Lesson 4: Solving Magnetic Circuits with Electrical Analogies

ET 332a

Dc Motors, Generators and Energy Conversion Devices

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Learning Objectives

After this presentation you will be able to:

- Convert a magnetic structure to a electric circuit analogy
- Solve a complex magnetic circuit using the mathematical relationships of magnetic circuits
- Compute the inductance of a coil
- Define Hysteresis power loss in magnetic circuits and determine power losses.

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Magnetic-Electric Circuit Analogies

Sources = windings and current flowing into coils

Core Reluctances = length, area and permeability of core carrying a given flux

Air Gap Reluctances = length, area and permeability of free space (air) used to compute these quantities

Known flux or Flux Density
One of these quantities must be given to find the permeability of core sections. Remember, reluctance is non-linear and depends on the level of flux carried by a core section.

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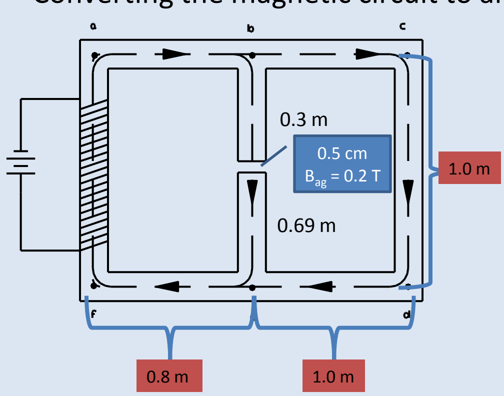
Magnetic Circuit Example 3

Converting the magnetic circuit to an electrical analogy



The magnetic core at the left has the following core segment lengths
$L_{af} = L_{cd} = L_{bc} = L_{ed} = 1.0$ m
$L_{ab} = L_{fe} = 0.8$ m
The air gap length is $L_{ag} = 0.5$ cm
Flux density in the air gap is $B_{ag} = 0.2$T
Coil turns: N =80 t
Core cross sectional area: A = 0.04 m^2
Coil Resistance: R=2.05 Ω
Fringing negligible

1.) Find battery V to produce B_{ag}
2.) Compute μ_r for each core leg

Using magnetization curve (B-H) from text

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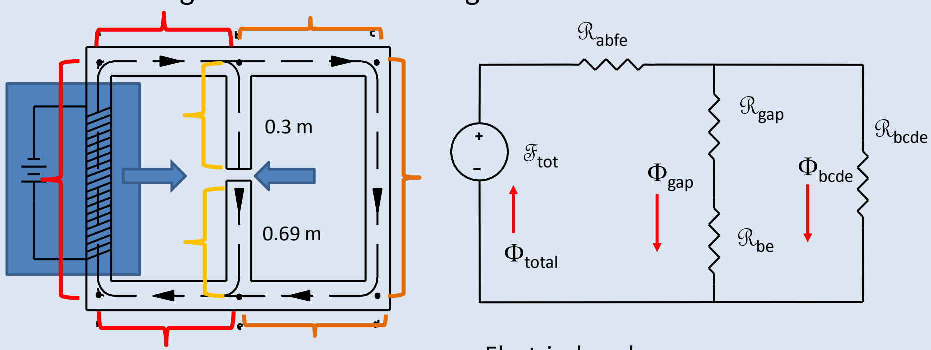
Example 3

Convert diagram to schematic diagram



Electrical analog

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Example 3 Solution Method

Part 1: Find flux in air gap
Determine H of air gap
Determine H for center core
Find MMF to drive flux in center core
Find flux in right leg
Find $\mathcal{F}_{tot}$
Use Ohms law to find V

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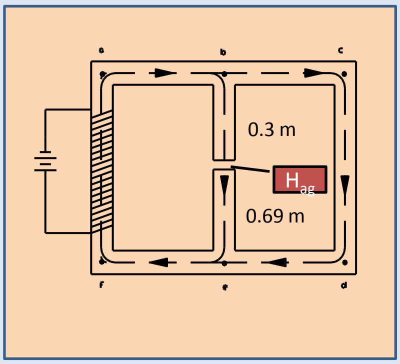
Example 3 Solution –Part 1

