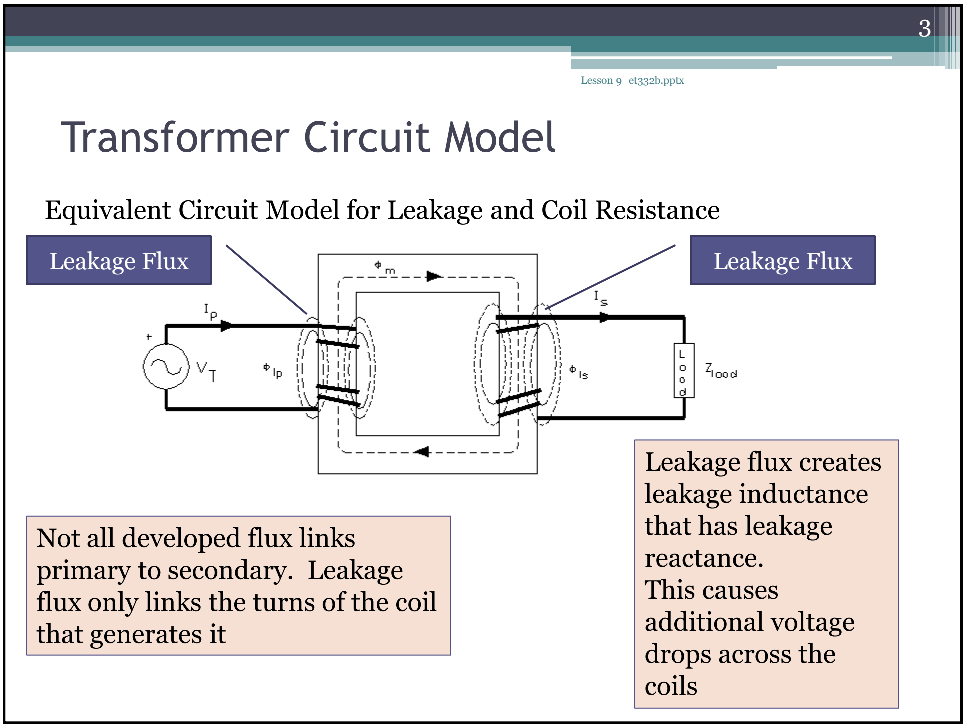
Lesson 9: Practical Transformer Model and Calculations

ET 332b Ac Motors, Generators
and Power Systems

Learning Objectives

After this presentation you will be able to:

- Identify factors that contribute to non-ideal operation of power transformers
- Draw the schematic model of a non-ideal transformer and include all parameters
- Reflect impedances through a non-ideal transformer
- Identify step-up and step down transformer connections
- Compute transformer voltages and currents using the full circuit model.



Transformer Circuit Model

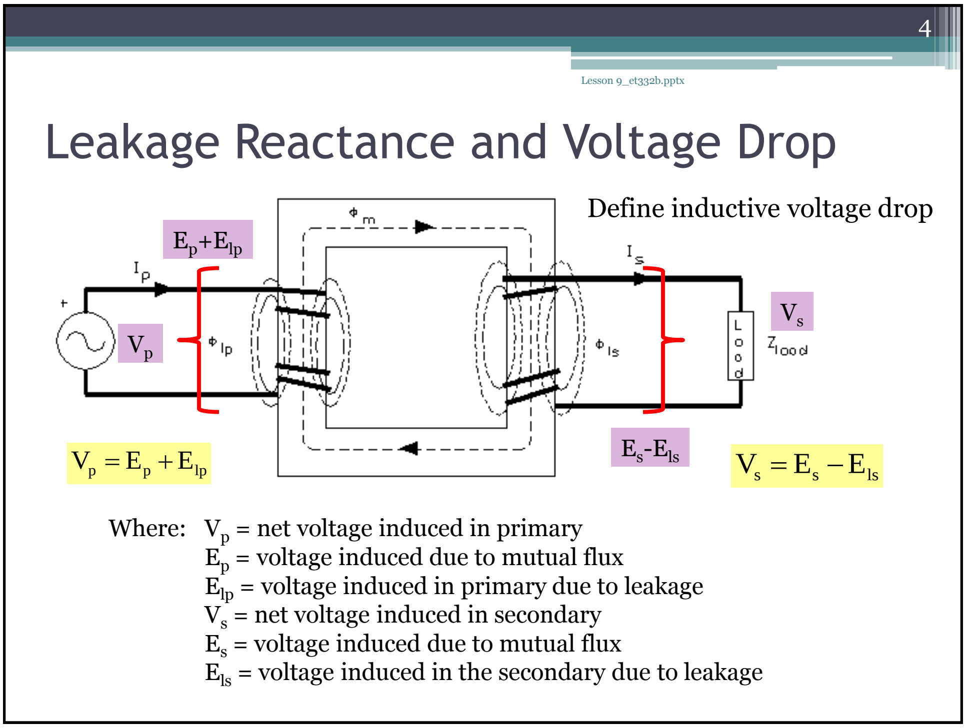
Equivalent Circuit Model for Leakage and Coil Resistance

