Lesson 4: Solving Magnetic Circuits with Electrical Analogies

ET 332a
Dc Motors, Generators and Energy Conversion Devices

Learning Objectives

After this presentation you will be able to:

- Convert a magnetic structure to a electric circuit analogy
- Solve a complex magnetic circuit using the mathematical relationships of magnetic circuits
- Compute the inductance of a coil
- Define Hysteresis power loss in magnetic circuits and determine power losses.
Magnetic-Electric Circuit Analogies

**Sources** = windings and current flowing into coils

**Core Reluctances** = length, area and permeability of core carrying a given flux

**Air Gap Reluctances** = length, area and permeability of free space (air) used to compute these quantities

**Known flux or Flux Density**
One of these quantities must be given to find the permeability of core sections. Remember, reluctance is non-linear and depends on the level of flux carried by a core section.

Magnetic Circuit Example 3

Converting the magnetic circuit to an electrical analogy

The magnetic core at the left has the following core segment lengths
- $L_{af} = L_{cd} = L_{bc} = L_{ed} = 1.0 \text{ m}$
- $L_{ab} = L_{fe} = 0.8 \text{ m}$

The air gap length is $L_{ag} = 0.5 \text{ cm}$

Flux density in the air gap is $B_{ag} = 0.2 \text{ T}$

Coil turns: $N = 80 \text{ t}$

Core cross sectional area: $A = 0.04 \text{ m}^2$

Coil Resistance: $R = 2.05 \Omega$

Fringing negligible

1.) Find battery $V$ to produce $B_{ag}$
2.) Compute $\mu_r$ for each core leg

Using magnetization curve (B-H) from text
Example 3

Convert diagram to schematic diagram

Electrical analog

Example 3 Solution Method

Part 1:
- Find flux in air gap
- Determine H of air gap
- Determine H for center core
- Find MMF to drive flux in center core
- Find flux in right leg
- Find $F_{\text{tot}}$
- Use Ohms law to find $V$
Example 3 Solution – Part 1

Find \( H_{aq} \), air gap field intensity

\[
\mu_0 = \frac{B_{aq}}{H_{aq}} \Rightarrow H_{aq} = \frac{B_{aq}}{\mu_0} = \frac{0.2 \text{ wb/m}^2}{4\pi \times 10^{-7} \text{ wb/At}} = 157,155 \text{ A-t/m}
\]

Remember, permeability is constant in air gap.

Solution continued

Find flux in air gap

\[
E_{aq} = B_{aq} A = (0.2 \text{ wb/m}^2)(0.69 \text{ m}) = 0.138 \text{ wb}
\]

\( H_{0.3} \) field intensity in 0.3 m core, \( H_{0.69} \) field intensity in 0.69 m core.

From B-N curve p 8 Text

\( H_{0.3} = H_{0.69} = 0.97 \text{ A-t/m} \)

Calculate total mmf for center leg

\( L_a = 0.5 \text{ cm} \times \frac{1 \text{ m}}{100 \text{ cm}} = 0.005 \text{ m} \)

\[
\Phi_a = H_{aq}(L_a) = 157,155 \text{ A-t/m}(0.005 \text{ m}) = 785.77 \text{ A-t} \quad \text{air gap}
\]

\[
\Phi_{0.3} = \text{mmf in 0.3 m section} = (37.4 \text{ A-t/m})(0.3 \text{ m}) = 11.2 \text{ A-t}
\]

\[
\Phi_{0.69} = \text{mmf in 0.69 m section} = (37.4 \text{ A-t/m})(0.69 \text{ m}) = 25.8 \text{ A-t}
\]
Example 3 Solution – Part 1

Solution continued

\[ H_{0.3} = H_{0.69} = 37.4 \text{ A-t/m} \]

\[ H_{ag} = 159,155 \text{ A-t/m} \]

\[ F_{0.3} \]

\[ F_{0.69} \]

Example 3 Solution – Part 1

Solution continued

\[ J_{gap} = \text{MMF in center core} \]

\[ J_{gap} = J_{ag} + J_{0.3} + J_{0.69} = (795.77 + 112.2 + 25.8) \text{ A-m} = 833 \text{ A-t} \]