Find H_{ag} Air gap field intensity

$$\mu_0 = \frac{B_{ag}}{H_{ag}} \Rightarrow H_{ag} = \frac{B_{ag}}{\mu_0} \qquad H_{ag} = \frac{0.2\ wb/m^2}{4\pi \times 10^{-7}\ wb/A\text{-}t\text{-}m}$$



$H_{ag} = 159{,}155$ A-t/m

Remember, permeability is constant in air gap

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Example 3 Solution –Part 1

Solution continued

Find Flux in air gap

$\Phi_{gap} = B_{ag}A = (0.2\ wb/m^2)(0.04\ m^2) = 0.008$ wb

$H_{0.3}$ = field intensity of 0.3 m core $H_{0.69}$ = field intensity of 0.69 m core

From B-H curve p 8 Text

$$H_{0.3} = H_{0.69} = 0.47\ \text{oersteds}\left(79.577\ \frac{A\text{-}t/m}{\text{oersteds}}\right) = 37.4\ A\text{-}t/m$$

Calculate total mmf for center leg $L_{ag} = 0.5\ cm\left(\frac{1\ m}{100\ cm}\right) = 0.005\ m$

$\mathcal{F}_{ag} = H_{ag}(L_{ag}) = 159{,}155\ A\text{-}t/m\,(0.005\ m) = 795.77\ A\text{-}t$ air gap

$\mathcal{F}_{0.3}$ = MMF in 0.3 m section $(37.4\ A\text{-}t/m)(0.3\ m) = 11.2\ A\text{-}t$

$\mathcal{F}_{0.69}$ = MMF in 0.69 m section $(37.4\ A\text{-}t/m)(0.69\ m) = 25.8\ A\text{-}t$

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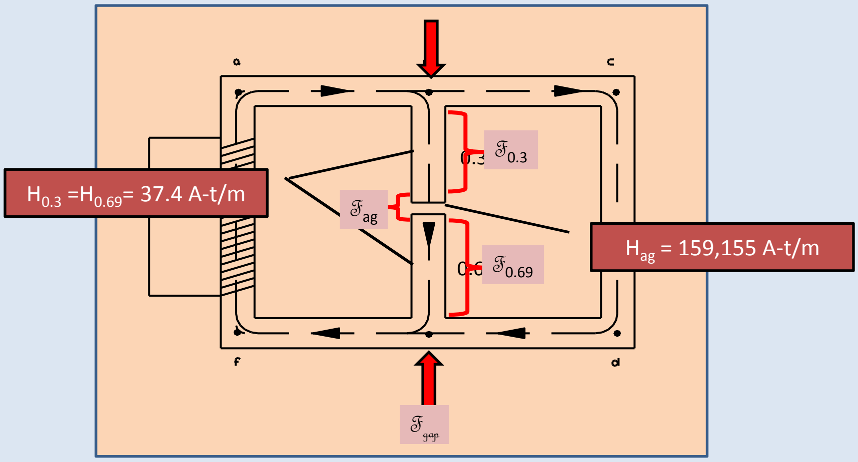
Example 3 Solution –Part 1

Solution continued



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Example 3 Solution –Part 1

Solution continued

$\mathcal{F}_{gap}$ = MMF in center core

$$\mathcal{F}_{gap} = \mathcal{F}_{ag} + \mathcal{F}_{0.3} + \mathcal{F}_{0.69} = (795.77 + 11.2 + 25.8)\text{ A-t} = 833\text{ A-t}$$

Determine what flux this MMF drives through path bcde (right core)

$$H_{bcde} = \frac{\mathcal{F}_{gap}}{L_{bc} + L_{cd} + L_{ed}} = \frac{833\text{ At}}{1.0\text{ m} + 1.0\text{ m} + 1.0\text{ m}} = 277.7\text{ A-t/m}$$