Leakage Flux

Leakage Flux

Not all developed flux links primary to secondary. Leakage flux only links the turns of the coil that generates it

Leakage flux creates leakage inductance that has leakage reactance.
This causes additional voltage drops across the coils



Leakage Reactance and Voltage Drop

Define inductive voltage drop

E_p+E_{lp}

V_p

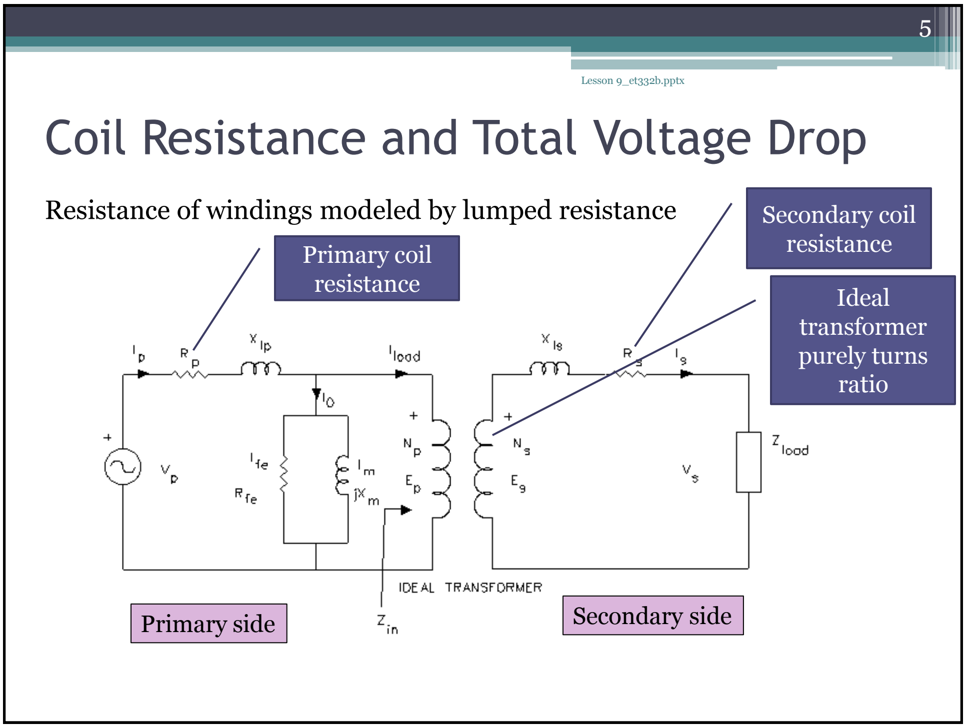
V_s

E_s-E_{ls}

$V_p = E_p + E_{lp}$

$V_s = E_s - E_{ls}$

Where: V_p = net voltage induced in primary
E_p = voltage induced due to mutual flux
E_{lp} = voltage induced in primary due to leakage
V_s = net voltage induced in secondary
E_s = voltage induced due to mutual flux
E_{ls} = voltage induced in the secondary due to leakage



