LESSON 8: IDEAL TRANSFORMER THEORY AND OPERATION

ET 332b Ac Motors, Generators and Power Systems

Learning Objectives

After this presentation you will be able to:

- Explain how an ideal transformer operates
- Find the voltages and currents on both sides of an ideal transformer using the turns ratio
- Reflect impedances through a transformer
- Identify and compute the no-load currents that flow in a non-ideal transformer
- Draw the no-load circuit model of a non-ideal transformer.
**Ideal Transformer Action**

**Principle**: Stationary coils, time varying flux due to ac current flow. Flux produced by one coil must link to other coil to induce voltage.

**Lentz's Law**

\[ e_1 = -N_1 \left( \frac{d\phi_m}{dt} \right) \]

\[ e_2 = -N_2 \left( \frac{d\phi_m}{dt} \right) \]

Induced voltage has opposite polarity from source.

For sinusoidal sources

\[ E'_p = 4.44 \cdot N_p \cdot f \cdot \phi_{max} \]

\[ E'_s = 4.44 \cdot N_s \cdot f \cdot \phi_{max} \]

Dividing the above equations gives:

\[ \frac{E'_p}{E'_s} = \frac{N_p}{N_s} \]

Voltage ratio equals the turns ratio.

Where:

- \( E'_p \) = voltage induced in the primary (V)
- \( E'_s \) = voltage induced in the secondary (V)
- \( N_p \) = turns in the primary coil
- \( N_s \) = turns in the secondary coil
Assumptions for Ideal Transformer Operation

1) All flux produced in the primary coil links to the secondary coil
2) no core losses due to hysteresis or eddy currents
3) no power losses
4) permeability is infinite (no saturation no magnetizing $\phi$)
5) windings have zero resistance
6) no current required to magnetize the iron core

For ideal transformer

$$a = \frac{E'_p}{E'_s} = \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

Where: $a = \text{turns ratio}$
$V_p = \text{nameplate rated primary voltage (higher V)}$
$V_s = \text{nameplate rated secondary voltage (lower V)}$
$E'_p = \text{induced primary voltage}$
$E'_s = \text{induced secondary voltage}$

Ideal Transformer Equations

Voltage Ratio

$$a = \frac{E_p}{E_s} = \frac{N_p}{N_s}$$

The turns ratio is a scalar. Introduces no phase shift

Apparent Power balance

$$E_p \cdot I_p = E_s \cdot I_s$$

No power losses in ideal transformer

Current Ratio

$$\frac{I_p}{I_s} = \frac{1}{a} \quad I_p = \left(\frac{1}{a}\right) \cdot I_s$$

Current ratio is the inverse of the voltage ratio
Impedances Reflected Through Ideal Transformers

**Load impedance as seen from primary side of transformer**

By Ohm's Law

\[ Z_{in} = \frac{E_p}{I_p} \]

Write \( E_s \) and \( I_s \) in terms of primary values

\[ E_s = \frac{E_p}{a} \quad I_s = a \cdot I_p \]

Load impedance is increased when viewed from primary side

\[ Z_{load} = Z_{in} \left( \frac{1}{a^2} \right) \Rightarrow Z_{load} \cdot a^2 = Z_{in} \]

Generally: Moving impedance from secondary to primary multiply by \( a^2 \). Moving from primary to secondary, divide by \( a^2 \).

\[ Z_p = Z_s \cdot a^2 \quad \frac{Z_p}{a^2} = Z_s \]

Derive equation when impedances are connected to the primary side and viewed from the secondary side.

