Lesson 4: Three Phase Sources and Loads

ET 332b

ET 332b Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Identify circuit impedances, voltages and currents using the double subscript notation.
- Perform calculations on wye connected three-phase sources and loads.
- Perform calculations on delta connected three-phase sources and loads.
- Construct phasor diagrams of three-phase sources and loads.
- Identify the time and phasor plots of a three phase set of voltages and currents.
Double Subscript Notation

Sources and voltage drops are defined by the terminal letter. Voltage drop and polarity defined by order of subscripts.

Voltages considered positive if first node subscript is higher potential than second node subscript.

Identify current flow from point b to c

\[ I_{bc} = \frac{V_{bc}}{Z_{bc}} \]

\[ V_{bc} = \text{difference in potential between points b and c. If voltage at point b is taken as the reference point, then the polarity is reversed.} \]
Three-Phase Power Systems

Balanced three-phase voltage sources

Characteristics:
- Three phasor voltages
- Equal voltage magnitudes
- Phase shift equally spaced 120 degrees apart

Time equations for balanced three-phase voltage sources

\[
\begin{align*}
    v_a(t) &= V_p \cdot \sin(2\pi f t + 0^\circ) \\
    v_b(t) &= V_p \cdot \sin(2\pi f t - 120^\circ) \\
    v_c(t) &= V_p \cdot \sin(2\pi f t - 240^\circ)
\end{align*}
\]

Three-Phase Power Systems

Time Plots of Three Phase Voltages

![Time Plots of Three Phase Voltages](image)
Three-Phase Three-Phase

Time Plots of Three Phase Voltages and Phasors

![Graph of three-phase voltages and phasors](image1)

Three-Phase Source Connections

**Wye – Connected, three-phase 3φ sources**

![Diagram of wye connection](image2)

In wye connection:

\[ I_L = I_p \]

Where:

- \( I_L \) = line current
- \( I_p \) = phase current

Determine the relationship between the magnitude and phase shift of each source voltage and the current and voltage at the terminals of the connection.
Voltage relationships

\[
\begin{align*}
V_{ca} &= +V_{an} - V_{bn} \\
V_{bc} &= +V_{bn} - V_{cn} \\
V_{cb} &= +V_{cn} - V_{an}
\end{align*}
\]

In balanced systems, \( |V_{an}| = |V_{bn}| = |V_{cn}| \)

**Perform phasor subtraction to find the values**

Wye Connected Sources

**Phase shifts**

Line-to-line (line) voltages lead phase voltages by 30 degrees for CCW rotation

\[
\begin{align*}
\bar{V}_{ab} &= V_{ab} \angle 30^\circ \\
\bar{V}_{bc} &= V_{bc} \angle -90^\circ \\
\bar{V}_{ca} &= V_{ca} \angle -210^\circ
\end{align*}
\]
Wye Connect Systems

**Example 4-1**: a) Find the line-to-line voltage phasors for the wye connected source. b) Find the line current phasors in each of the three phases of the resistive load. c) Find the current flowing in the neutral.

\[ V_{an} = 100 \angle 0^\circ \quad V_{bn} = 100 \angle -120^\circ \quad V_{cn} = 100 \angle -240^\circ \]

Example 4-1 Solution (1)

Balanced 3φ systems can be analyzed using a single phase to neutral and the other quantities determined by the appropriate phase shifts.

\[ V_{ab} = \sqrt{3} V_{an} \]
\[ V_{ab} = \sqrt{3} (100V) = 173.2V \]

For this sequence of phasors, line-to-line voltages lead phase voltages by 30 degrees.

\[ V_{ab} = 173.2 \angle 30^\circ \]
\[ V_{bc} = 173.2 \angle -90^\circ \]
\[ V_{ca} = 173.2 \angle -210^\circ \]

**Ans**
Example 4-1 Solution (2)

b) In wye connection line current equals phase current

For “a” phase

\[ I_a = \frac{\sqrt{3} I_{an}}{R_a} = \frac{1.857/60\, \text{A}}{35/\text{ohm}} = 2.857/60\, \text{A} \]

Ans

For other phases

\[ I_b = \frac{1.857/120\, \text{A}}{35/\text{ohm}} = 2.857/120\, \text{A} \]

Ans

\[ I_c = \frac{1.857/240\, \text{A}}{35/\text{ohm}} = 2.857/240\, \text{A} \]

Ans

Example 4-1 Solution (3)

c) To find the neutral current, sum the currents at the load neutral point

\[ I_n = I_a + I_b + I_c \]

\[ I_n = 2.857/60 + 2.857/120 + 2.857/240\, \text{A} \]

Convert to rectangular form to add

\[ I_n = (2.857 + 2.857 + 2.857) + (0 + 0 + 0) \, \text{A} \]

\[ I_n = 0 \, \text{A} \]

In balanced 3φ system, neutral conductor carries no current. It is used for safety and to handle unbalances.
Example 4-1 Solution (4)

Phasor diagram

All line-to-line voltages lead the phase voltages by 30 degrees. For resistive load, the phase current is in phase with \( V_{pn} \), the phase-to-neutral voltage.

