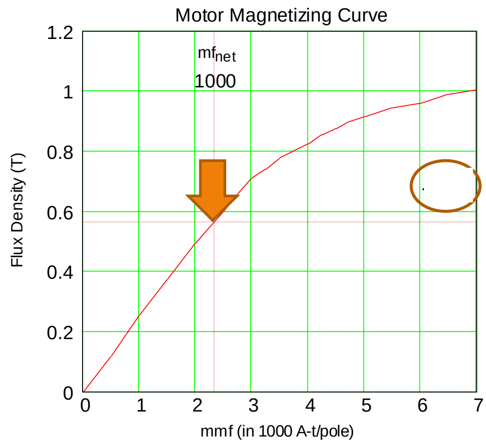
$$T_D = k_T \cdot B_p \cdot I_a$$

Where:
$B_p$ = field flux/pole
$k_T$ = torque constant
$I_a$ = armature current

Note: effects of field current are included in $B_p$

## Example 19-1 Solution (5)

Use the graph to find pole flux density for case 1, $B_{p1}$ .




$B_{p1} = 0.567 \quad I_a = 255\,A$

$$k_T = \frac{T_{D1}}{B_{p1} \cdot I_a}$$

472

255

0.567

$$k_T = 3.265 \frac{(N \cdot m)}{A}$$

Ans Part b

This is the value of $k_T$ for the first operating condition

## Example 19-1 Solution (6)



**Part c**: Find the torque when the armature current increases to 250% of rated value and determine if the motor is in saturation.

$I_{rated} = 255 \cdot A$

$I_{a2} = 2.5 \cdot I_a$

$I_{a2} = 637.5\,A$ Armature current when the motor is overloaded

Number of series field windings $N_S = 10\,T$

The mmf developed by the series field in this case is given by.....

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$mf_2 = N_S \cdot I_{a2}$ $\quad mf_2 = 6375\,A \cdot T$

637.5

## Example 19-1 Solution (7)

$$mf_{net2} = mf_2 - 0.08 \cdot mf_2$$

6375 6375

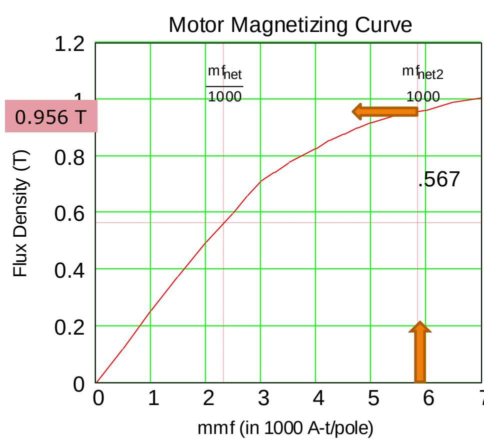
This is the net mmf including the armature reaction effects for the overloaded case

$$mf_{net2} = 5865 \, A \cdot T$$

Plot this value on motor magnetization curve and read the new value of $B_p$ from the curve

## Example 19-1 Solution (8)






The motor has gone into the saturation region of the magnetization curve. The flux density in no longer proportional to the mmf. (so flux is not proportional to the field current)

$$B_{p2} := 0.956$$

Find the torque for case 2 with the effects of saturation included.

## Example 19-1 Solution (9)

In saturation, the torque is proportional to the product of the field flux density (B) and $I_a$.

$$\frac{T_{D1}}{T_{D2}} = \frac{B_{p1} \cdot I_{a1}}{B_{p2} \cdot I_{a2}}$$

Rearranging to solve for $T_{D2}$ gives....

$$T_{D2} := \frac{B_{p2} \cdot 2.5 \cdot I_a}{B_{p1} \cdot I_a} \cdot T_{D1}$$

$I_{a2} = 2.5 \cdot I_{rated}$

$I_{a2} = 637.5\,A$

$$\frac{0.956 \cdot 637.5 \cdot A}{0.567 \cdot 255 \cdot A} \cdot 472.0 \cdot N \cdot m = 1989.6\,N \cdot m$$

$T_{D2} = 1989.6\,N \cdot m$ ← Ans Part c

This includes the effects of saturation on the series motor operation.

## Example 19-1 Solution (10)



Assume a linear magnetization curve from the first case and compute the torque value neglecting saturation effects.

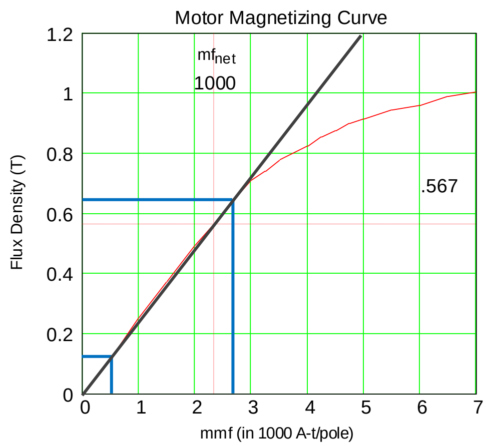
If $B_p$ is proportional to the mmf, increasing the field current 250% increases B proportionally (series motor connection $I_f = I_a$)

Estimate the slope of a linear curve by picking two values in the linear region and computing the slope

$B_{p1} = 0.567\,T$

$B_{p2} = ?$

## Comparison to Linear Operation




$mmf_1 = 0.5$ x1000 A-t

$B_1 = 0.12\,T$

$mmf_6 = 2.75$ x1000 A-t

$B_6 = 0.66\,T$

Slope of linear approximation

$$\frac{B_6 - B_1}{mmf_6 - mmf_1} = 0.24\,T$$

Compute value of $B_{p2}$ using linear approximation of curve

$$B_{p2} = 0.24 \cdot \frac{T}{A} \cdot \frac{mf_{net2}}{1000}$$

# Linear Comparison

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Flux density if linear magnetization curve is assumed

$B_{p2} = 1.408$ $\quad mf_{net2} = 5865\,A \cdot T$

Compute the torque using this value of flux

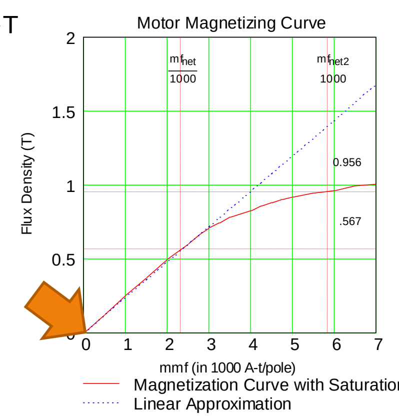
$$T_{D2l} := \frac{B_{p2} \cdot 2.5 \cdot I_a}{B_{p1} \cdot I_a} \cdot T_{D1}$$

$$T_{D2l} := \frac{1.408 \cdot T \cdot 2.5 \cdot 255 \cdot A}{0.567 \cdot T \cdot 255 \cdot A} \cdot 472.0 \cdot N \cdot m$$

$T_{D2l} = 2930.2\,N \cdot m$

Linear approximation torque value

$T_{D2} = 1989.6\,N \cdot m$ Torque with saturation




# Linear Comparison

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Saturation reduces the amount of flux that can be produced by the motor field, which reduces the amount of torque the motor can develop. Linear approximation good if motor is not in saturation

Include saturation effects when motor operates outside of rated conditions such as during start-up and overload. Analysis includes magnetization curve.

## Linear Comparison



# End Lesson 19

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