Determine what flux this MMF drives through path b c d e (right core)

\[ H_{b c d e} = \frac{J_{gap}}{L_e + L_d + L_e} = \frac{833 \text{ A-t}}{1.0 \text{ m} + 1.0 \text{ m} + 1.0 \text{ m}} = 297.7 \text{ A-t/m} \]

Use B-H curve to find corresponding B

Convert to oersteds: 297.7 A-t/m (Joersted/3.077 A-t/m) = 3.49 oersteds

\[ B_{b c d e} = 1.95 \text{ wb/m}^2 \]

\[ \Phi_{b c d e} = 1.95 \text{ wb/m}^2 (0.04 \text{ m}^2) = 0.078 \text{ wb}. \]
Example 3 Solution – Part 1

Solution continued

\[ H_{bcde} = 277.7 \text{ A} \cdot \text{t/m} \]
\[ B_{bcde} = 1.45 \text{ T} \]

\[ 0.008 \text{ Wb} = \Phi_{\text{gap}} \]
\[ 0.058 \text{ Wb} = \Phi_{bcde} \]

\[ 0.3 \text{ m} \]
\[ 0.69 \text{ m} \]

Example 3 Solution – Part 1

Solution continued

*Find total flux in left core*

\[ \Phi_{\text{tot}} = \Phi_{\text{gap}} + \Phi_{\text{bcde}} \]

\[ \Phi_{\text{tot}} = 0.008 \text{ Wb} + 0.058 \text{ Wb} = 0.064 \text{ Wb} \]

*Use B-H curves to find H required*

\[ B_{\text{tot}} = \frac{\Phi_{\text{tot}}}{A} = \frac{0.064 \text{ Wb}}{0.09 \text{ m}^2} = 1.65 \text{ Wb/m}^2 (T) \]

From curve, \( H_{\text{eb}} = 37 \) oersteds

\[ H_{\text{eb}} \cdot \frac{79577 \text{ A} \cdot \text{ym}}{1 \text{ oersteds}} = 2944 \text{ A} \cdot \text{ym} \]

Find MRR from left path
Example 3 Solution – Part 1

Solution continued

\[ H_{bcde} = 2944 \text{ A-t/m} \]
\[ B_{bcde} = 1.65 \text{ T} \]

\[ 0.008 \text{ Wb} = \Phi_{\text{exp}} \]
\[ 0.058 \text{ Wb} = \Phi_{\text{bcde}} \]
\[ 0.066 \text{ Wb} = \Phi_{\text{total}} \]

\[ 0.3 \text{ m} \]
\[ 0.69 \text{ m} \]

\[ 0.008 \text{ Wb} = \Phi_{\text{exp}} \]
\[ 0.058 \text{ Wb} = \Phi_{\text{bcde}} \]
\[ 0.066 \text{ Wb} = \Phi_{\text{total}} \]

\[ J_{bc} = 7659.4 \text{ A-t} \]

\[ J_{bc} = \frac{H_{bc} \cdot (L_{ab} + L_{ta} + L_{al})}{\mu_0} \]
\[ J_{bc} = 2944 \text{ A-t/m} \times (0.8 \text{ m} + 0.8 \text{ m} + 1.0 \text{ m}) = 3944 \text{ A-t/m (2.6 m)} \]

Total MMF drop sum of drops of \( R_{bc} \) and parallel paths

\[ J_{bc} = J_{bc} + J_{yy} = 7659.4 \text{ A-t} + 833 \text{ A-t} \]
\[ J_{bt} = 8492 \text{ A-t} \]

\[ J_{bt} = NI \text{ so } I = \frac{J_{bt}}{N} = \frac{8492 \text{ A-t}}{80} = 106 \text{ A} \]

use Ohm's Law to find \( V \)

\[ V = I \cdot R = 106(2.05 \text{ A}) = 217.6 \text{ V} \]

Answering
Example 3 Solution – Part 2 Computing Relative Permeabilities

Part 2 Solution Method:

Find B and H for each section (From Part 1)
Compute permeability of each section
Compute relative permeability

From Part 1, in Center Core
\[ B_{ag} = 0.2 \text{ T} \]
\[ H_{ag} = H_{ag} = 37.4 \text{ A} \cdot \text{m} \]

For right core,
\[ B = 1.95 \text{ T} \]
\[ H_{ode} = 277.7 \text{ A} \cdot \text{m} \]