Delta Connected Sources

Phase voltages are equal to line-to-line voltages in delta connections

\[ |V_{LL}| = |V_p| \]

Where: \( V_{LL} \) = line-to-line voltage
\( V_p \) = phase voltage

So
\[ |V_{ab}| = |V_{bc}| = |V_{ca}| \]

To find relationship between phase and line currents, perform KCL at every corner node.
**Delta Connected Sources**

**Current relationships between line and phase**

For node B

\[ \vec{I}_{bc} - \vec{I}_{ab} - \vec{I}_b = 0 \]

\[ \vec{I}_{bc} - \vec{I}_{ab} = \vec{I}_b \]

For node A

\[ \vec{I}_{ab} - \vec{I}_{ca} - \vec{I}_a = 0 \]

\[ \vec{I}_{ab} - \vec{I}_{ca} = \vec{I}_a \]

For node C

\[ \vec{I}_{ca} - \vec{I}_{bc} - \vec{I}_c = 0 \]

\[ \vec{I}_{ca} - \vec{I}_{bc} = \vec{I}_c \]

Phasor subtraction gives the current magnitude

\[ |\vec{I}_L| = \sqrt{3} \cdot |\vec{I}_p| \]

Where:

- \( \vec{I}_L \) = line current
- \( \vec{I}_p \) = phase current

**Phasor diagram of delta currents**

Line current phasors lag phase currents by 30 degrees in balanced delta connection

\[ \vec{I}_L = \sqrt{3} \cdot |\vec{I}_p| \angle -30^\circ \]

Above hold for all phases with \( \vec{I}_p \) as reference phasor

Phase rotation is ABC in this case
Delta Connection Example

Example 4-2: For the delta connected load shown, a) find the phasor values of phase and line currents for the circuit.: b) draw a phasor diagram of the computed currents and given voltages  R = 15 ohms in each phase.

\[ V_{ab} = 240 \angle 0^\circ \]
\[ V_{bc} = 240 \angle -120^\circ \]
\[ V_{ca} = 240 \angle -240^\circ \]

Example 4-2 Solution (1)

For this balanced system, compute the values for single phase and then shift the angles for the other values

\[ I_{ab} = \frac{V_{ab}}{R} = \frac{240/0^\circ}{15/0^\circ} = 16/0^\circ \text{A} \]

Now compute the line current

\[ I_a = \sqrt{3} I_{ab}/-30^\circ \]
\[ I_c = \sqrt{3} (16)/-30^\circ \]
\[ I_b = 27.7/2^\circ \text{A} \]

\[ I_{bc} = -120^\circ \text{ and } I_{ca} = -240^\circ \text{ for symmetry} \]

\[ I_{ac} = 16/0^\circ \text{ A} \]

\[ I_{bc} = 16/-120^\circ \text{ A} \]

\[ I_{ca} = 16/-240^\circ \text{ A} \]
Example 4-2 Solution (2)

Similarly for the line current:
\[
\vec{I}_b = 27.72 \angle -150^\circ \ A \\
\vec{I}_c = 27.72 \angle -210^\circ \ A
\]

Phasor diagram for delta connected load Example 4-2

Resistive circuit, so phase currents are in phase with voltages that produce it.

Line currents lags the phase currents by the 30 degrees

Delta Connected Load-General Impedances

Example 4-3: a) Find the phasor values of phase and line currents for the circuit shown: b) draw a phasor diagram of the computed currents and given voltages. \( Z = 30 \angle -39^\circ \) ohms.

\[
\begin{align*}
V_{ab} &= 240 \angle 0^\circ \\
V_{bc} &= 240 \angle 120^\circ \\
V_{ca} &= 240 \angle -240^\circ \\
\end{align*}
\]
Example 4-3 Solution (1)

Phase current of $V_{ab}$

\[ \vec{I}_{ab} = \frac{\vec{V}_{ab}}{Z} = \frac{29.8/A^*}{30 \angle 36^\circ} \]

\[ \vec{I}_{a b} = 8/39^\circ \, \text{A} \]

Line current

\[ \vec{I}_a = \sqrt{3} \cdot I_{ab} \angle -36^\circ + 9^\circ = 13.85 \angle 9^\circ \, \text{A} \]

Other phases

\[ \vec{I}_b = \frac{\sqrt{3}}{2} \cdot I_{bc} \angle -28.8^\circ + 9^\circ = 8/80^\circ \, \text{A} \]

\[ \vec{I}_c = \frac{1}{2} \cdot I_{ca} \angle -50^\circ + 9^\circ = 6/20^\circ \, \text{A} \]

\[ \vec{I}_b = 13.85/31.1^\circ \, \text{A} \]

\[ \vec{I}_c = 13.85/231^\circ \, \text{A} \]

Example 4-3 Solution (2)

Example 4-3 phasor diagram

Phase current leads voltage by 39 degrees

Line current leads voltage by 9 degrees
End Lesson 4: Three Phase Sources and Loads

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