Use B-H curve to find corresponding B

Convert to oersteds $277.7\text{ A-t/m}\left(\frac{1\text{ oersted}}{79.577\text{ A-t/m}}\right) = 3.49$ oersteds

$B_{bcde} \approx 1.45\text{ wb/m}^2$ $\quad \Phi_{bcde} = B_{bcde} A$

$$\Phi_{bcde} = 1.45\text{ wb/m}^2 (0.04\text{ m}^2) = 0.058\text{ wb.}$$

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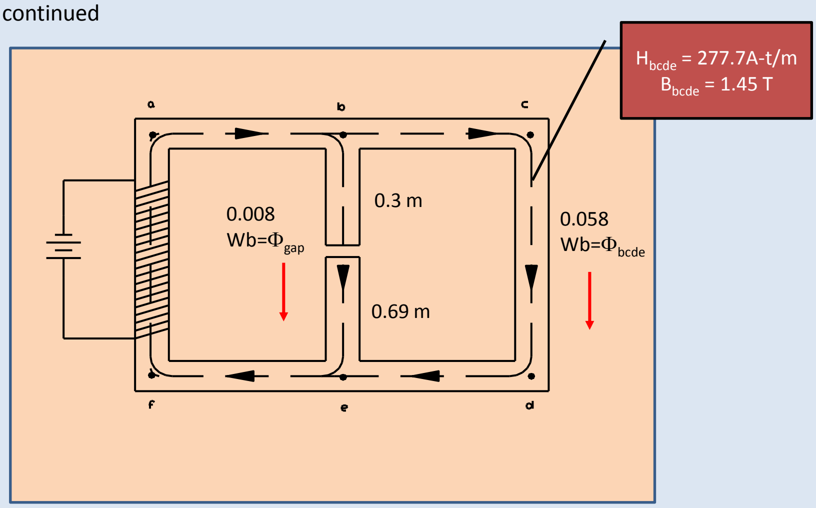
Example 3 Solution –Part 1

Solution continued



Example 3 Solution –Part 1

Solution continued

Find total flux in left core

$$\Phi_{tot} = \Phi_{gap} + \Phi_{bcde}$$

$$\Phi_{tot} = 0.008 \text{ wb} + 0.058 \text{ wb} = 0.066 \text{Wb}$$

Use B-H curves to find H required

$$B_{tot} = \frac{\Phi_{tot}}{A} = \frac{0.066 \text{ wb}}{0.04 \text{ m}^2} = 1.65 \text{ wb/m}^2 (T)$$

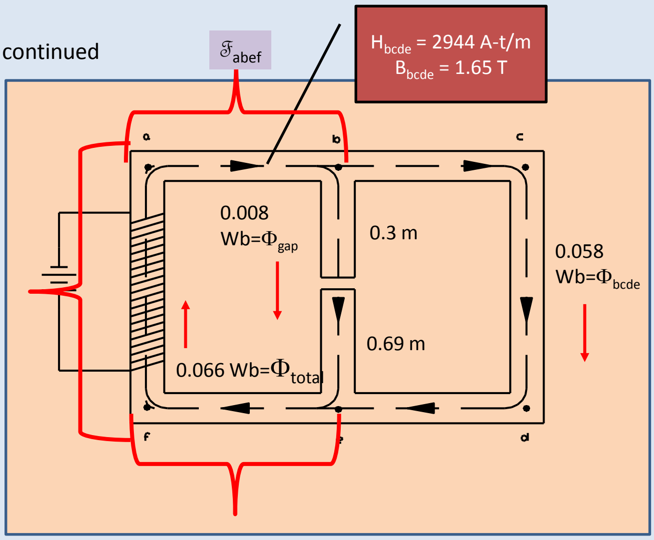
From Curve, H_{efab} = 37 oersteds

$$H_{efab} = 37 \text{ oersteds} \left(\frac{79.577 \text{ A-t/m}}{1 \text{ Oersteds}}\right) = 2944 \text{ A-t/m}$$