Write primary values in terms of secondary and substitute in the \( Z_{load} \) equation.

\[ E_p = a \cdot E_s \quad I_p = \frac{I_s}{a} \]

\[ Z_{load} = \frac{E_p}{I_p} = \frac{a \cdot E_s}{\frac{I_s}{a}} = a \cdot I_s \cdot \left( \frac{a}{I_s} \right) = a^2 \cdot \left( \frac{E_s}{I_s} \right) \]

\[ Z_{load} = Z_{in} \cdot a^2 \Rightarrow \frac{Z_{load}}{a^2} = Z_{in} \]
Example 8-1: A 25 kVA, 7200 - 240/120 center-tap single phase transformer operates at rated voltage. It supplies a single phase load that has an equivalent impedance of $7.2 \angle +36.9^\circ$ ohms. Assume Ideal operation and find:

a.) turns ratio
b.) secondary current
c.) primary current
d.) load Z as seen from primary side
e.) $P_T$, $S_T$, $Q_T$, and $F_p$

Example 8-1 Solution (1)

a) For ideal transformers

\[
\frac{N_p}{N_s} = \frac{V_p}{V_s} \quad V_p = 7200 \text{ V} \quad V_s = 240 \text{ V}
\]

Find the turns ratio and primary current:

\[
\alpha = \frac{N_p}{N_s} = \frac{7200\text{ V}}{240\text{ V}} = 30 \quad \text{Ans}
\]

b) Secondary current

Use Ohm's law to find $I_s$

\[
\frac{I_p}{I_s} = \frac{1}{\alpha} \quad \Rightarrow \quad I_p = \frac{1}{30} I_s \quad \overline{E_p} = 7200/l^\circ \text{ V} \quad \overline{E_s} = 240/l^\circ \text{ V}
\]

Find the secondary current:

\[
\overline{I_s} = \frac{240/l^\circ \text{ V}}{7.2/36.9^\circ \Omega} \quad \overline{I_s} = 33.33/l^\circ 36.9^\circ \text{ A} \quad \text{Ans}
\]
Example 8-1 Solution (2)

c) Find the primary current
\[ I_p = \frac{1}{a} I_s \]
\[ I_p = \left( \frac{L}{3a} \right) \left( 33.33 \angle -36.9^\circ \right) \]
\[ I_p = 1.11 \angle -36.9^\circ \ A \quad \text{Ans} \]

d) Find the input impedance as seen from the primary side
\[ Z_{in} = a^2 Z_{load} \]
\[ Z_{in} = \left( 3a \right)^2 \left( 7.2 \angle 36.9^\circ \right) \]
\[ Z_{in} = 64.8 \angle 36.9^\circ \ \Omega \quad \text{Ans} \]

Example 8-1 Solution (3)

e) Find the power and the power factor of the load

Using secondary side quantities
\[ S_{TS} = I_s^* V_s \]
\[ S_{TS} = \left( 2.90 \angle 0^\circ \right) \left( 33.33 \angle 36.9^\circ \right)^* \]
\[ S_{TS} = 8000 \angle 36.9^\circ \ \text{VA} \quad \text{Ans} \]

Using primary side quantities
\[ S_{TP} = I_p^* V_p \]
\[ S_{TP} = \left( 1.11 \angle -36.9^\circ \right)^* \left( 7200 \angle 0^\circ \right) \]
\[ S_{TP} = 1.11 \angle 36.9^\circ \left( 7200 \right) \]
\[ S_{TP} = 8000 \angle 36.9^\circ \ \text{VA} \quad \text{Ans} \]

Power equal on both sides of ideal transformer

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Example 8-1 Solution (4)

Now find the power factor and the active and reactive powers

\[ P_T = S_T \cos \theta \quad \theta = 36.9^\circ \]
\[ Q_T = S_T \sin \theta \]
\[ P_T = 8000 \cos (36.9^\circ) \quad \text{Ans} \]
\[ Q_T = 8000 \sin (36.9^\circ) \quad \text{Ans} \]
\[ P_T = 6397.5 \text{ W} \quad \text{Ans} \]
\[ Q_T = 4803 \text{ VAR} \quad \text{Ans} \]
\[ F_p = \frac{P_T}{S_T} \]
\[ F_p = \frac{6397.5}{8000} = 0.80 \quad \text{Lagging} \quad \text{Ans} \]