Coil Resistance and Total Voltage Drop

Resistance of windings modeled by lumped resistance

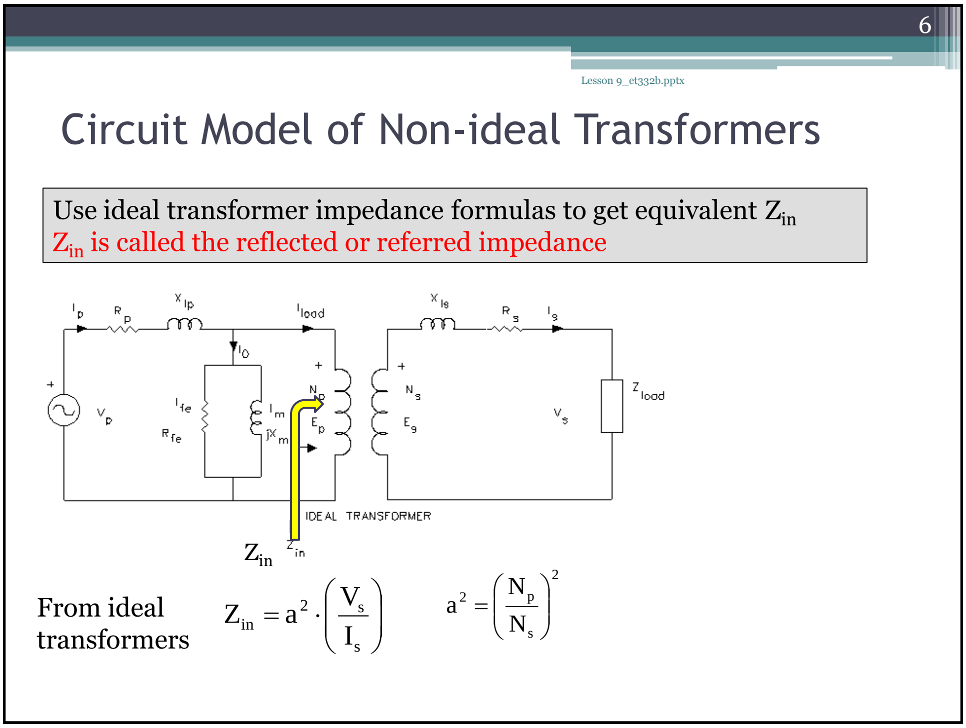
Primary coil resistance

Secondary coil resistance

Ideal transformer purely turns ratio

Primary side

Secondary side



Circuit Model of Non-ideal Transformers

Use ideal transformer impedance formulas to get equivalent Z_{in}

Z_{in} is called the reflected or referred impedance

From ideal transformers

$$Z_{in} = a^2 \cdot \left(\frac{V_s}{I_s}\right) \qquad a^2 = \left(\frac{N_p}{N_s}\right)^2$$

Circuit Model of Non-ideal Transformers

From Ohm's Law

$$\left(\frac{V_s}{I_s}\right) = (R_s + j\cdot X_{ls}) + Z_{load}$$

$$Z_{in} = a^2\cdot(R_s + j\cdot X_{ls}) + a^2\cdot Z_{load}$$

$$Z_{in} = a^2\cdot R_s + j\cdot a^2\cdot X_{ls} + a^2\cdot Z_{load}$$

Where: $a^2\cdot R_s$ = secondary R **referred to primary**
$a^2 \cdot X_{ls}$ = secondary X_{ls} **referred to primary**
$a^2 \cdot Z_{load}$ = load impedance **referred to primary**

Referring the secondary impedances to the primary side effectively removes the need for the ideal transformer. The turns ratio captures all the effects

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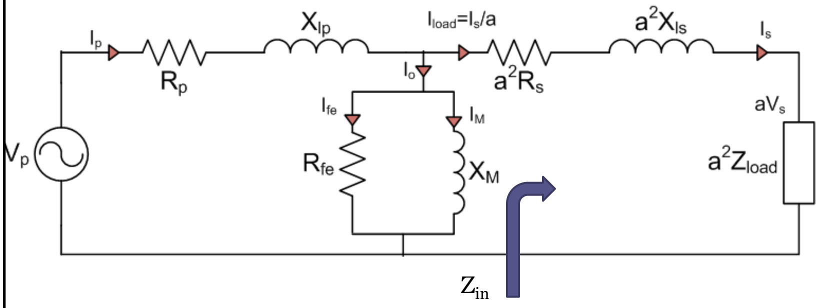
Circuit Model of Non-ideal Transformers