Find MMF drop for abef path

Example 3 Solution –Part 1

Solution continued



Example 3 Solution –Part 1

Solution continued

$$\mathcal{F}_{abef} = H_{efab}(L_{ab} + L_{fe} + L_{af})$$

$$\mathcal{F}_{abef} = 2944 \text{ A-t/m } (0.8 \text{ m} + 0.8 \text{ m} + 1.0 \text{ m}) = 2944 \text{ A-t/m } (2.6 \text{ m})$$

$$\mathcal{F}_{abef} = 7654.4 \text{ A-t}$$

Total MMF drop sum of drops of $\mathcal{R}_{abef}$ and parallel paths

$$\mathcal{F}_{tot} = \mathcal{F}_{abef} + \mathcal{F}_{gap} = 7654.4 \text{ A-t} + 833 \text{ A-t}$$

$$\mathcal{F}_{tot} = 8488 \text{ A-t}$$

$$\mathcal{F}_{tot} = NI \text{ so } I = \frac{\mathcal{F}_{tot}}{N} = \frac{8488 \text{ A-t}}{80 \text{ t}} = 106 \text{ A}$$

use Ohm's Law to find V

$$V = I \cdot R = 106\,(2.05\,\Omega) = \underline{217.5 \text{ V}} \quad \text{ANSW}$$

Example 3 Solution –Part 2 Computing Relative Permeabilities

Part 2 Solution Method: Find B and H for each section (From Part 1)
Compute permeability of each section
Compute relative permeability

From Part 1, in Center Core $B_{ag} = 0.2\ T$

$H_{0.3} = H_{0.69} = 37.4\ A\text{-}t/m$

For right core, $B = 1.45\ T$

$H_{bcde} = 277.7\ A\text{-}t/m$

For left core, $B = 1.65\ T$

$H_{abef} = 2944\ A\text{-}t/m$

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Example 3 Solution –Part 2 Computing Relative Permeabilities

Part 2 solution continued

$$\mu_{center} = \frac{B_{ag}}{H_{0.3}} = \frac{0.2\ Wb/m^2}{37.4\ A\text{-}t/m} = 0.0053\ Wb/A\text{-}t\text{-}m \quad \text{ANSW}$$

Relative μ

$$\mu_{rcenter} = \frac{\mu_{center}}{\mu_0} = \frac{0.0053\ Wb/A\text{-}t\text{-}m}{4\pi \times 10^{-7}\ Wb/A\text{-}t\text{-}m} = 4255 \quad \text{ANSW}$$

$$\mu_{right} = \frac{1.45\ Wb/m^2}{277.7\ A\text{-}t/m} = 0.0052\ Wb/A\text{-}t\text{-}m \quad \text{ANSW}$$

Relative $\mu_{rright} = \frac{\mu_{right}}{\mu_0} = \frac{0.0052\ Wb/A\text{-}t\text{-}m}{4\pi \times 10^{-7}\ Wb/A\text{-}t\text{-}m} = 4138$ ANSW

$$\mu_{Left} = \frac{1.65\ Wb/m^2}{2944\ A\text{-}t/m} = 5.60 \times 10^{-4}\ Wb/A\text{-}t\text{-}m \quad \text{ANSW}$$

Relative $\mu_{rleft} = \frac{\mu_{Left}}{\mu_0} = \frac{5.6 \times 10^{-4}\ Wb/A\text{-}t\text{-}m}{4\pi \times 10^{-7}\ Wb/A\text{-}t\text{-}m} = 446$ ANSW

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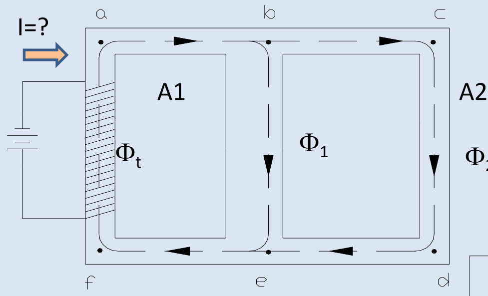
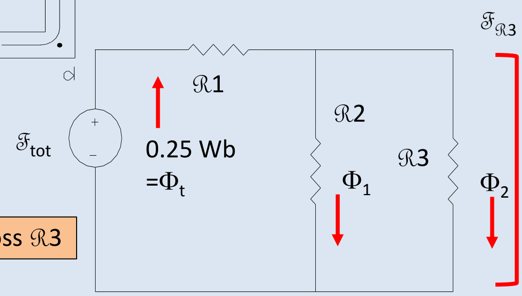
Example 4- Electric Circuit Analogy with Given Reluctances