Ideal Transformer Calculations

Example 8-2: 300 kVA 2400-120, 60 Hz single phase transformer operates at 2300 volts on the primary side. It supplies 115 kVA to a load that has a power factor of 0.723 lagging. Assume ideal operation and find:

a.) secondary voltage at operating voltage
b.) secondary current
c.) impedance of the load as seen on the secondary side
d.) impedance of the load as seen on the primary side
Example 8-2 Solution (1)

a) Find secondary voltage at operating voltage

\[ V_p = \alpha \cdot V_s \]
\[ \alpha = \frac{2900 \text{V}}{120 \text{V}} = 20 \]

Use rated values to find turns ratio

\[ V_s = \frac{2300 \text{V}}{20} = 115 \text{V} \]

Ans

b) Find secondary current at operating voltage

\[ S_p = S_s \]

Power is equal on both sides of ideal transformers

\[ I_s = \frac{S_s}{V_s} = \frac{115,000 \text{VA}}{115 \text{V}} = 1000 \text{A} \]

Ans

Example 8-2 Solution (2)

c) Find load impedance seen on secondary side

\[ |Z_s| = \frac{V_s}{I_s} = \frac{115 \text{V}}{1000 \text{A}} = 0.115 \Omega \]

Next find impedance angle

\[ F_p = 0.723 \text{ lagging} \]

\[ \Theta = \cos^{-1} \left( F_p \right) \]

\[ \Theta = \cos^{-1} \left( 0.723 \right) \]

\[ \Theta = -43.7^\circ \]

\[ \Theta = 43.7^\circ \]

\[ Z_s = 0.115 / 43.7^\circ \]

Ans
Example 8-2 Solution (3)

d) Find load impedance seen on primary side of transformer

Reflecting impedance from secondary to primary-multiply by $a^2$.

$$\tilde{Z}_1 = a^2 \tilde{Z}_s$$

$$\tilde{Z}_p = (2\tilde{Z}_s)^2 (0.115/43.7^\circ)$$

$$\tilde{Z}_p = 46 \angle 43.7^\circ \Omega$$

Ans

Non-Ideal Operation-No-load

Practical transformers draw current with no load connected to secondary winding. Current caused by two non-ideal conditions: power losses and core magnetization

**Hysteresis losses** - power losses due to repeated change in magnetic polarity. It takes more mmf $(NI)$ to demagnetize core in one direction than the other.

**Eddy currents** - ac currents induced in iron core due to changing magnetic field

**Active power losses**

**Control**

- Control hysteresis losses - use alloy steels designed for magnetic circuits
- Control eddy current losses - laminate core, insulate laminates

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Non-Ideal Operation-No-load

A finite amount of current is necessary to drive mutual flux between coils. Permeability is finite so reluctance is finite. Some $NI = \phi$ needed.

$$\mathcal{R} = \frac{l}{\mu \cdot A}$$

$$\phi_m = \frac{N \cdot I}{\mathcal{R}}$$

$\phi_m$ = mutual flux, $\mathcal{R}$ = reluctance

In terms of inductance:

$$L = \frac{N^2}{\mathcal{R}}$$

so core has inductance with associated inductive reactance

Define above as the magnetizing inductance with associated magnetizing reactance $X_m$

No-Load Circuit Model

$V_T$ = the primary voltage

$I_{fe}$ = core-loss component

$I_0$ = exciting current

$I_M$ = magnetizing component

$R_{fe}$ = resistance that represents the core losses

$X_m$ = inductive reactance that represents the core magnetizing $L$
No-Load Circuit Model

Model equation using phasors

\[ I_{fe} = \frac{V_T}{R_{fe}} \quad I_M = \frac{V_T}{jX_M} \quad I_o = I_{fe} + I_M \]

\[ I_o = |I_{fe}| + j|I_M| \]  
Add current magnitudes at 90 degrees

No-load apparent power

\[ S_M = V_T \cdot I_o \]