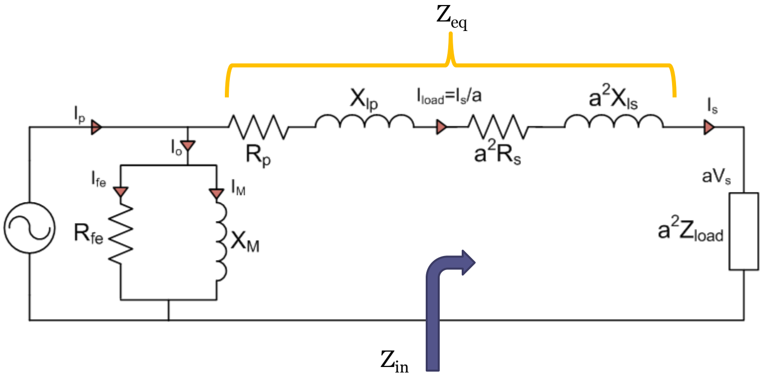
Transformer circuit with ideal turns ratio removed



Can solve this circuit to find anything about non-ideal transformer operation. Voltage drop, power losses, primary and secondary currents and voltages.

Simplified Circuit Model

Another simplifying assumption: $I_o << I_{load}$. Combine primary and secondary impedances

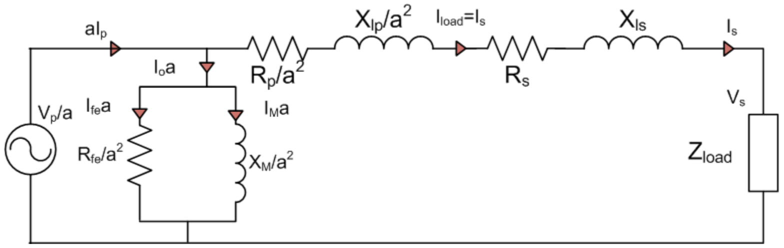


Equivalent series impedance referred to primary side

$$Z_{eq} = (R_p + a^2 \cdot R_s) + j \cdot (X_{lp} + a^2 \cdot X_{ls})$$

Simplified Model-Referred to Secondary

The impedance Z_{eq} can be referred to the secondary side instead. Just divide the impedance components by a².

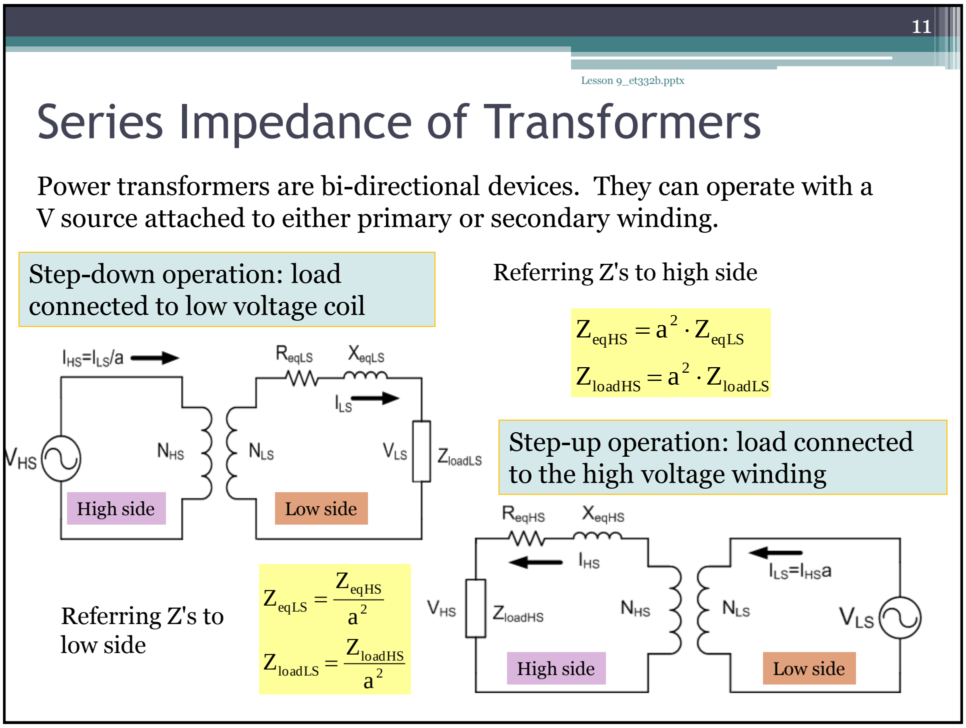


Resistance of the primary winding referred to secondary

Leakage reactance of the primary winding referred to secondary

Equivalent impedance referred to the secondary side

$$Z_{eqS} = \left(\frac{R_p}{a^2} + R_s\right) + j \cdot \left(\frac{X_{lp}}{a^2} + X_{ls}\right)$$



Series Impedance of Transformers

Power transformers are bi-directional devices. They can operate with a V source attached to either primary or secondary winding.