For left core,
\[ B = 1.63 \text{ T} \]
\[ H_{ode} = 2949 \text{ A} \cdot \text{m} \]

Example 3 Solution – Part 2 Computing Relative Permeabilities

Part 2 solution continued

Relative \( \mu \)
\[ \frac{\mu_{center}}{\mu_0} = \frac{B_{ag}}{H_{ag}} = \frac{0.2 \text{ Wb/m}^2}{37.4 \text{ A} \cdot \text{m}} = \frac{0.0053 \text{ Wb/A-m}}{\text{Ans}} \]

Relative \( \mu \)
\[ \frac{\mu_{right}}{\mu_0} = \frac{B_{right}}{H_{right}} = \frac{1.95 \text{ Wb/m}^2}{277.7 \text{ A} \cdot \text{m}} = \frac{0.007 \text{ Wb/A-m}}{\text{Ans}} \]

Relative \( \mu \)
\[ \frac{\mu_{left}}{\mu_0} = \frac{B_{left}}{H_{left}} = \frac{1.63 \text{ Wb/m}^2}{2949 \text{ A} \cdot \text{m}} = \frac{0.0005 \text{ Wb/A-m}}{\text{Ans}} \]
Example 4- Electric Circuit Analogy with Given Reluctances

Coil has 140 turns

\[ \Phi_1 = 0.25 \text{ Wb} \]

Find coil I, \( \Phi_2 \) and MMF drop across \( R_3 \)

Example 4 Solution

\[ I = \frac{\Phi}{R} \]

\[ I_{eq} = \frac{R_1}{R_1 + R_2 + R_3} = \frac{10,500}{40,000 + 30,000 + 40,000} = 0.17, 143 \text{ A-t/wb} \]

\[ R_{eq} = 10,500 \text{ A-t/wb} + \frac{30,000 \text{ A-t/wb} \times (40,000 \text{ A-t/wb})}{30,000 \text{ A-t/wb} + 40,000 \text{ A-t/wb}} = 27,643 \text{ A-t/wb} \]
Example 4 Solution

Solution continued

\[ \mathbf{F}_{\text{tot}} = (0.25 \text{ \Omega}) (27,693 \text{ A-t/\Omega}) = 6910 \text{ A-t} \]
\[ \mathbf{F}_{\text{tot}} = \frac{\mathbf{F}_{\text{ext}}}{N} \Rightarrow \mathbf{F}_{\text{ext}} = \frac{6910 \text{ A-t}}{190} = 35.9 \text{ A} \quad \text{Answer} \]

Use division of mmf to find \( \mathbf{F}_{\mathbf{R}_3} \).
\( \mathbf{R}_2 \) and \( \mathbf{R}_3 \) so \( \mathbf{F}_{\mathbf{R}_3} = \frac{\mathbf{F}_{\mathbf{R}_2}}{121 \mathbf{R}_3} \)
\[ \mathbf{F}_{\mathbf{R}_3} = \frac{\mathbf{F}_{\text{ext}}}{121 \mathbf{R}_3} = \frac{6910 \text{ A-t}}{121 \mathbf{R}_3} \left[ \frac{17,193 \text{ A-t/\Omega}}{19,000 \text{ A-t/\Omega} + 17,193 \text{ A-t/\Omega}} \right] \]
\[ \mathbf{F}_{\mathbf{R}_3} = 4284.2 \text{ A-t} \quad \text{Answer} \]

Example 4 Solution

Check the previous result using flux division

Use division of flux to find \( \mathbf{\Phi}_2 \)
\[ \mathbf{\Phi}_2 = \frac{\mathbf{\Phi}}{t} \left[ \frac{\mathbf{R}_2}{\mathbf{R}_2 + \mathbf{R}_2} \right] = 0.25 \text{ \Omega} \left[ \frac{39,000 \text{ A-t/\Omega}}{39,000 + 49,000 \text{ A-t/\Omega}} \right] \]
\[ \mathbf{\Phi}_2 = 0.107 \text{ \Omega} \quad \text{Answer} \]
Magnetic Circuits and Inductance

Magnetic structures are modeled as inductors. These structures also have a dc resistance due to winding resistance.