$\Re1$ = 10,500 A-t/Wb
$\Re2$ = 30,000 A-t/Wb
$\Re3$ = 40,000 A-t/Wb



Coil has 140 turns

Find coil I, Φ_2 and MMF drop across $\Re3$

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Example 4 Solution

$$\Phi = \frac{\mathcal{F}}{\Re}$$

$$\mathcal{F}_{tot} = \Phi_t \Re_{eq}$$

$\Re1 = 10{,}500$ A-t/wb
$\Re2 = 30{,}000$ A-t/wb
$\Re3 = 40{,}000$ A-t/wb
$\Phi_t = 0.25$ wb

$$\Re_{eq} = \Re1 + \Re2 \,||\, \Re3 =$$

$$\Re_{eq} = 10{,}500 \text{ A-t/wb} + \underbrace{\frac{30{,}000 \text{ A-t/wb} \,(40{,}000 \text{ A-t/wb})}{30{,}000 \text{ A-t/wb} + 40{,}000 \text{ A-t/wb}}}_{17{,}143 \text{ A-t/wb}}$$

$$\Re_{eq} = 27{,}643 \text{ A-t/wb}$$

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Example 4 Solution

Solution continued

$$\mathcal{F}_{tot} = (0.25\,wb)(27{,}643\,A\text{-}t/wb) = 6910\,A\text{-}t$$

$$\mathcal{F}_{tot} = NI \rightarrow \frac{\mathcal{F}_{tot}}{N} = I \qquad \frac{6910\,A\text{-}t}{140\,t} = \underline{49.4\,A} \quad \underline{\text{ANSW}}$$

Use Division of MMF to find $\mathcal{F}_{\mathcal{R}3}$. $\mathcal{R}2 \parallel \mathcal{R}3$ so $\mathcal{F}_{\mathcal{R}3} = \mathcal{F}_{\mathcal{R}2}$

$$\mathcal{F}_{\mathcal{R}3} = \mathcal{F}_{tot}\left[\frac{\mathcal{R}2 \parallel \mathcal{R}3}{\mathcal{R}1 + \mathcal{R}2 \parallel \mathcal{R}3}\right] = 6910\,A\text{-}t\left[\frac{17{,}143\,A\text{-}t/wb}{10{,}500\,A\text{-}t/wb + 17{,}143\,A\text{-}t/wb}\right]$$

$$\mathcal{F}_{\mathcal{R}3} = \underline{4284.2\,A\text{-}t} \quad \underline{\text{ANSW}}$$

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Example 4 Solution

Check the previous result using flux division

Use division of flux to find Φ_2

$$\Phi_2 = \Phi_t\left[\frac{\mathcal{R}_2}{\mathcal{R}_2 + \mathcal{R}_3}\right] = 0.25\,wb\left[\frac{30{,}000\,A\text{-}t/wb}{30{,}000 + 40{,}000\,A\text{-}t/wb}\right]$$

$$\Phi_2 = \underline{0.107\,wb} \quad \underline{\text{ANSW}}$$

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Magnetic Circuits and Inductance

Magnetic structures are modeled as inductors. These structures also have a dc resistance due to winding resistance

Define inductance in terms of coil parameters

Product of flux and turns -flux linkages

λ= N Φ λ (lambda)

Remember $\Phi = \frac{\mathscr{F}}{\Re} \quad \mathscr{F} = N \cdot I \quad \Re = \frac{l}{\mu \cdot A}$

Substitute $\Phi = \frac{N \cdot I}{\frac{l}{\mu \cdot A}} = \frac{\mu \cdot N \cdot I \cdot A}{l}$