Step-down operation: load connected to low voltage coil

Referring Z's to high side

$$Z_{eqHS} = a^2 \cdot Z_{eqLS}$$

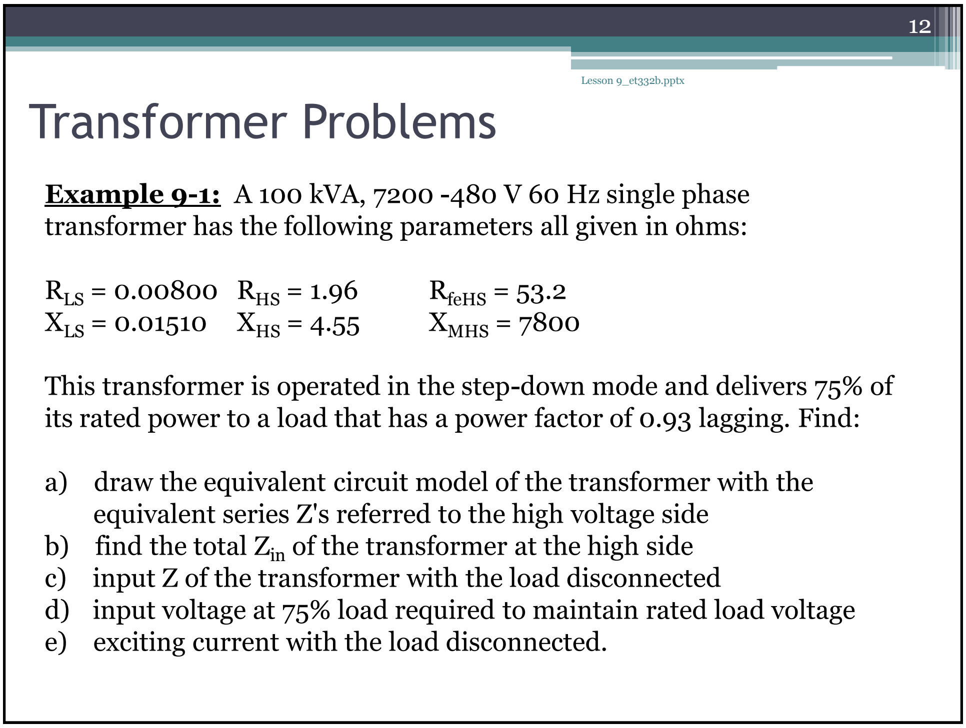
$$Z_{loadHS} = a^2 \cdot Z_{loadLS}$$

Step-up operation: load connected to the high voltage winding

Referring Z's to low side

$$Z_{eqLS} = \frac{Z_{eqHS}}{a^2}$$

$$Z_{loadLS} = \frac{Z_{loadHS}}{a^2}$$



Transformer Problems

Example 9-1: A 100 kVA, 7200 -480 V 60 Hz single phase transformer has the following parameters all given in ohms:

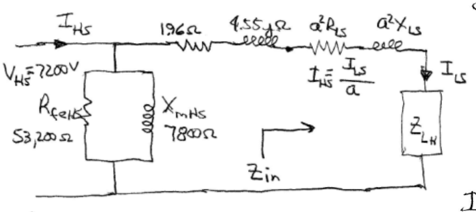
R_{LS} = 0.00800 R_{HS} = 1.96 R_{feHS} = 53.2
X_{LS} = 0.01510 X_{HS} = 4.55 X_{MHS} = 7800

This transformer is operated in the step-down mode and delivers 75% of its rated power to a load that has a power factor of 0.93 lagging. Find:

a) draw the equivalent circuit model of the transformer with the equivalent series Z's referred to the high voltage side
b) find the total Z_{in} of the transformer at the high side
c) input Z of the transformer with the load disconnected
d) input voltage at 75% load required to maintain rated load voltage
e) exciting current with the load disconnected.