Define inductance in terms of coil parameters

Product of flux and turns - flux linkages

\[ \lambda = N \Phi \]  

(\lambda\) (lambda)

Remember

\[ \Phi = \frac{\mathcal{F}}{R} \]

\[ \mathcal{F} = N \cdot I \]

\[ R = \frac{1}{\mu \cdot A} \]

Replace

\[ \Phi = \frac{N \cdot I}{1} = \frac{\mu \cdot N \cdot I \cdot A}{1} \]

Inductance, \( L \), defined as flux linkages per amp so .......

\[ L = \frac{\lambda}{I} = \frac{\mu \cdot N^2 \cdot A}{1} \]

Where:

- \( L \) = inductance (H)
- \( \mu \) = permeability of core material (Wb/A-t-m)
- \( N \) = number of turns in coil
- \( \Phi \) = flux
- \( A \) = cross-sectional area of core (m²)
- \( l \) = length of core (m)

Note:

\[ R = \frac{1}{\mu \cdot A} \]

so

\[ L = \frac{N^2}{R} \]
Inductance Calculation Example

A 100 turn coil with a cross-section area of 0.025 m\(^2\) is 20 cm long. The core material has a relative permeability of 2750. Find the inductance of this coil.

\[
\mu_r = \frac{\mu_r}{\mu_0} \quad \mu_r \mu_0 = \mu \\
L = \frac{\mu N^2 A}{\ell} \quad \ell = 0.20 \text{ m} \quad A = 0.025 \text{ m}^2 \\
L = \frac{2750 (4\pi \times 10^{-7} \text{ Wb/A-m}) (100)^2 (0.025 \text{ m}^2)}{0.20 \text{ m}} \\
L = \frac{4.32 \text{ H}}{\text{m}^3} \quad A_{\text{SW}} \text{ Wb}
\]

Hysteresis in Magnetic Circuits

- Initial magnetization: oa
- Demagnetization: abc
- Change poles: cd
- Reverse magnetization: defa

Hysteresis occurs in ac circuits where the area inside the loop represents the power lost. A smaller area indicates less losses (J/cycle/m\(^3\)).
Power Loss Due to Hysteresis

Hysteresis losses depend on:
- frequency
- flux density
- mass of core iron

\[ P_h = k_n \cdot f \cdot B_{max}^n \]

Where:
- \( P_h \) = hysteresis losses (W/unit mass)
- \( f \) = frequency of flux wave (Hz)
- \( B_{max} \) = Maximum flux density (T)
- \( k_n \) = constant (depends on material and unit system)
- \( n \) = exponent varies with material (1.4 -1.6)

Hysteresis Power Loss Example

Knowing one set of conditions, can use proportions to find another.

A power transformer has a silicon steel core (\( n = 1.6 \)) This power transformer operates at 60 Hz with a \( P_h \) of 2.5 kW. What are the hysteresis losses when it operates at 50 Hz
Hysteresis Power Loss Example
Solution

Set up proportion
\[ \frac{P_{h60}}{P_{h50}} = \frac{k_n \cdot f \cdot B_{\text{max}}^{1.6}}{k_n \cdot f \cdot B_{\text{max}}^{1.6}} \]

Cross multiply and solve for \( P_{h50} \)
\[ P_{h60} \cdot \frac{k_n \cdot f \cdot B_{\text{max}}^{1.6}}{k_n \cdot f \cdot B_{\text{max}}^{1.6}} = P_{h50} \cdot \frac{k_n \cdot f \cdot B_{\text{max}}^{1.6}}{k_n \cdot f \cdot B_{\text{max}}^{1.6}} \]
\[ P_{h60} \cdot \frac{k_n \cdot 60 \cdot B_{\text{max}}^{1.6}}{k_n \cdot 50 \cdot B_{\text{max}}^{1.6}} = P_{h50} \]
\[ 2.5 \text{ kW} \cdot \frac{k_n \cdot 50 \cdot B_{\text{max}}^{1.6}}{k_n \cdot 60 \cdot B_{\text{max}}^{1.6}} = P_{h50} \]
\[ 2.083 \text{ kW} = P_{h50} \]

ET 332a
Dc Motors, Generators and Energy Conversion Devices

END LESSON 4: SOLVING MAGNETIC CIRCUITS WITH ELECTRICAL ANALOGIES