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Magnetic Circuits and Inductance

Define flux linkages in terms of magnetic coil parameters

$$\lambda = N \cdot \Phi = N\left[\frac{\mu \cdot N \cdot I \cdot A}{l}\right] = \frac{\mu \cdot N^2 \cdot I \cdot A}{l}$$

Inductance, L, defined as flux linkages per amp so ……

$$L = \frac{\lambda}{I} = \frac{\mu \cdot N^2 \cdot A}{l}$$

Where: L = inductance (H)
μ = permeability of core material (Wb/A-t-m)
N = number of turns in coil
A = cross-sectional area of core (m^2)
l = length of core (m)

Note:

$\Re = \frac{l}{\mu \cdot A}$ so $L = \frac{N^2}{\Re}$

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Inductance Calculation Example

A 100 turn coil with a cross-section area of 0.025 m^2 is 20 cm long. The core material has a relative permeability of 2750. Find the inductance of this coil.

$$\mu_r = \frac{\mu}{\mu_0} \qquad \mu_r \mu_0 = \mu \qquad l = 0.20\,m$$

$$A = 0.025\,m^2$$

$$L = \frac{\mu N^2 A}{l}$$

$$L = \frac{2750\,(4\pi \times 10^{-7}\,wb/A\text{-}t\text{-}m)(100)^2(0.025\,m^2)}{0.20\,m}$$

$$L = \underline{4.32\,H} \quad \underline{ANSW}$$

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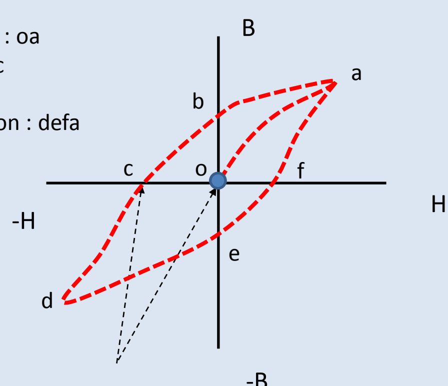
Hysteresis in Magnetic Circuits

Initial magnetization : oa
Demagetization: abc
Change poles: cd
Reverse magnetization : defa



Coercive
Force - force required to bring flux to zero

Hysteresis occurs in ac circuits area inside loop represents power lost smaller area less losses (J/cycle/m^3)

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Power Loss Due to Hysteresis

Hysteresis losses depend on:
- frequency
- flux density
- mass of core iron

Relationship $$P_h = k_n \cdot f \cdot B_{max}^n$$

Where:
- P_h = hysteresis losses (W/unit mass)
- f = frequency of flux wave (Hz)
- B_{max} = Maximum flux density (T)
- k_h = constant (depends on material and unit system
- n = exponent varies with material (1.4 -1.6)

Hysteresis Power Loss Example

Knowing one set of conditions, can use proportions to find another.

A power transformer has a silicon steel core (n = 1.6) This power transformer operates at 60 Hz with a P_h of 2.5 kW. What are the hysteresis losses when it operates at 50 Hz

Hysteresis Power Loss Example Solution

Set up proportion

$$\frac{P_{h60}}{P_{h50}} = \frac{\left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{60}}{\left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{50}}$$

Cross multiply and solve for P_{h50}

$$P_{h60} \cdot \left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{50} = P_{h50} \cdot \left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{60}$$

$$P_{h60} \cdot \frac{\left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{50}}{\left[k_n \cdot f \cdot B_{max}^{1.6}\right]_{60}} = P_{h50}$$

$$2.5\,\mathrm{kW} \cdot \frac{\left[\cancel{k_n} \cdot 50 \cdot \cancel{B_{max}^{1.6}}\right]_{50}}{\left[\cancel{k_n} \cdot 60 \cdot \cancel{B_{max}^{1.6}}\right]_{60}} = P_{h50}$$

$$2.083\,\mathrm{kW} = P_{h50}$$

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END LESSON 4: SOLVING MAGNETIC CIRCUITS WITH ELECTRICAL ANALOGIES