Example 9-1 Solution (1)

Draw circuit model and find Z_{eq}



$R_{LS}=0.008\Omega$ $X_{LS}=0.01510\Omega$

$a=\frac{V_P}{V_S}=\frac{7200V}{480V}$

$a=15$

$F_p=0.93$

$V_H=aV_{LS}$

$Z_{LHS}=a^2Z_{LS}$

Find Load Current

$S_L=100kVA\,(0.75)$

$S_L=75\text{ kVA}$

$I_{LS}=\frac{75\text{ kVA}}{0.48\text{ kV}}$

$I_{LS}=156.25A$

$\theta=\cos^{-1}(0.93)$

$\theta=-21.57°$ Lagging

$R_{eq}=R_{HS}+a^2R_{LS}$

$R_{eq}=1.96+(15)^2(0.008)$

$R_{eq}=3.76\Omega$

$X_{eq}=X_{HS}+a^2X_{LS}$

$X_{eq}=4.55+(15)^2(0.01510)$

$X_{eq}=7.9475\Omega$

$Z_{eq}=3.76+j7.9475\Omega$ ← Ans

Example 9-1 Solution (2)

b) Find the Z_{in} as seen on the HV side

$\bar{Z}_L=\frac{\bar{V}_L}{\bar{I}_L}$

$\bar{Z}_L=\frac{480\angle 0°}{156.25\angle -21.57°}$

$\bar{Z}_L=3.072\angle 21.57°\Omega$

$\bar{Z}_{LHS}=a^2\bar{Z}_L$

$\bar{Z}_{LHS}=(15)^2(3.072\angle 21.57°)$

$\bar{Z}_{LHS}=691\angle 21.57°\ \Omega$

$\bar{Z}_{LHS}=642.8+j254\ \Omega$

$\bar{Z}_{in}=\bar{Z}_{eq}+\bar{Z}_{LHS}$

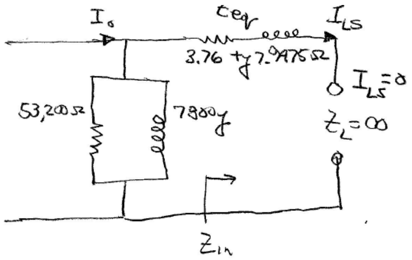
$\bar{Z}_{in}=(3.76+j7.9475)+(642.8+j254\Omega)$

$\bar{Z}_{in}=646.6+j262\ \Omega$

$\bar{Z}_{in}=697.7\angle 22.06°$ ← Ans

Example 9-1 Solution (3)

c) Z_{in} with the load disconnected



$$Z_L = \frac{V_{LS}}{I_{LS}}$$

$$Z_L = \frac{480}{0} = \infty$$

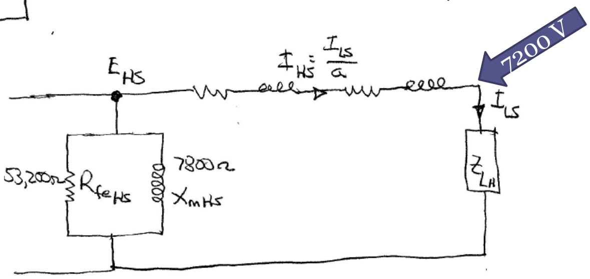
$$\bar{Z}_{in} = \bar{Z}_{eq} + \bar{Z}_L$$

$$\bar{Z}_{in} = 3.76 + j7.9475 + \infty$$

$$\bar{Z}_{in} = \infty$$

Ans

d) Input voltage required at load to maintain V_s=480 V



$V_{HS} = V_{LS}a$ $\quad$ $V_{HS} = 480(15)$

$V_{HS} = 7200\text{V}$

Example 9-1 Solution (4)

$$\bar{E}_{HS} = \bar{I}_{HS}\bar{Z}_{in}$$

$$\bar{I}_{HS} = \frac{156.25\angle -21.57^\circ\text{A}}{15}$$

$$\bar{I}_{HS} = 10.42\angle -21.57^\circ\ \text{A}$$

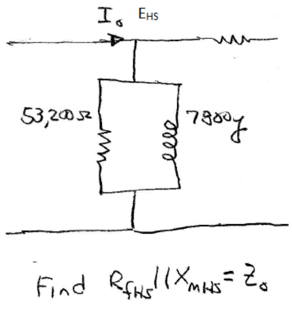
$$\bar{E}_{HS} = (10.42\angle -21.57^\circ)(697.7\angle 22.06^\circ)$$