Model parameter formulas

\[ P_{fe} = \frac{V_{fe}^2}{R_{fe}} \Rightarrow R_{fe} = \frac{V_{fe}^2}{P_{fe}} \]

\[ X_M = \frac{V_T}{I_M} \]

Core loss resistance

Magnetizing reactance

No-Load Transformer Example

Example 8-3: Computing the values of magnetizing reactance and core loss resistance. A 50 kVA 7200-240 V, 60 Hz single phase transformer is operating with no load. With the primary connected to a 7200 V system, it draws 248 W and has a power factor of 0.187 lagging. Find:

a) the exciting current and its components
b) the magnitudes of magnetizing reactance, \( X_M \) and core loss \( R \)
c) Repeat parts a and b if the transformer is energized from the secondary (low voltage) side.
Example 8-3 Solution (1)

a) Find current \( I_0 \)

\[ P_c = 248 \text{ W} \quad F_p = 0.187 \quad \text{Lagging} \]

\[ S_m = \text{magnetizing apparent power} \]

\[ S_m = \frac{P_c}{F_p} \]

\[ S_m = \frac{248 \text{ W}}{0.187} = 1326.2 \text{ VA} \]

\[ S_m = V_T I_o \]

\[ \frac{S_m}{V_T} = \frac{1326.2 \text{ VA}}{7200 \text{ V}} = I_o \]

\[ I_o = 0.1842 \text{ A} \]

\[ \theta = \cos^{-1}(F_p) \]

\[ \theta = \cos^{-1}(0.187) \]

\[ \theta = 79.2^\circ \]

\[ I_0 = 0.1842 / 79.2^\circ \]

Example 8-3 Solution (2)

b) Find the value of core loss resistance and magnetizing reactance

\[ P_c = \frac{V_p^2}{R_{fe}} \]

\[ R_{fe} = \frac{P_c}{V_p^2} \]

\[ R_{fe} = \left( \frac{7200}{248 \text{ W}} \right) = 209.032 \text{ \Omega} \]

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Example 8-3 Solution (3)

\[ X_m = \frac{V_p}{I_m} = \frac{7200 \text{V}}{0.1809 \text{A}} = 39,801 \text{ohms} \]

c) Find same parameters on secondary side

Power constant through transformer

\[ P_c = 298 \text{W} \quad S_m = 1324.2 \text{VA} \]
\[ V_s = 240 \text{V} \quad S_m = V_s I_s \]
\[ \frac{S_m}{V_s} = I_s = \frac{1324.2 \text{VA}}{240 \text{V}} = I_s \]

Ans

Example 8-3 Solution (4)

\[ \theta = \text{angle between } V_s \text{ and } I_o \]
\[ \theta = \cos^{-1}(0.187) \]
\[ \theta = 79.2^\circ \]
\[ I_{fe} = 5.525 \cos(79.2^\circ) \]
\[ I_{fe} = 1.0353 \text{ A} \]
\[ I_m = 5.525 \sin(79.2^\circ) \]
\[ I_m = 5.427 \text{ A} \]

\[ R_{fe} = \frac{V_s^2}{P_c} \quad R_{fe} = \frac{(240)^2}{298 \text{ W}} = 232.25 \text{ ohms} \]

Ans
Example 8-3 Solution (5)

\[ X_m = \frac{V_s}{I_m} \quad X_m = \frac{240\text{V}}{5.427\text{A}} \quad X_m = 44.2\ \Omega \]

Compare using turns ratio transfer

\[ a = \frac{V_p}{V_s} \quad a = \frac{7200\text{V}}{240\text{V}} \quad a = 30 \]

\[ R_{fep} = R_{fe} a^2 \quad R_{fep} = (232.2\ \Omega)(30)^2 \quad R_{fep} = 208,980\ \Omega \]

\[ X_{mph} = X_m a^2 \quad X_{mph} = 44.2 \times (30)^2 \quad X_{mph} = 39,780\ \Omega \]

209,032 on primary
39,801 on primary

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End Lesson 8: Ideal Transformer Theory and Operation

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