$$\bar{E}_{HS} = 7270\angle 0.49^\circ\ \text{V}$$

Ans

e) Exciting current with load disconnected

With load disconnected only current is I_o



Find $R_{fHS} \| X_{mHS} = Z_o$

Remember $I_o = Y_T E_{HS}$

Example 9-1 Solution (5)

$$Y_T = \frac{1}{R_{feHS}} + \frac{1}{\bar{X}_{mHS}}$$

$$Y_T = \frac{1}{53,200} + \frac{1}{7800j}$$

$$Y_T = 1.8797\times10^{-5} - j1.282\times10^{-4}\ S$$

$$Y_T = 1.295\times10^{-4}\angle -81.7^\circ\ S$$

$$\bar{I}_o = (1.295\times10^{-4}\angle -81.7^\circ)(7270\angle 0.49^\circ)$$

$$\bar{I}_o = 0.9415\angle -81.2^\circ\ A$$

Ans

Find I$_o$ as percent of rate load I

$$I_{pR} = \frac{S_{rated}}{V_{pr}} = \frac{100,000\ VA}{7200\ V} = 13.89\ A$$

$$I_o\% = \frac{I_o}{I_{pr}} = \frac{0.9415\ A}{13.89\ A} \times 100\%$$

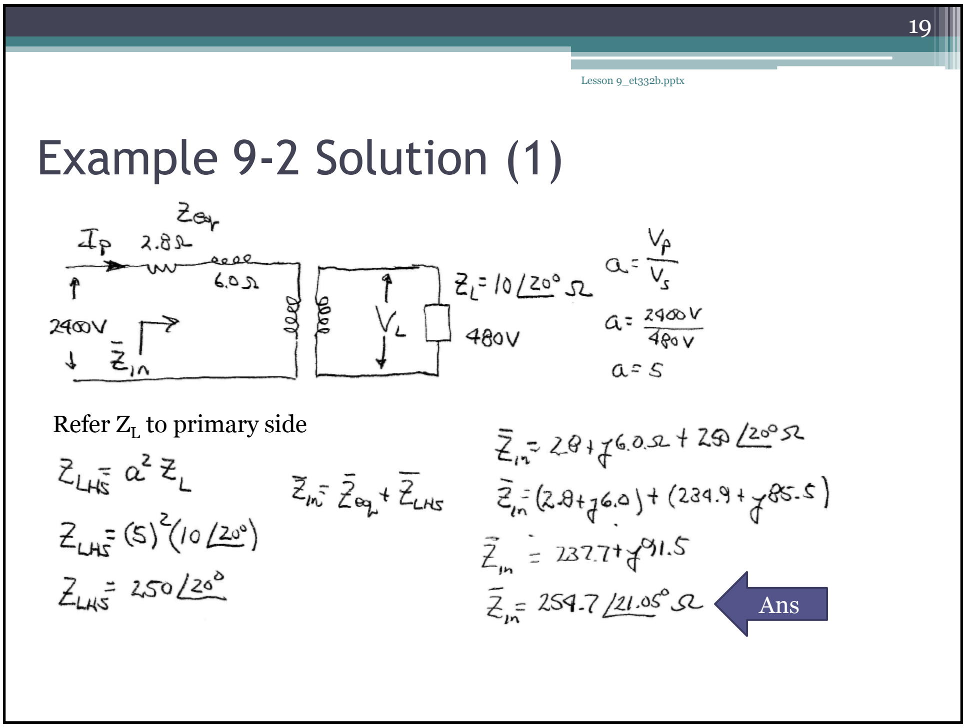
$$I_o\% = 6.8\%$$

Typical values: 3-5% of rated for large power transformers.

Transformer Voltage Drop and Impedance

Example 9-2: The equivalent resistance and reactance of a 50 kVA, 2400-480 V transformer's windings are R = 2.80 Ω and X = 6.00 Ω. (high side). A load of $10\angle 20^\circ$ is connected to the low voltage side. Determine:

a) equivalent impedance of the transformer and load combined
b) primary current if rated voltage is applied to primary
c) voltage across the load.



Example 9-2 Solution (1)

$$a = \frac{V_p}{V_s}$$

$$a = \frac{2400\,V}{480\,V}$$

$$a = 5$$

Refer Z_L to primary side

$$Z_{LHS} = a^2 Z_L$$

$$Z_{LHS} = (5)^2(10\angle 20°)$$

$$Z_{LHS} = 250\angle 20°$$

$$\bar{Z}_{in} = \bar{Z}_{eq} + \bar{Z}_{LHS}$$

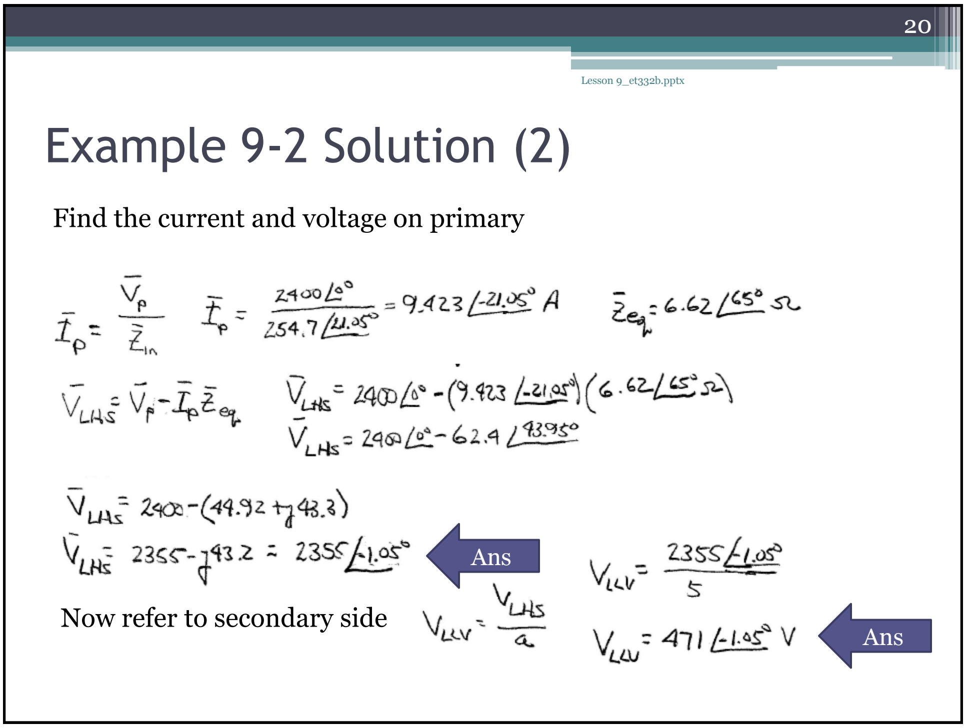
$$\bar{Z}_{in} = 2.8 + j6.0\,\Omega + 250\angle 20°\,\Omega$$

$$\bar{Z}_{in} = (2.8 + j6.0) + (234.9 + j85.5)$$

$$\bar{Z}_{in} = 237.7 + j91.5$$

$$\bar{Z}_{in} = 254.7\angle 21.05°\,\Omega$$

Ans



Example 9-2 Solution (2)

Find the current and voltage on primary

$$\bar{I}_p = \frac{\bar{V}_p}{\bar{Z}_{in}} \qquad \bar{I}_p = \frac{2400\angle 0°}{254.7\angle 21.05°} = 9.423\angle -21.05°\ A \qquad \bar{Z}_{eq} = 6.62\angle 65°\ \Omega$$

$$\bar{V}_{LHS} = \bar{V}_p - \bar{I}_p \bar{Z}_{eq}$$

$$\bar{V}_{LHS} = 2400\angle 0° - (9.423\angle -21.05°)(6.62\angle 65°\,\Omega)$$

$$\bar{V}_{LHS} = 2400\angle 0° - 62.4\angle 43.95°$$

$$\bar{V}_{LHS} = 2400 - (44.92 + j43.3)$$

$$\bar{V}_{LHS} = 2355 - j43.2 = 2355\angle -1.05°$$

Ans

Now refer to secondary side $V_{LLV} = \frac{V_{LHS}}{a}$

$$V_{LLV} = \frac{2355\angle -1.05°}{5}$$

$$V_{LLV} = 471\angle -1.05°\ V$$

Ans

End Lesson 9: Practical Transformer Model and Calculations

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